3D Models

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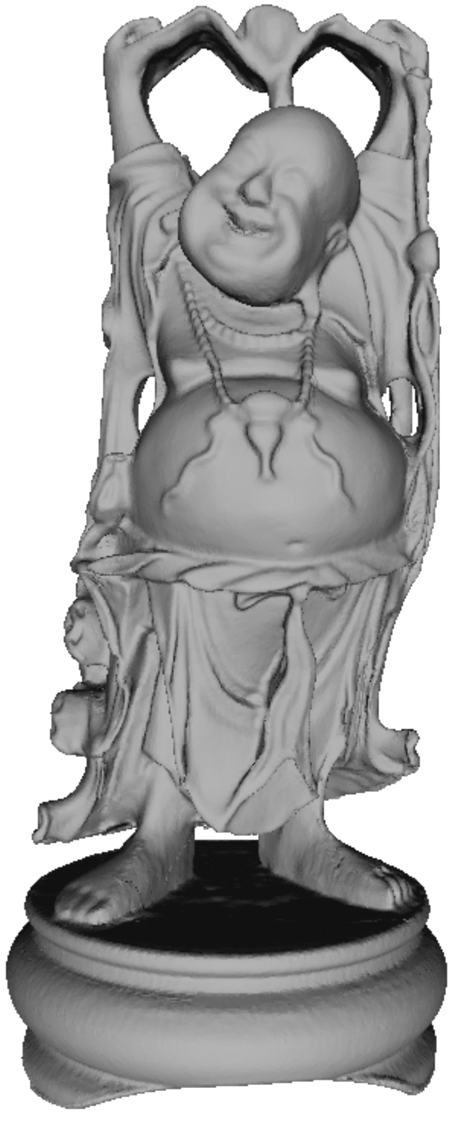
3D Models

- A 3D model is a computational representation of a real-world object. This is typically:
  - C0
  - Closed (not always!)
  - Discretized



3D Models

- Two main representations:
    - **Boundary representations** (b-rep): a 3D object is represented as a collection of connected surface elements; i.e., the boundary between solid and non-solid
    - **Volume representations**: a 3D object is represented by its interior volume. For example, 3D volumes or volume mesh (FEM)

Our focus is on

boundary representations

Polygonal Meshes

Surfaces

- A 2-dimensional region of 3-dimensional space
- A portion of space having length and breadth but no thickness

3D Representation: Polygonal Meshes

- Discretize the surface in a set of simple primitives:
  - Many points
  - **Triangles**
  - Quads
  - Polygons
  - Our focus is on:
    - simplicial complexes, e.g., triangles!

Why triangular meshes?

- Two main practical reasons:
    - Data-structures are straightforward
    - Graphics hardware (e.g., a GPU) uses triangles;

Why triangular meshes?

- Two main theoretical reasons:
    - Nice theory, i.e., simplicial complexes
    - Less limiting cases:
        - a triangle is always planar!
        - if we remove a vertex, we get another simplicial!

Simplex

- A k-simplex, $\boldsymbol{\sigma}$, is convex combination of $k+1$ points, $\mathbf{p}_i$, that are **linearly independent** in the k-dimensional Euclidian space, $\mathbb{R}^k$:

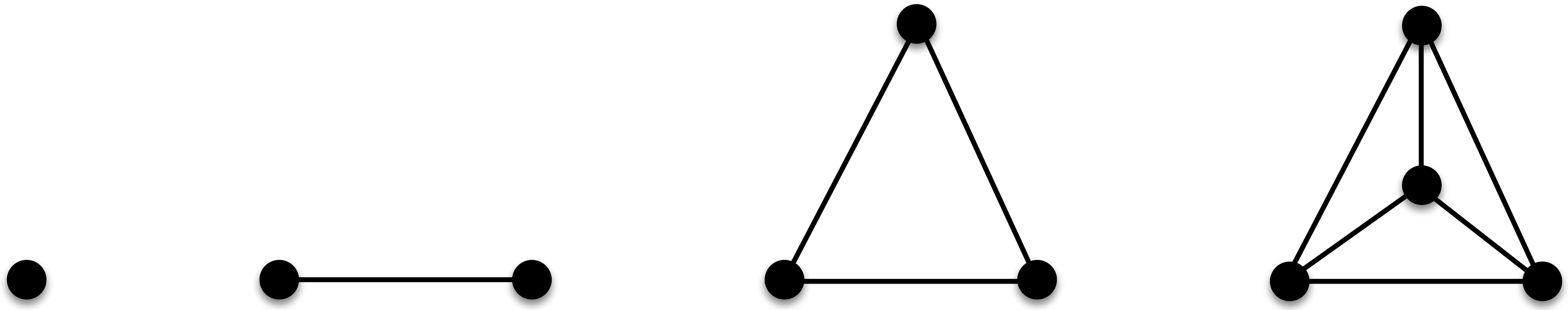
$$\mathbf{x} = \sum_{\mathbf{p}_i \in \sigma} \alpha_i \cdot \mathbf{p}_i$$

$$\sum_i \alpha_i = 1 \ \wedge \ \alpha_i \geq 0 \ \forall i$$

- A point $\mathbf{p}_i$ is called a vertex.

- k is the order of the simplex.

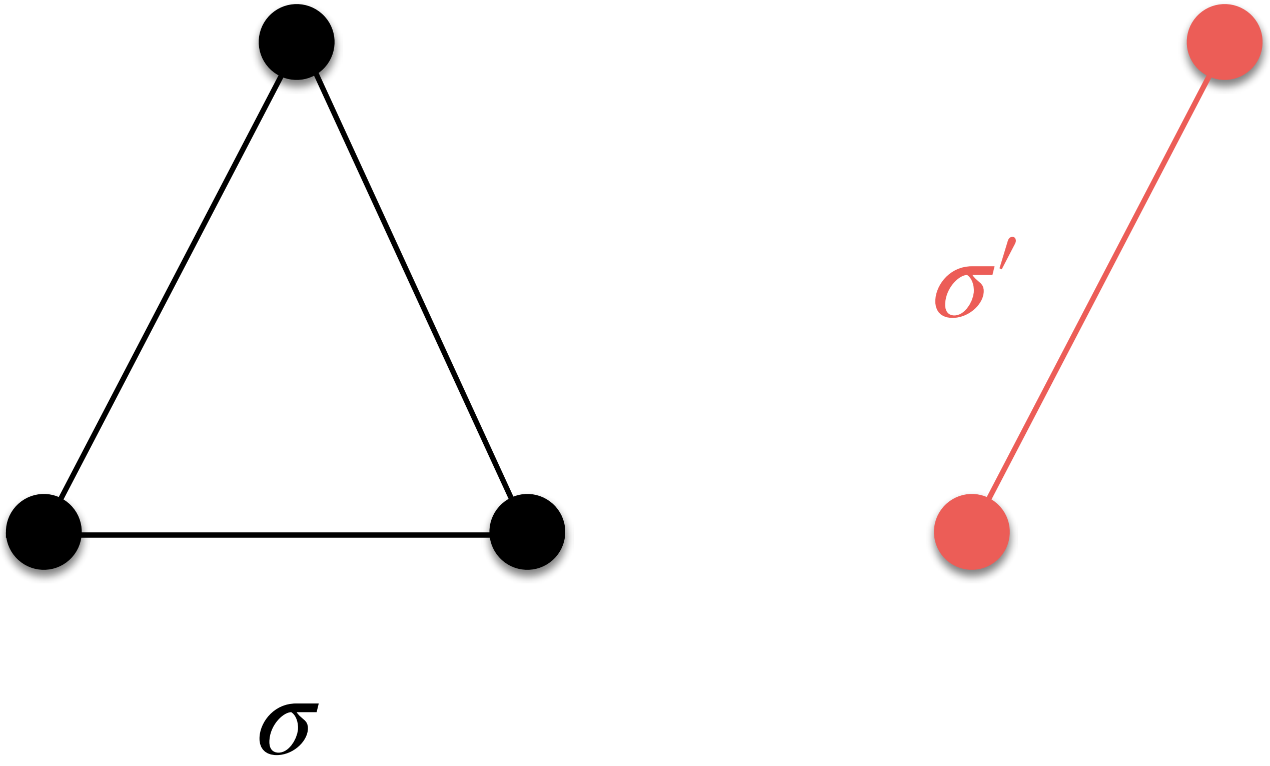
Simplices Example



$k = 0$ $k = 1$ $k = 2$ $k = 3$

Sub-Simplex

- A sub-simplex σ' is called a **face** of a simplex $\boldsymbol{\sigma}$ if it is a sub-set of vertices of $\boldsymbol{\sigma}$.



A Simplicial k-Complex

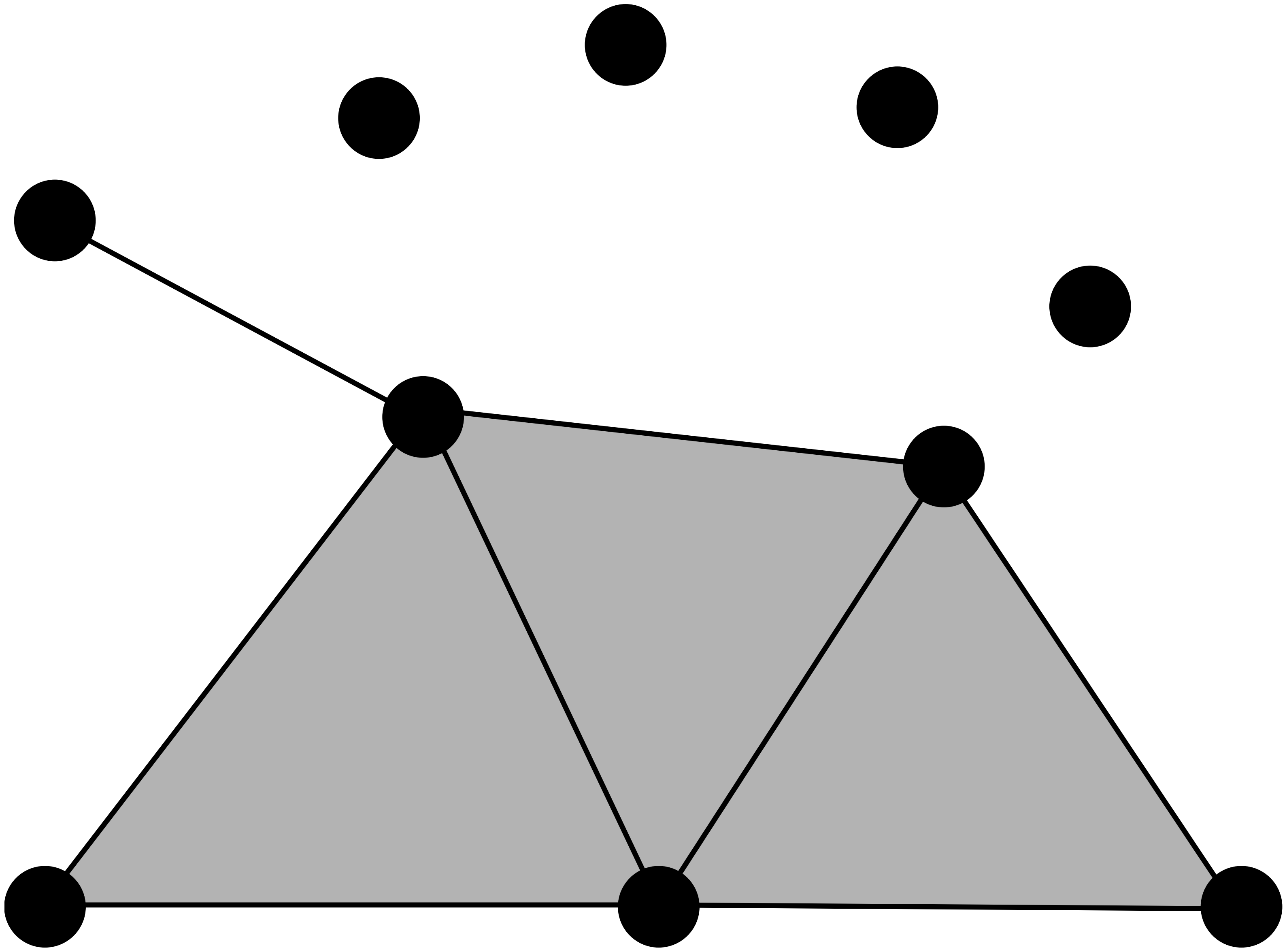
- A simplicial k-complex, Σ, is a finite collection of n simplices such that:

  (i) The intersection of any two simplices of Σ is a face of each of them

  (ii) Every face of a simplex, σ, of is in Σ

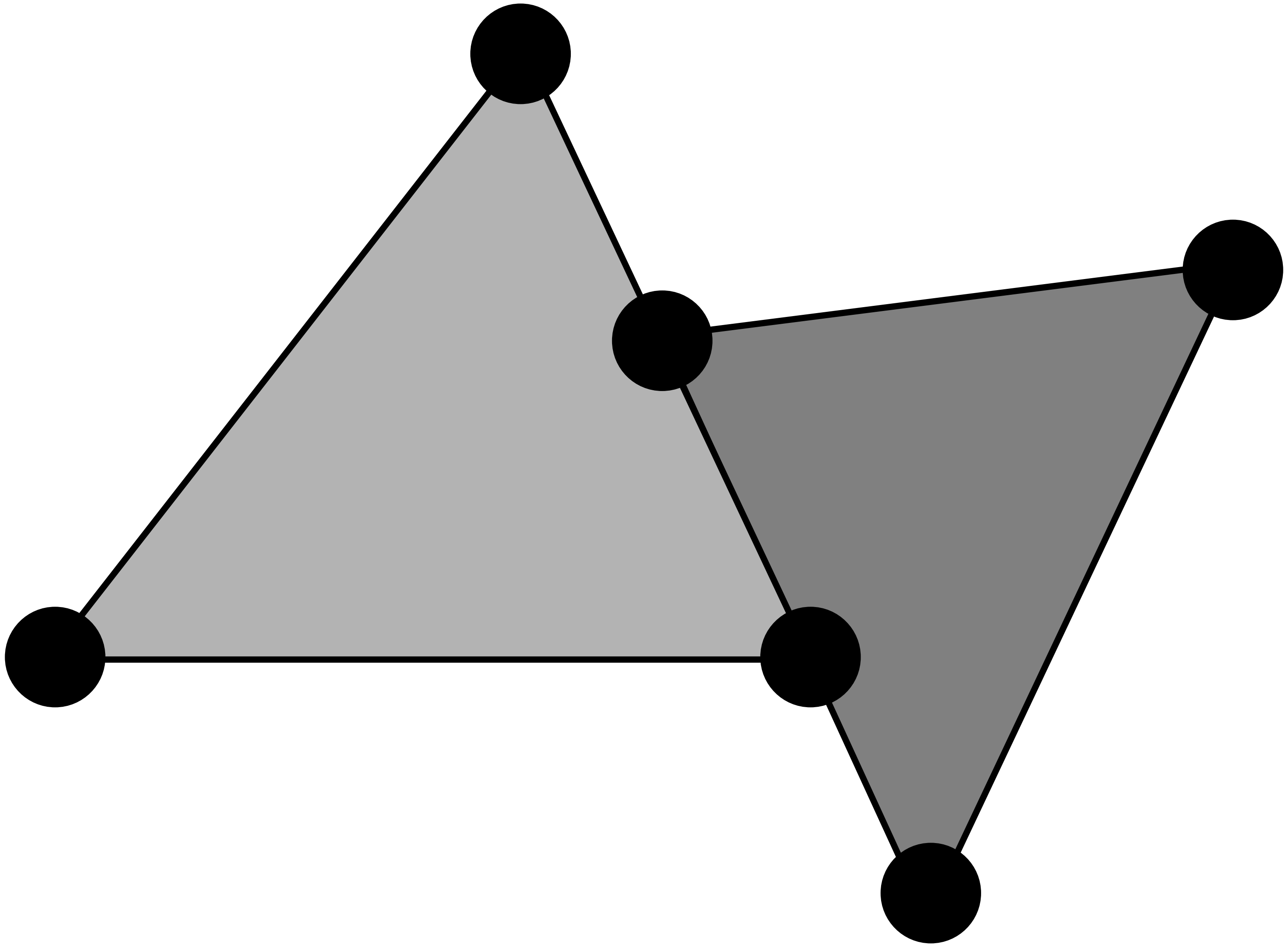
  NOTE: k is the maximum order of all σ in Σ

Simplicial Complexes Example



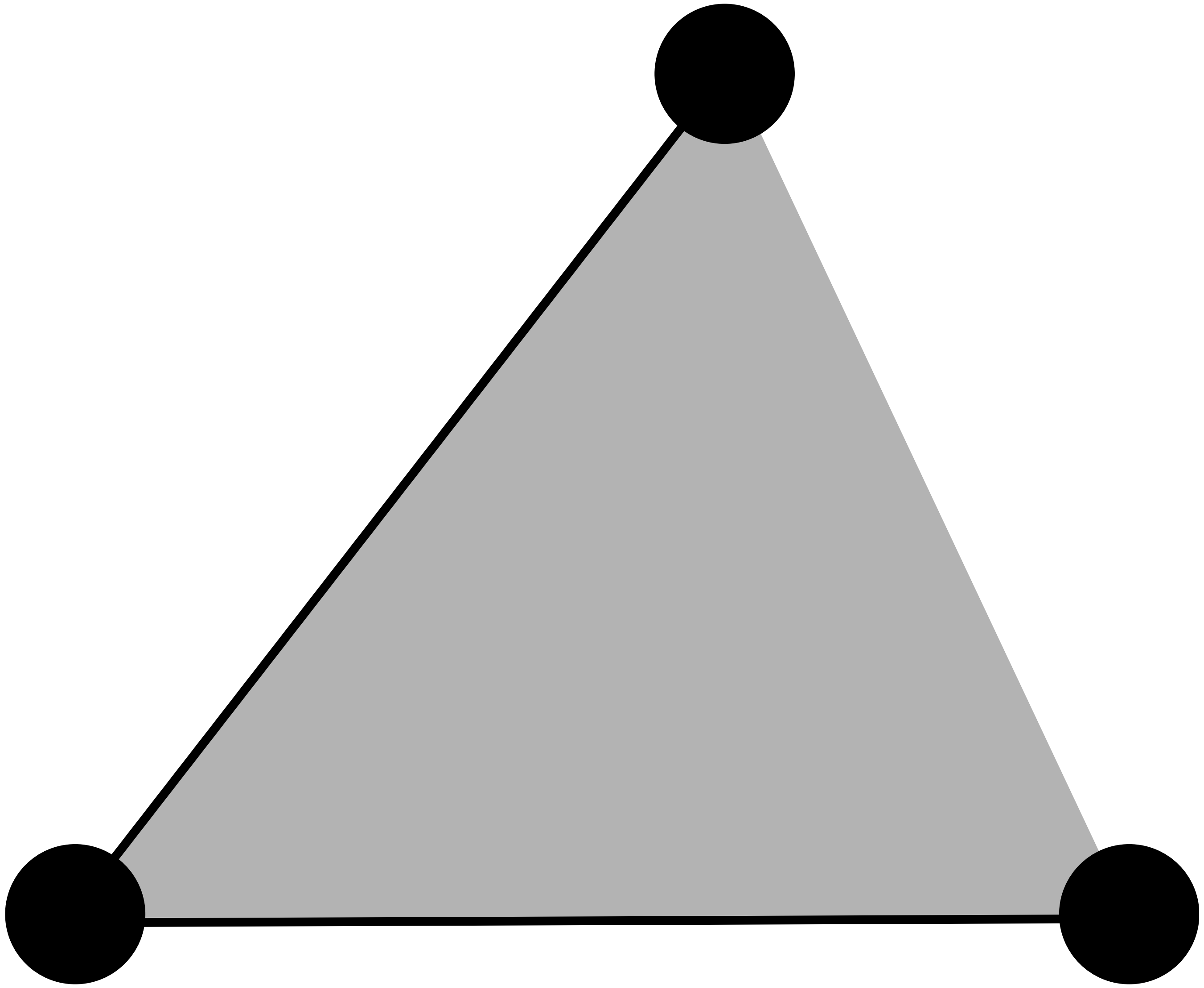
GOOD!

Simplicial Complexes Example



BAD: is not valid! —> Condition (i)

Simplicial Complexes Example

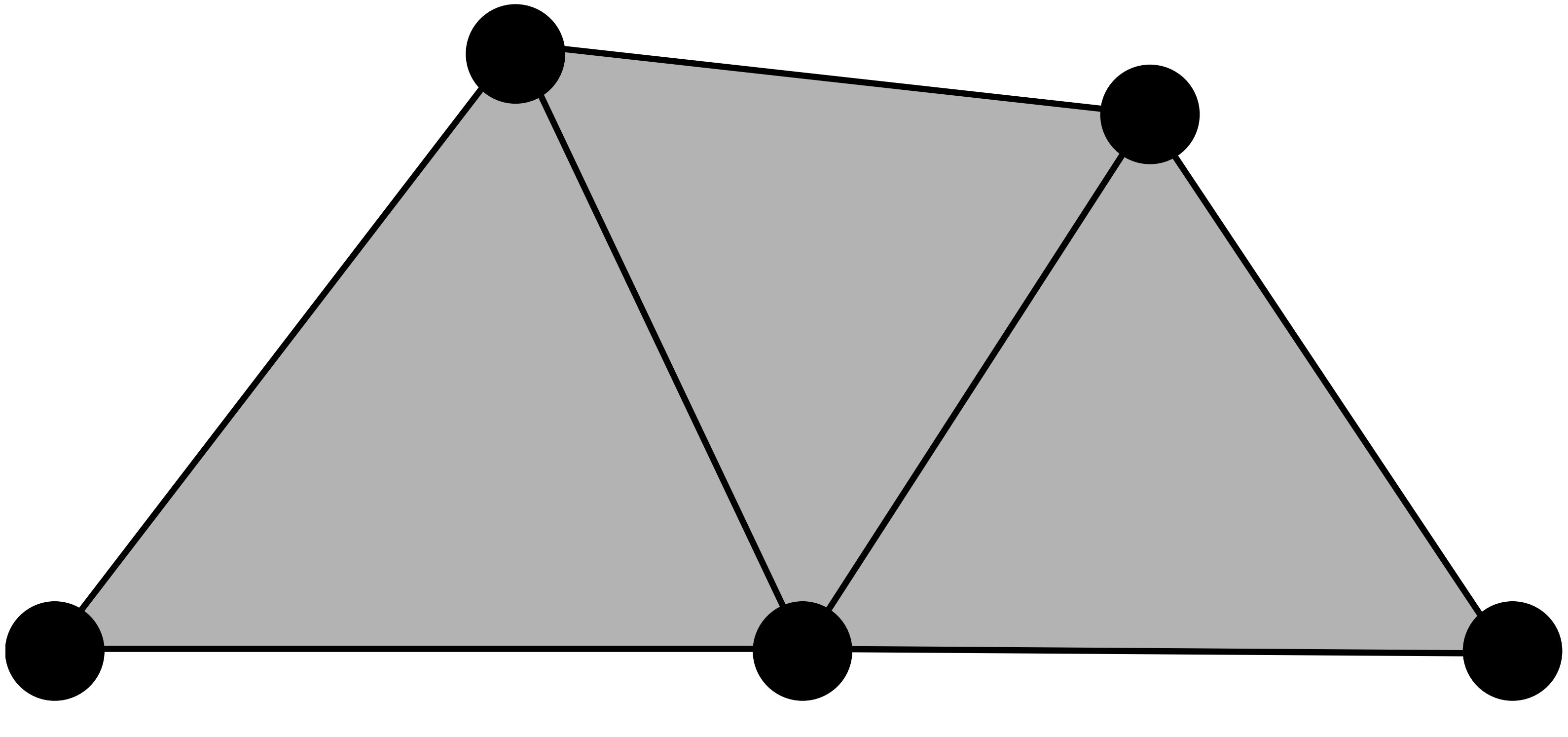


BAD: is not valid! —> Condition (ii)

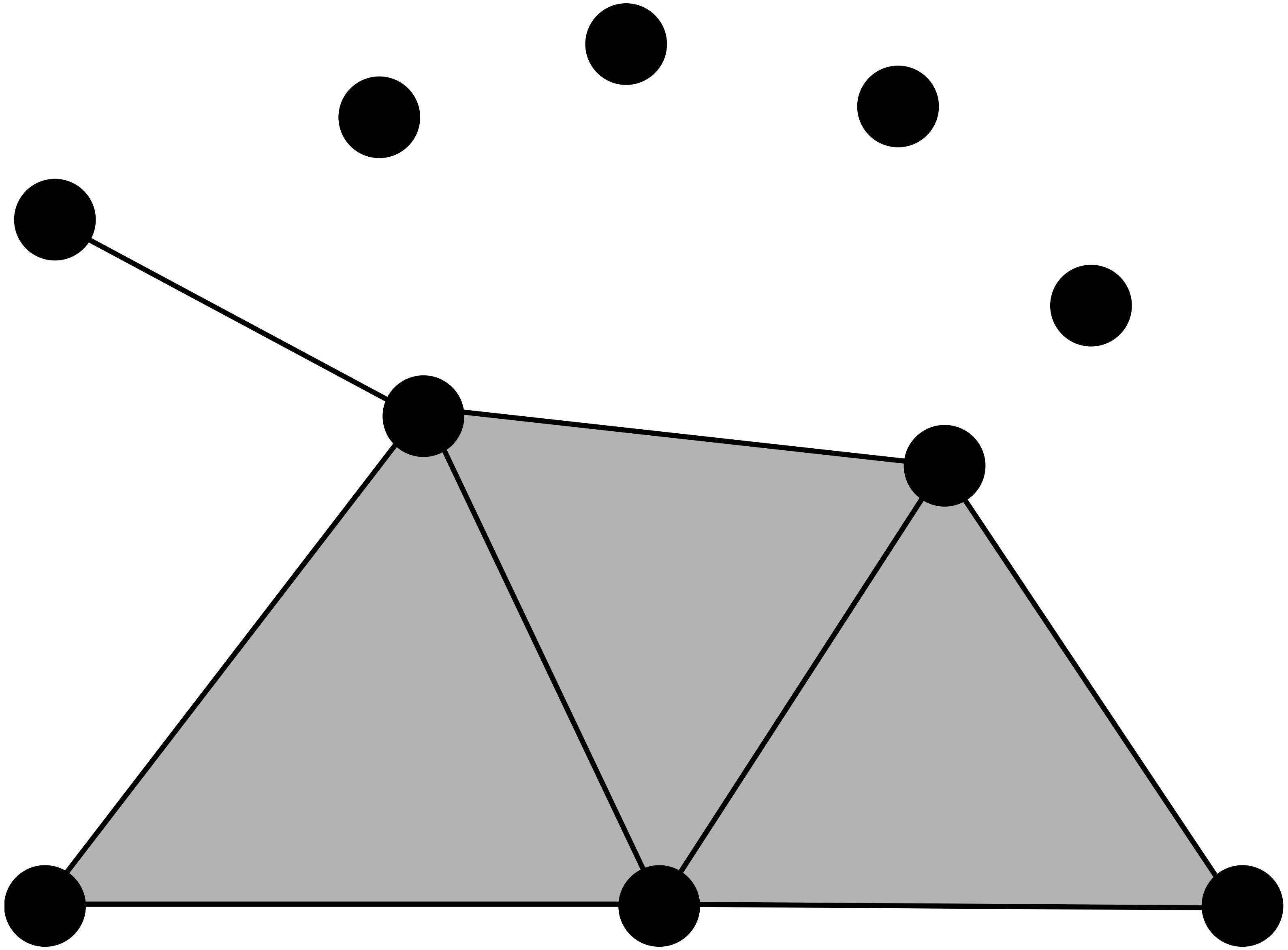
Simplicial Complexes

- A simplex, σ, is maximal in a simplicial complex, Σ, if it does not belong to any other simplex σ_2 of Σ.
- A *k*-simplicial complex, Σ, is maximal if all maximal simplices have order k.

A Maximal Simplicial Complex Example



A Non-Maximal Simplicial Complex Example



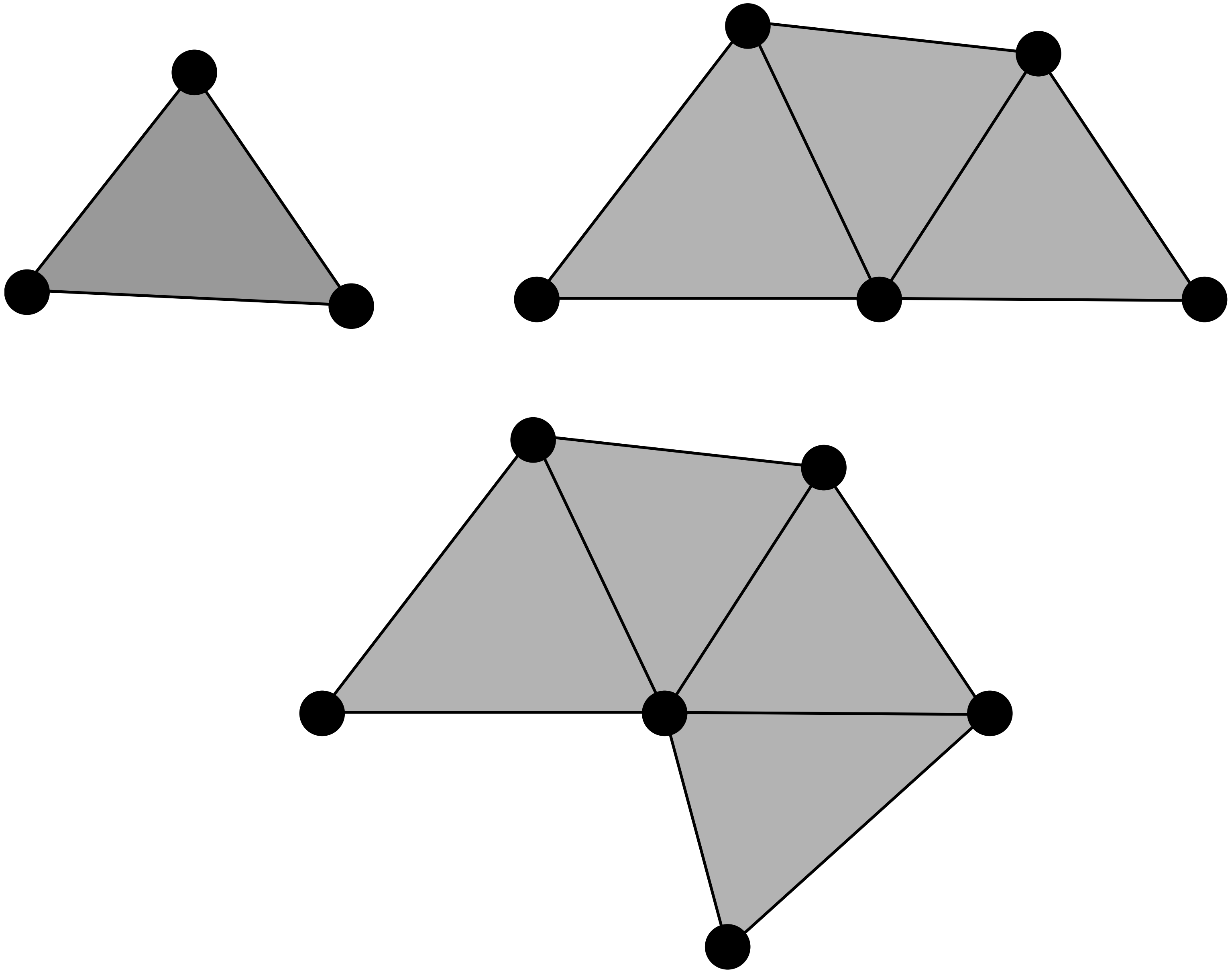
Manifoldness

- A surface, $S \in \mathbb{R}^3$ is manifold if and only if:
  - The neighborhood of each point is homeomorphic to an Euclidean space in two dimension or in other words:
    - The neighborhood of each point is homeomorphic to a disk or a semi-disk if the surface has boundaries!

Manifoldness

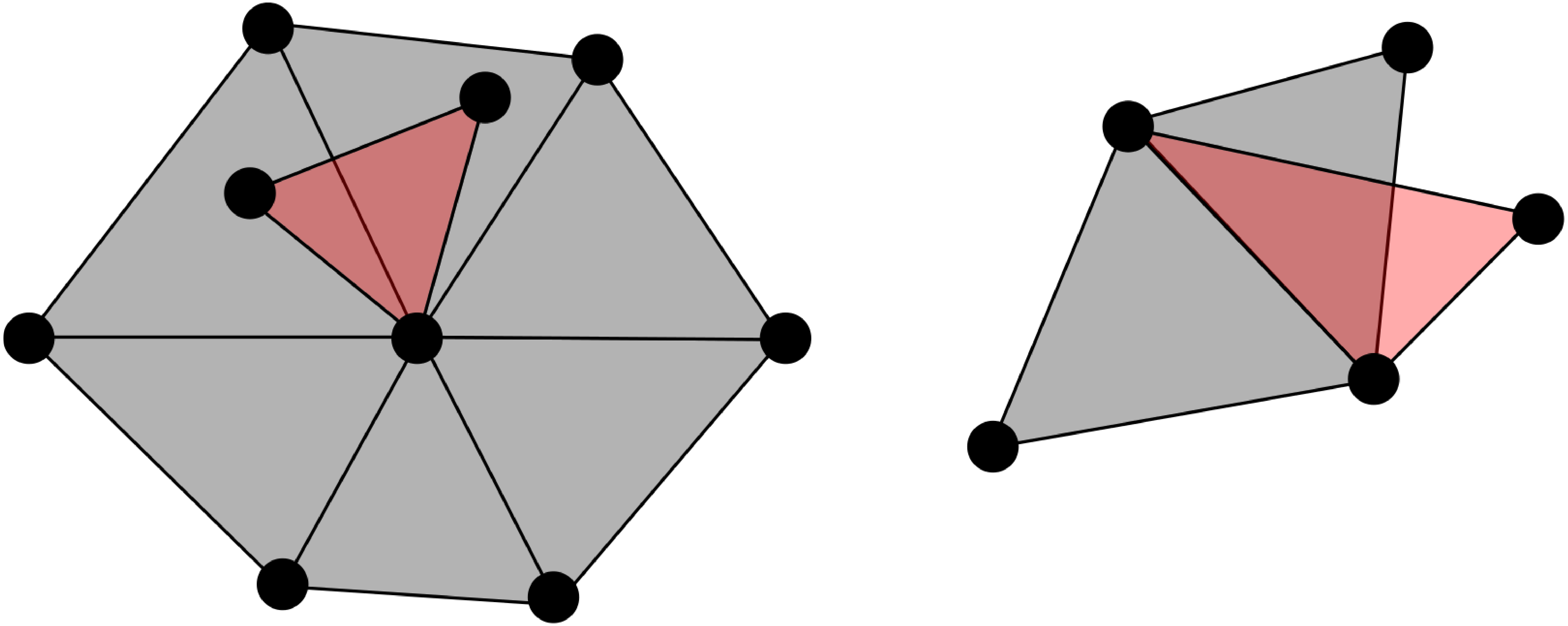
- In other words:
  - Each edge, E, is incident to only one or two faces!
  - The faces that are incident to a vertex form a closed or an open fan

Manifoldness Example



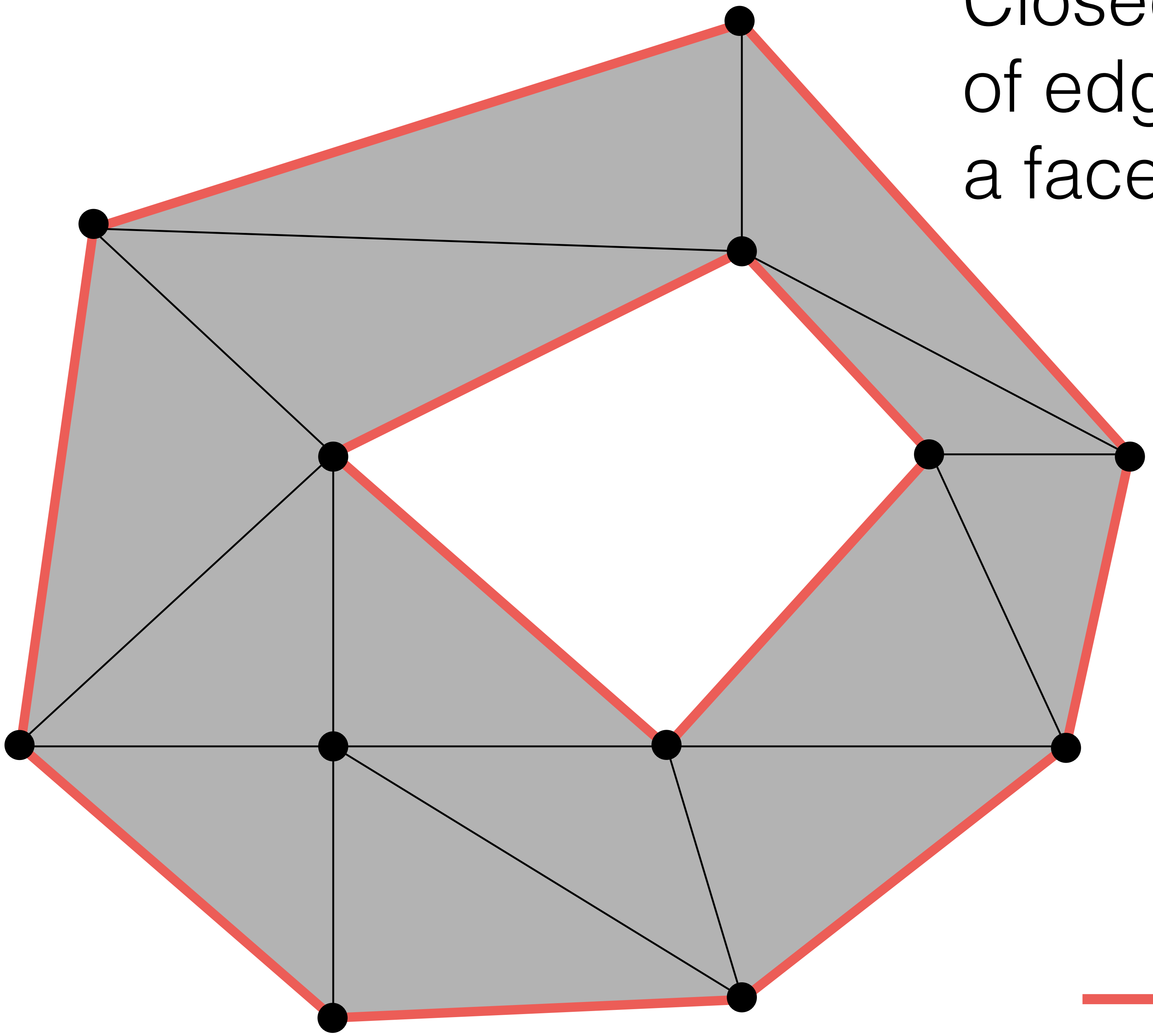
GOOD!

Manifoldness Example



BAD!

Borders



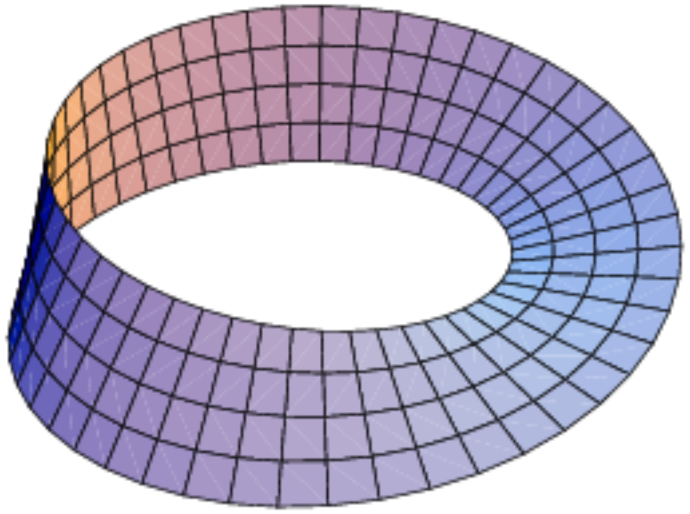
Closed sequence of edges with only a face

🟥 Border

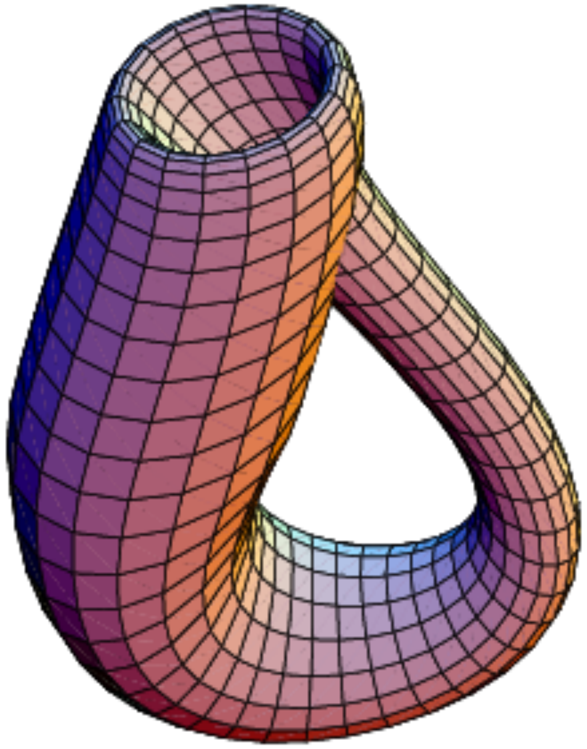
Orientability

- A surface, S, is orientable if it is possible to set a coherent normal to each point of the surface

- **NOTE**: Möbius strip and Klein bottle and non-manifold surfaces are not orientable:

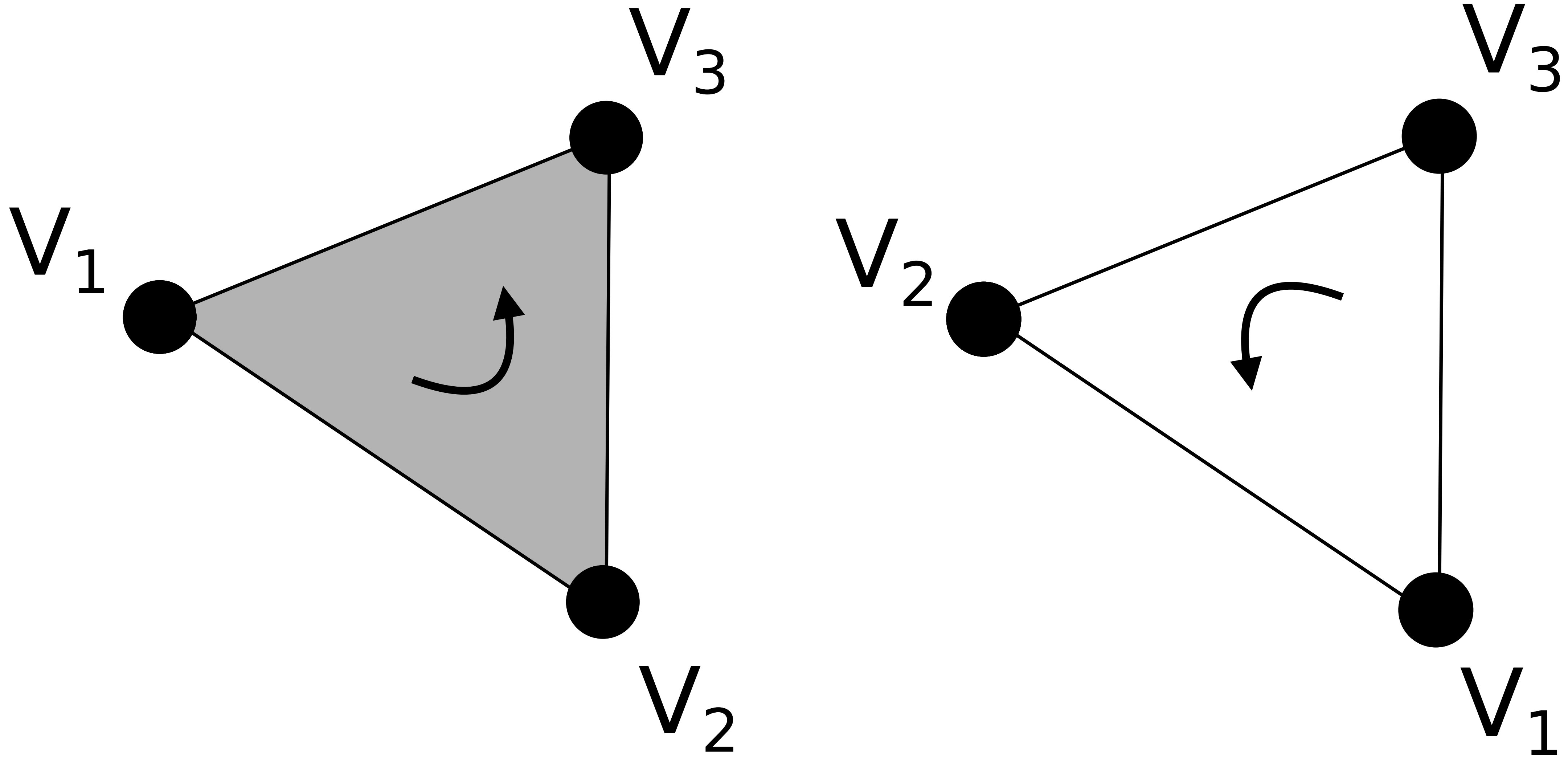


Möbius strip



Klein bottle

Orientability



Front
(counter-clockwise)

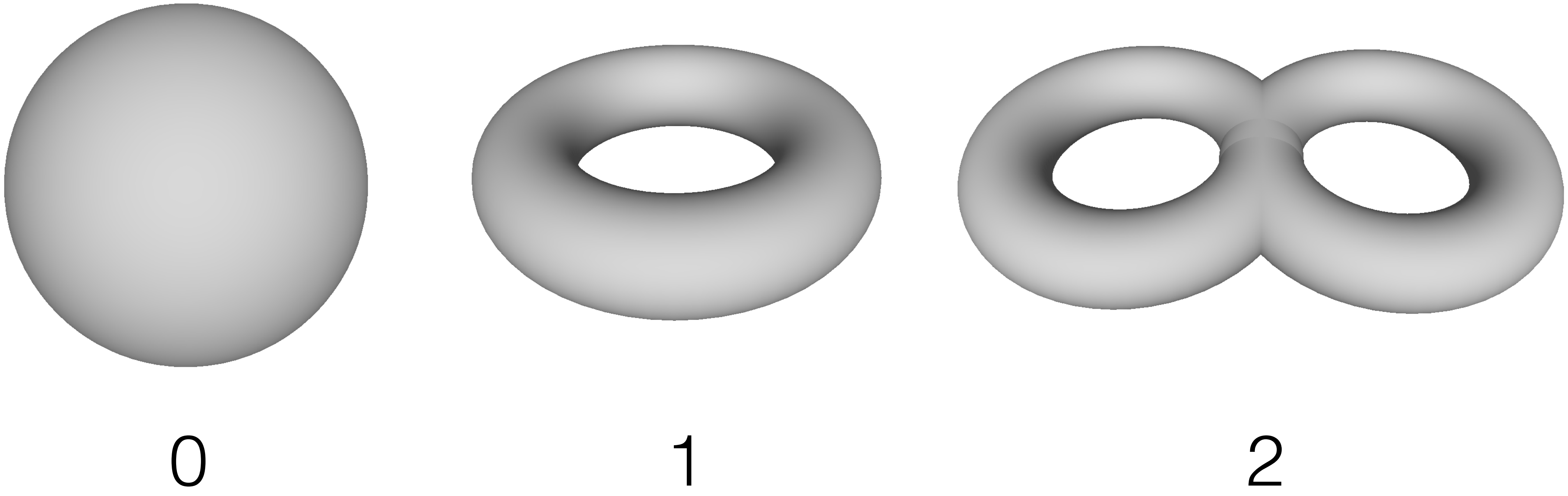
Back
(clockwise)

Mesh

- A mesh is maximal 2-simplicial complexes that is a 2-manifold orientable surface.
    - We can have non 2-manifold meshes
    - We assume that they are maximal

Genus

- The genus, G , is the maximum number of cuttings along non-intersecting closed simple curves without rendering the resultant manifold disconnected



- Genus —> “the number of handles”

Euler Characteristic

- Given V vertices, E edges, and F faces of a polygonal closed and orientable surface with genus G, we have:

$$2 - 2G = V - E + F$$

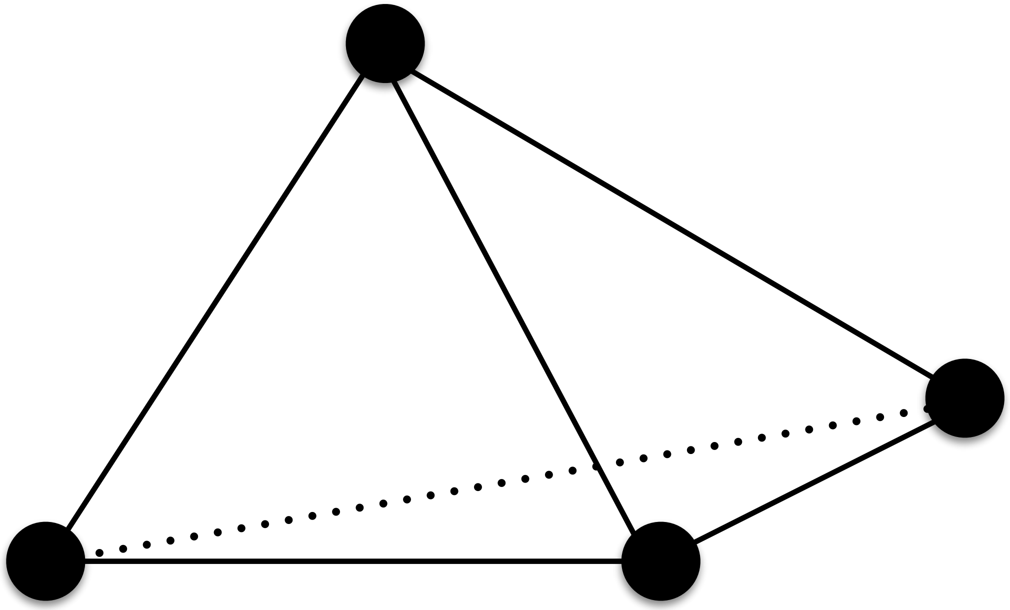
$$\chi = V - E + F$$

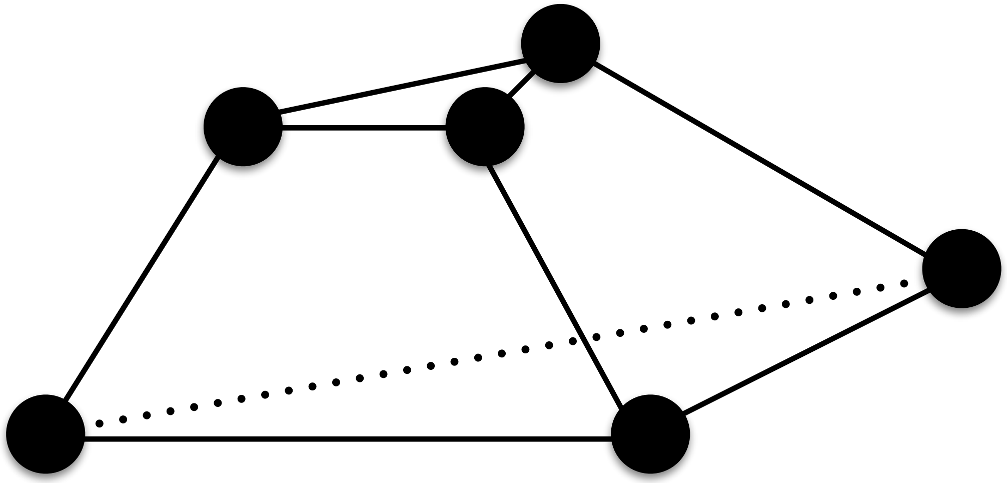
- More in general for a 2-manifold orientable polygonal mesh (with S connected components and B borders):

$$V - L + F = 2(S - G) - B$$

Euler Characteristic Example

The Euler characteristic is 2 for any simply connected polyhedron





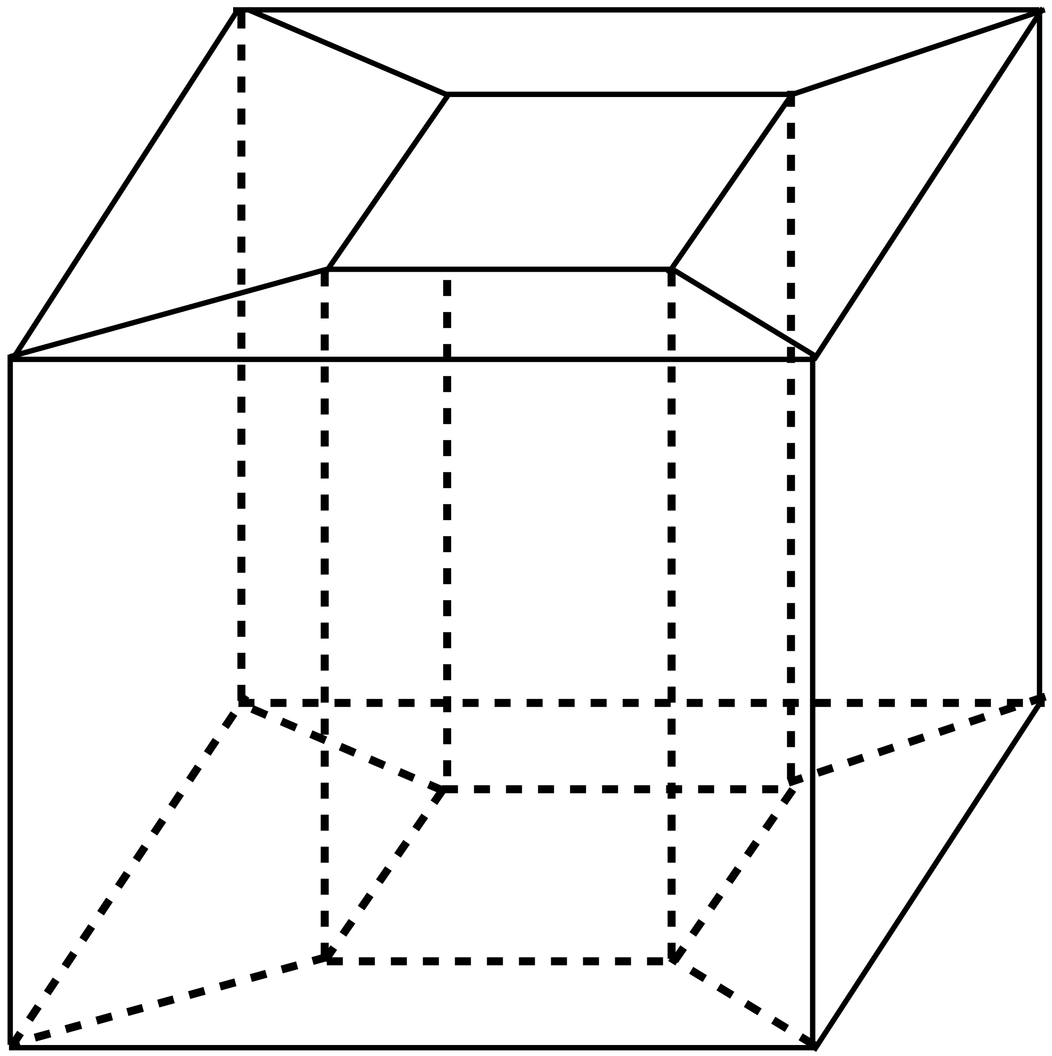
$$\chi = V - E + F$$

$$\chi = 4 - 6 + 4 = 2$$

$$\chi = V - E + F$$

$$\chi = 6 - 9 + 5 = 2$$

Euler Characteristic Example

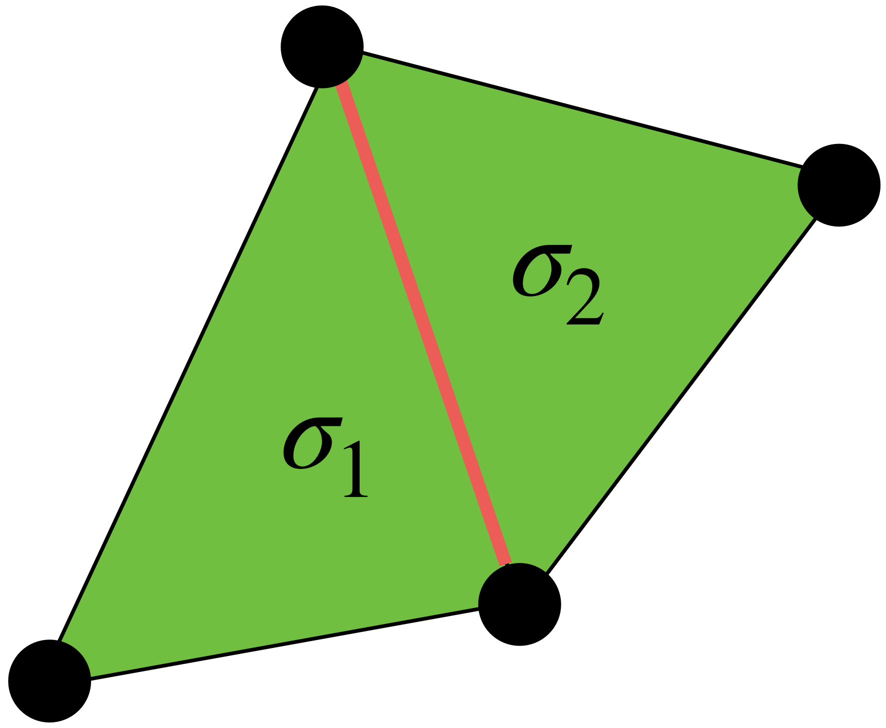


$$\chi = V - E + F$$

$$\chi = 16 - 32 + 16 = 0 = 2 - 2g$$

Adjacency Relations

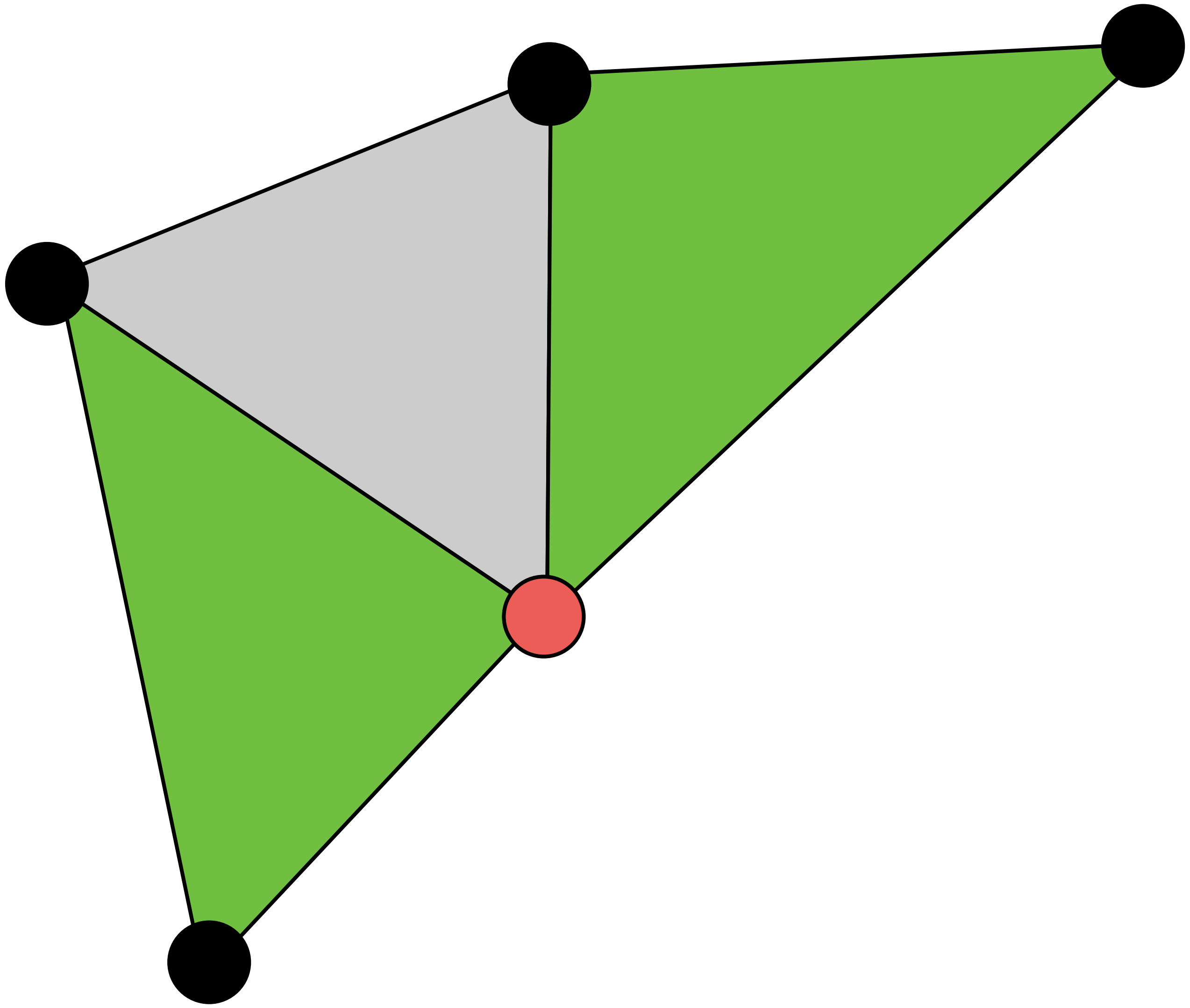
- Given two simplices, σ_1 and σ_2, they are incident if σ_1 is a face of σ_2 or vice-versa:



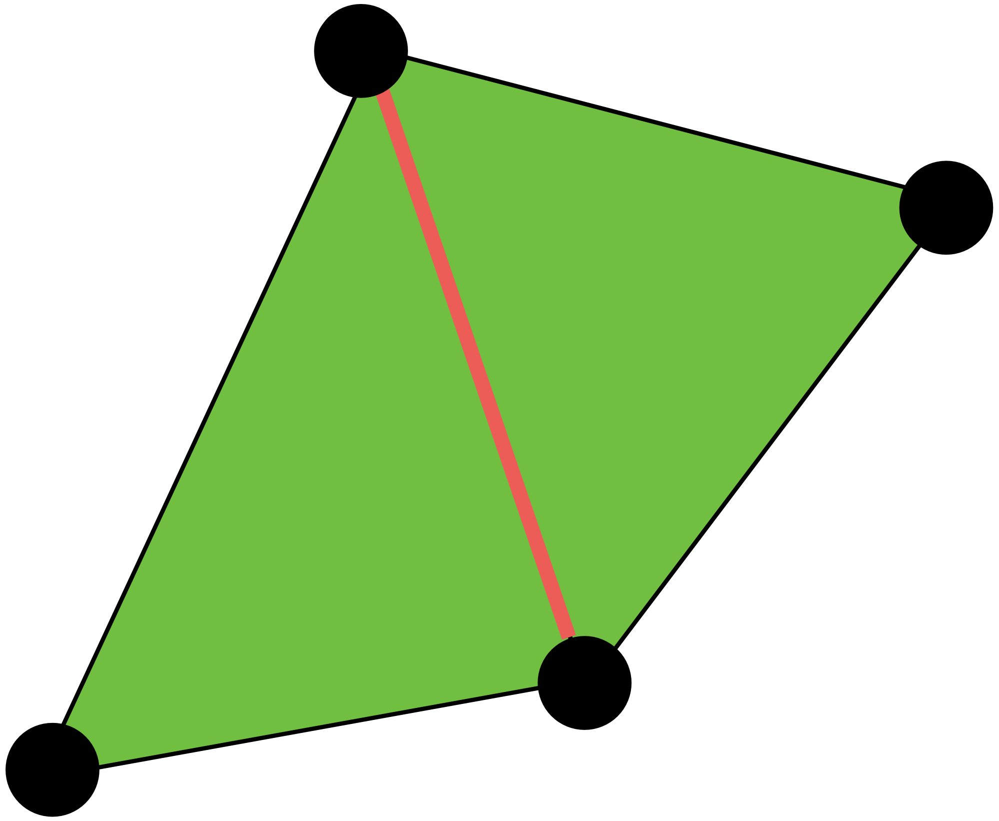
Adjacency Relations

- Two k-simplices are m-adjacent ($k > m$) if a m-simplex exists such that it is a face of both.
- For example:
  - Two triangle sharing a vertex are 0-adjacent
  - Two triangles sharing an edge are 1-adjacent

Adjacency Relations



0-Adjacent



1-Adjacent

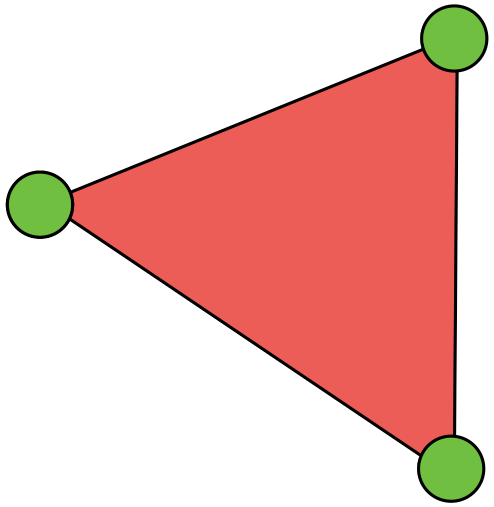
Adjacency Relations

- An adjacency relations is an ordered couple of the following elements:
    - E —> edge
    - F —> Face
    - V —> Vertex
- For example: (E,E), (V,V), (F,F), (E,F), (F,E), (E,V), (V,E), (F,V), (V,F), (E,V), and (V,E).

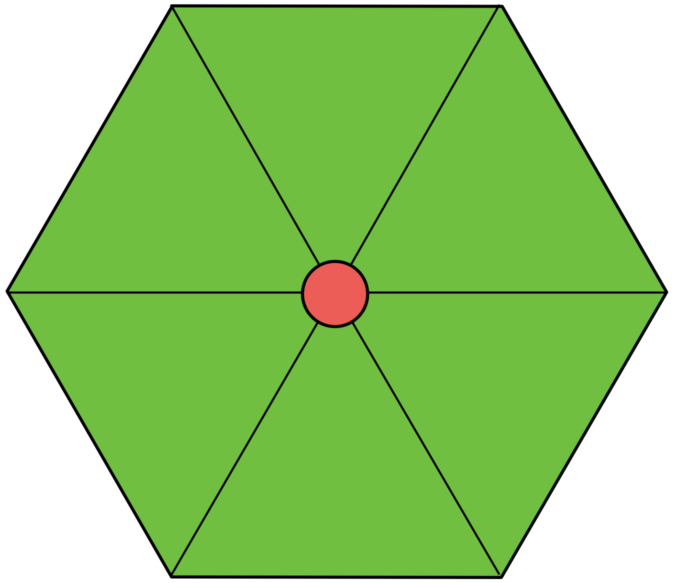
Adjacency Relations Example

- Meaning of some relations:
  - FF —> adjacency between triangles
  - FV —> vertices of a triangle
  - VF —> triangles sharing a vertex

Adjacency Relations Example

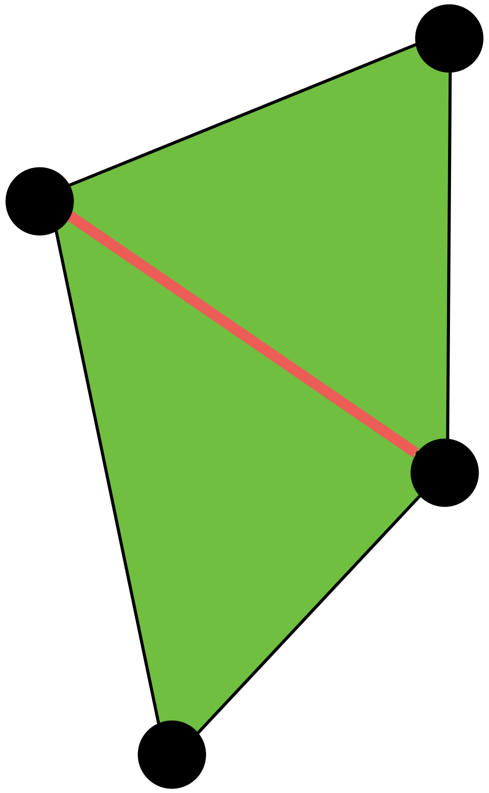


FV

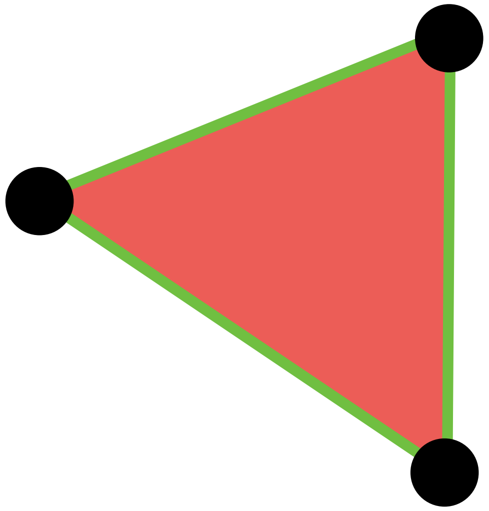


VF

Adjacency Relations Example



EF

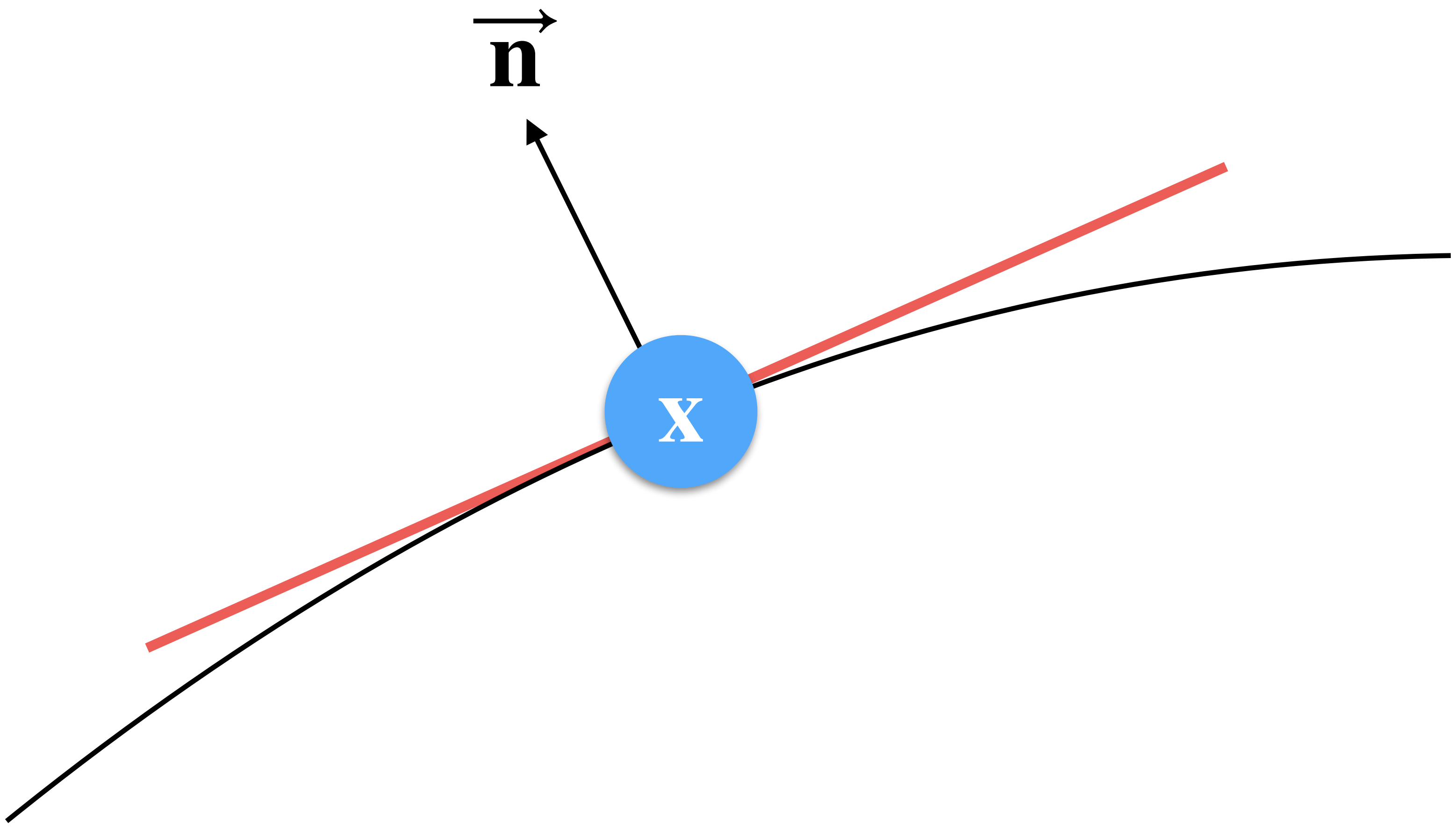


FE

Normals

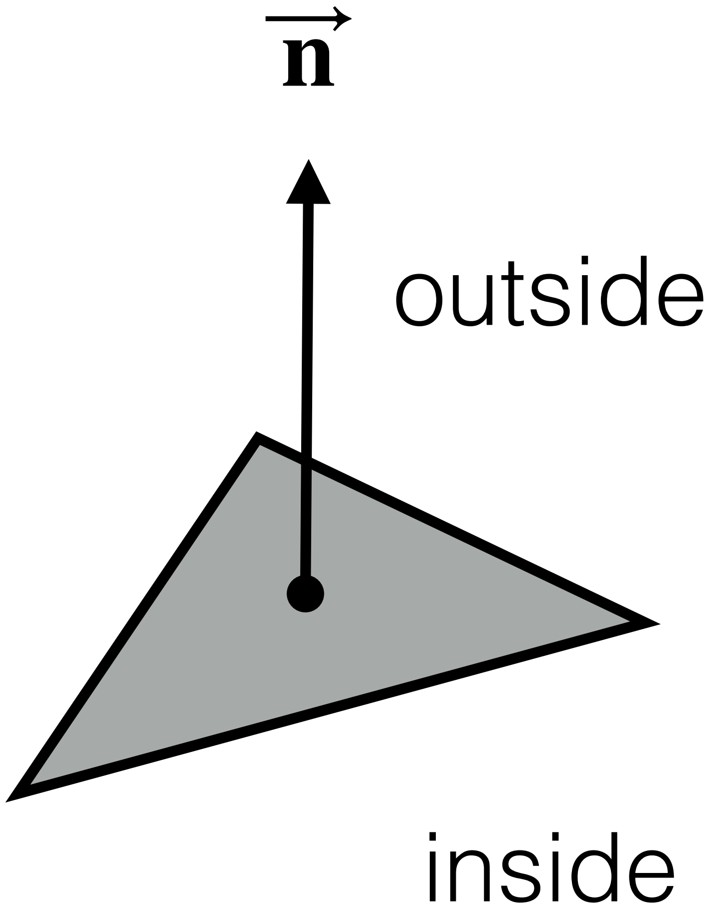
The Unit Normal

- The unit normal, $\vec{\mathbf{n}}$, to a point, $\mathbf{x}$, is the unit vector perpendicular to the tangent plane



The Unit Normal

- A normal is an important attribute for a vertex:
  - It defines the direction of the object boundary

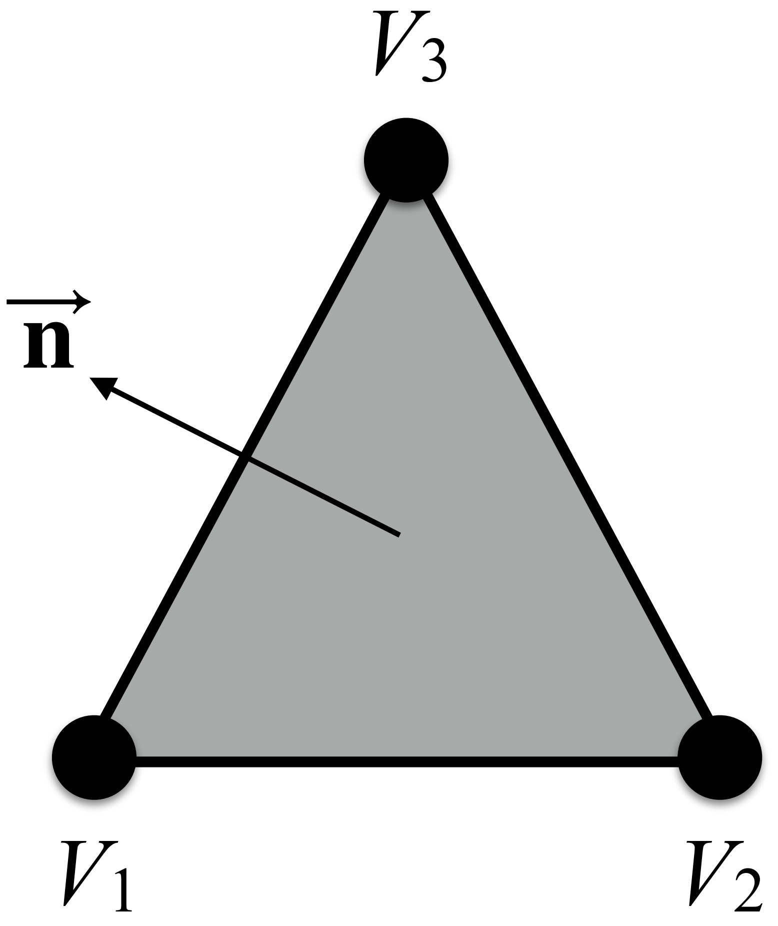


How to Compute per Triangle Normals?

- Given a triangle (V_1, V_2, and V_3), its normal (outer-pointing normal):

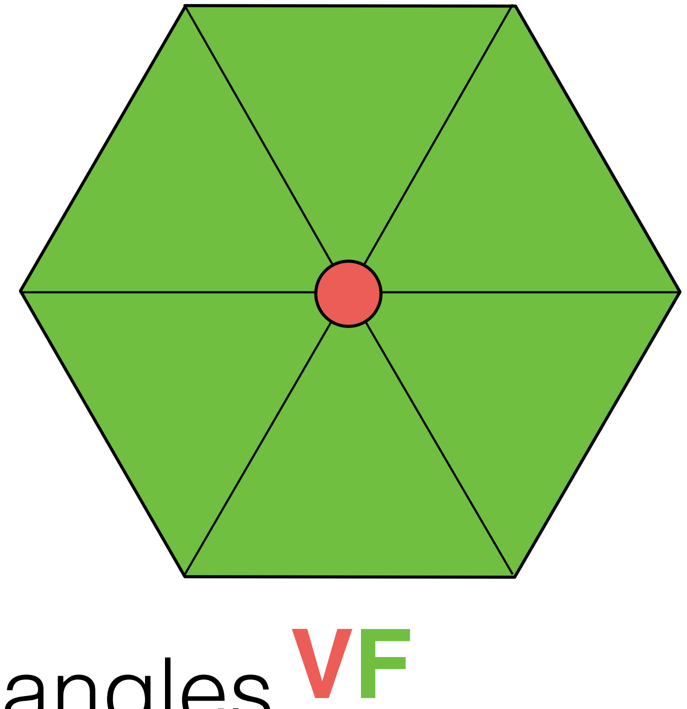
$$\overrightarrow{\mathbf{n}} = (V_3 - V_2) \times (V_1 - V_2)$$

$$\overrightarrow{\mathbf{n}} = \frac{\overrightarrow{\mathbf{n}}}{\| \overrightarrow{\mathbf{n}} \|}$$



- This means that vertices order is important! Typically is counter-clockwise

How to Compute per Vertex Normals?



- We compute normals for each triangle
- For each vertex:
  - We compute the sum of normals of all triangles sharing that vertex:

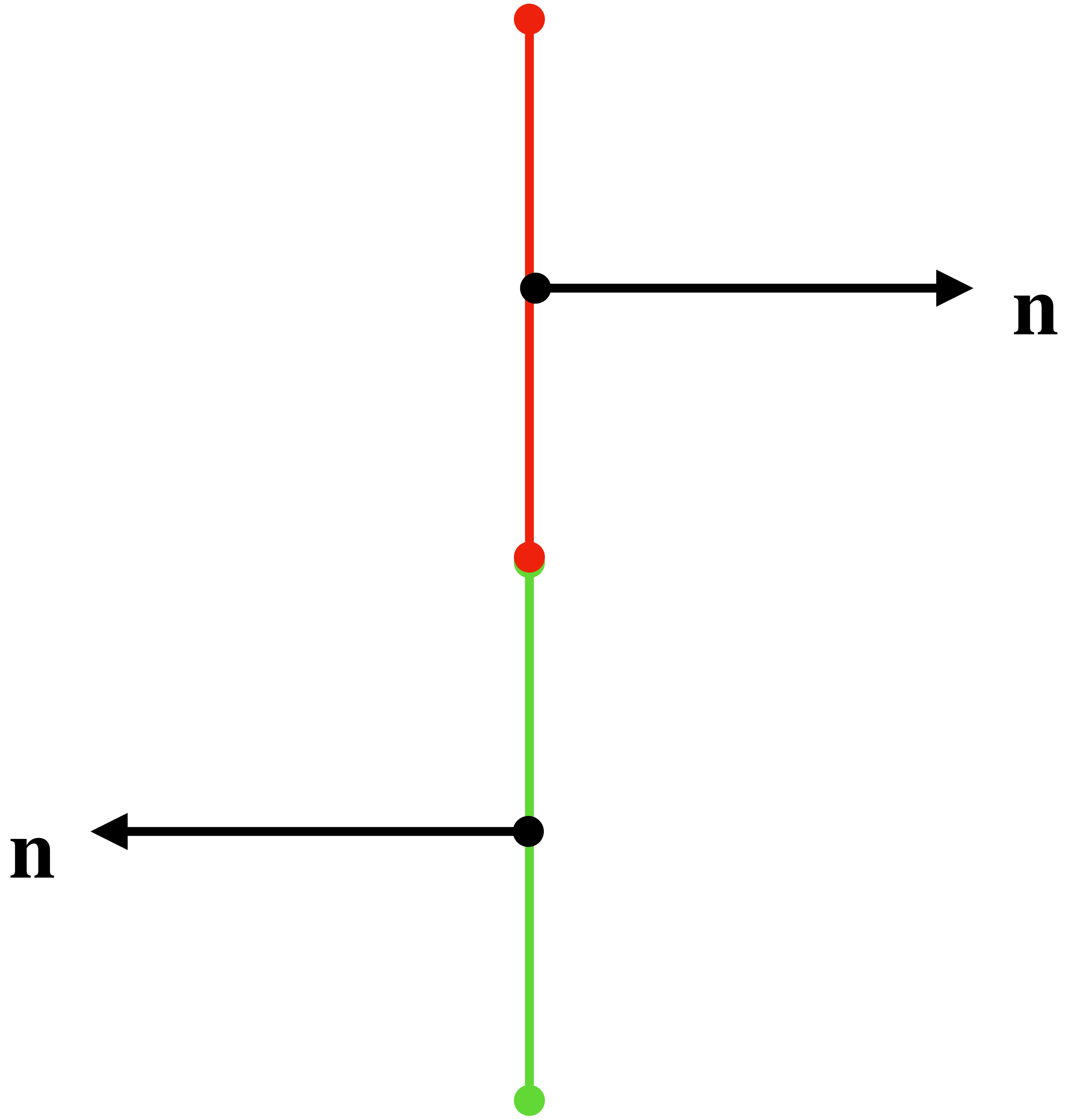
$$\vec{n}_s(V) = \sum_{\{i|V\in T_i\}} \vec{n}_{T_i}$$

  - We normalize this sum
- **Note**: per-vertex normals are useful but not correct!

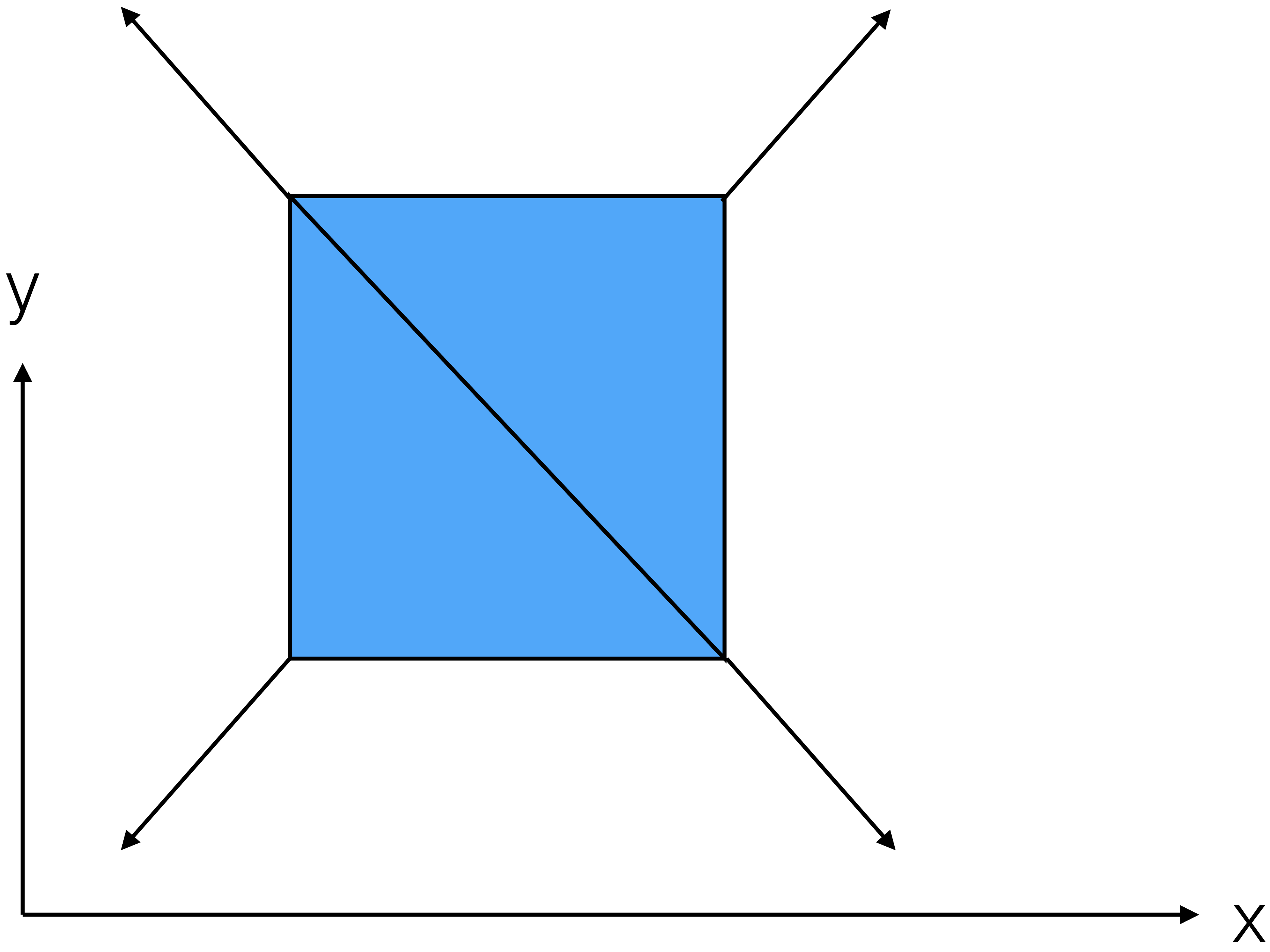
How to Compute per Vertex Normals?

- Problems:
  - We may end up with a null vector $\mathbf{n}_i = [0,0,0]^{\mathsf{T}}$:
    - Triangles with different orientation.
    - Non-manifold triangles.
  - If the model does not have too many triangles we may have a poor result. For example, for this cube (top view):

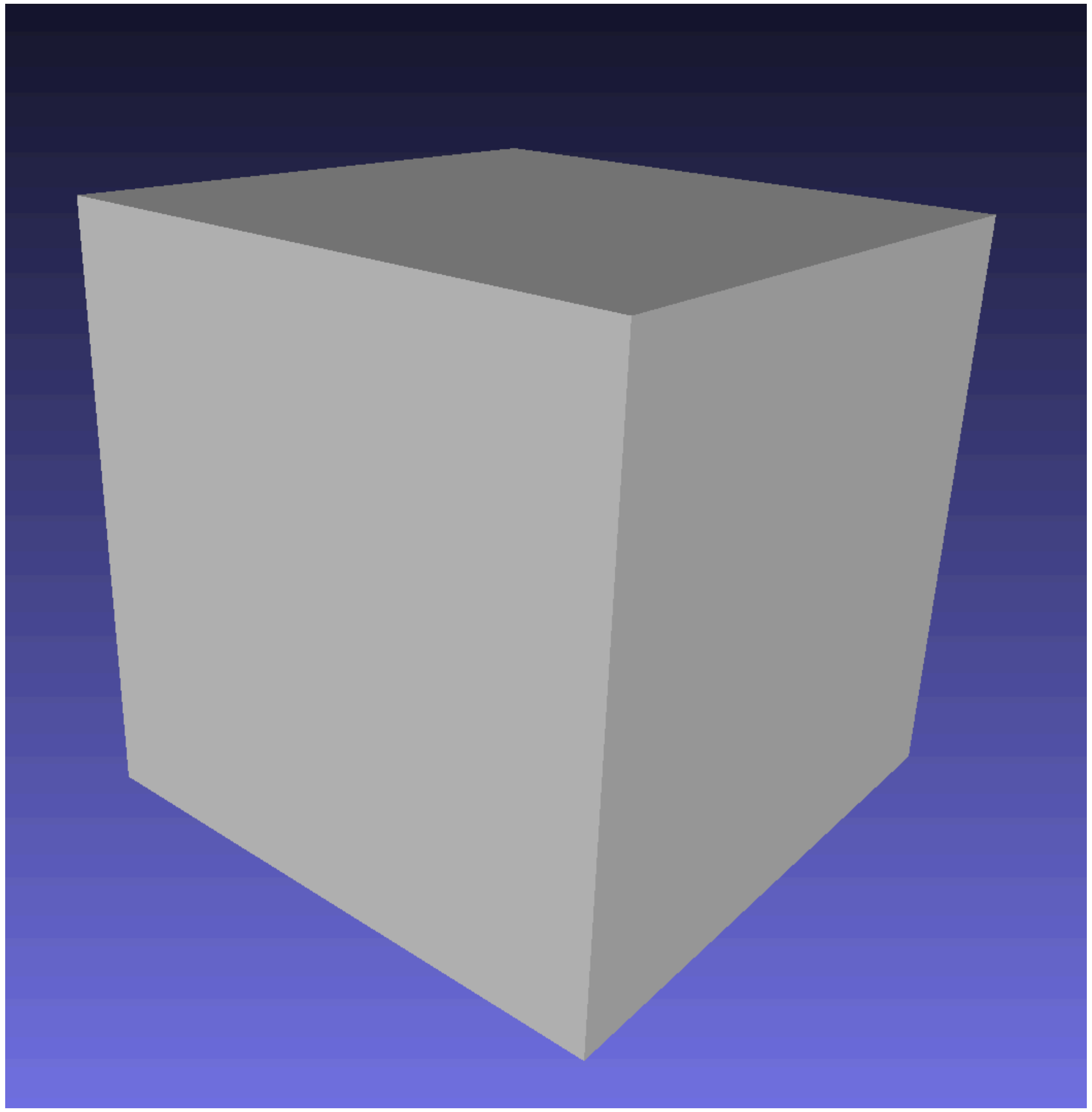
How to Compute per Vertex Normals?



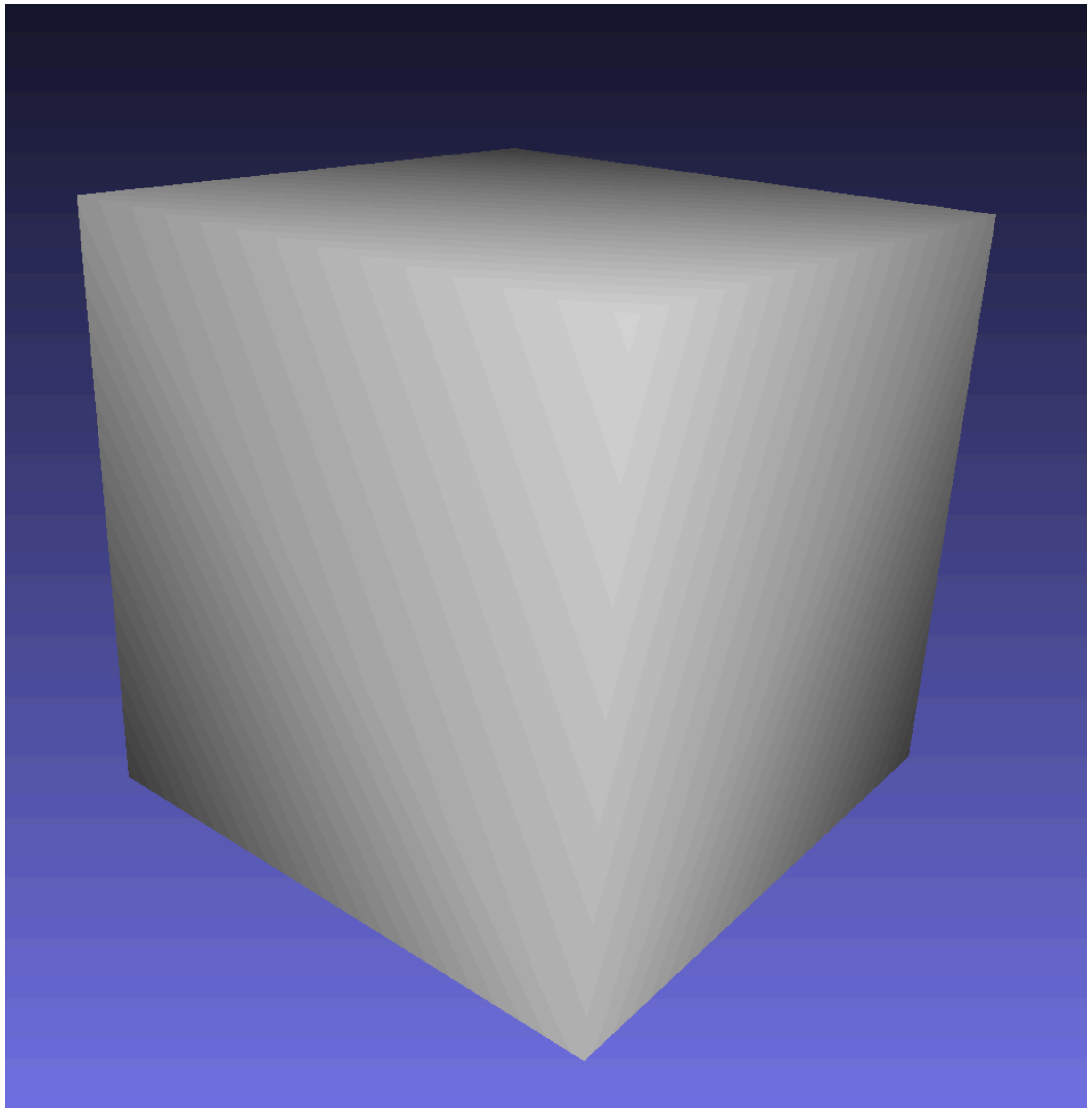
How to Compute per Vertex Normals?



How to Compute per Vertex Normals?

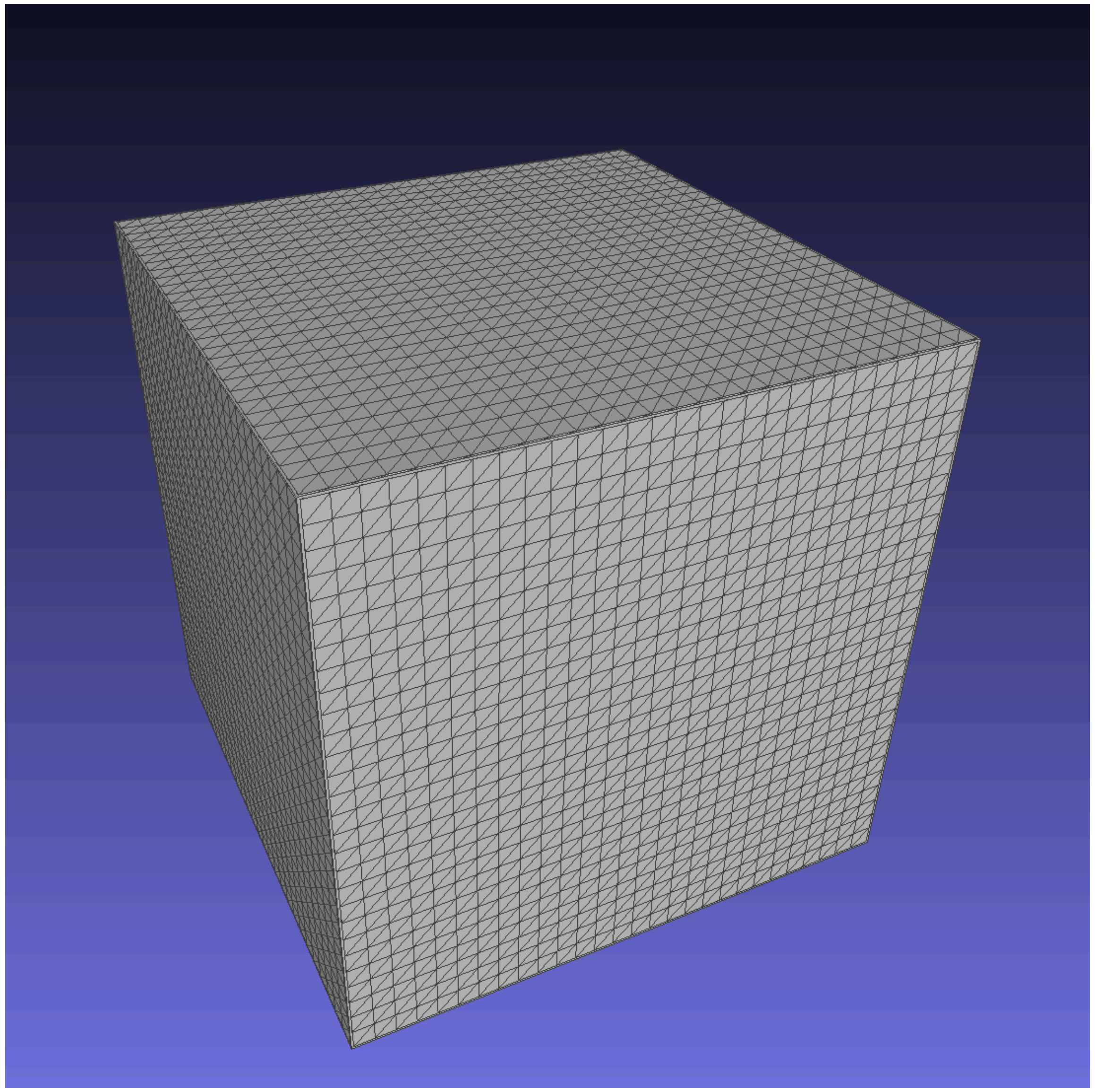


Normal per Triangle

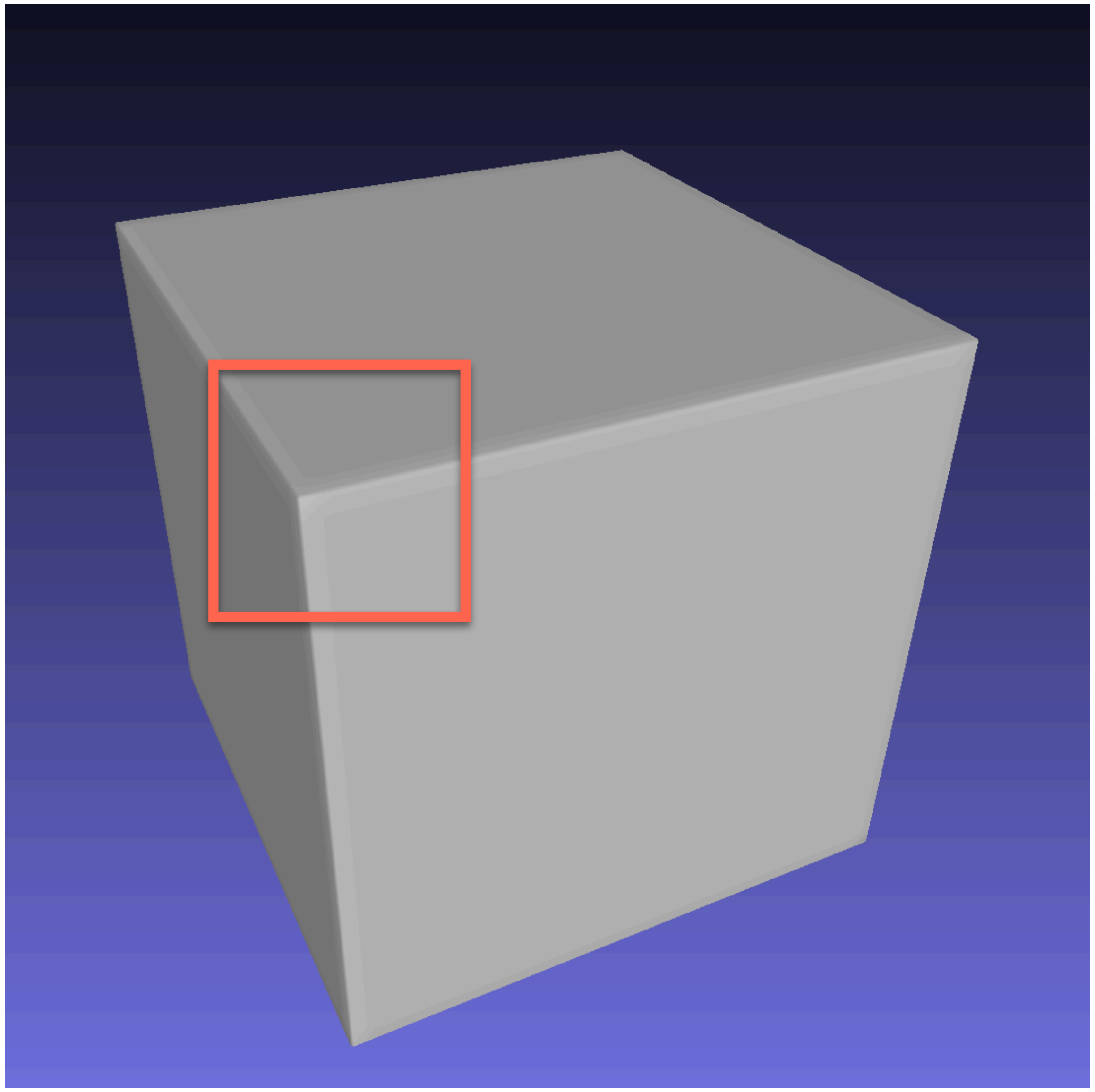


Normal per Vertex

How to Compute per Vertex Normals?

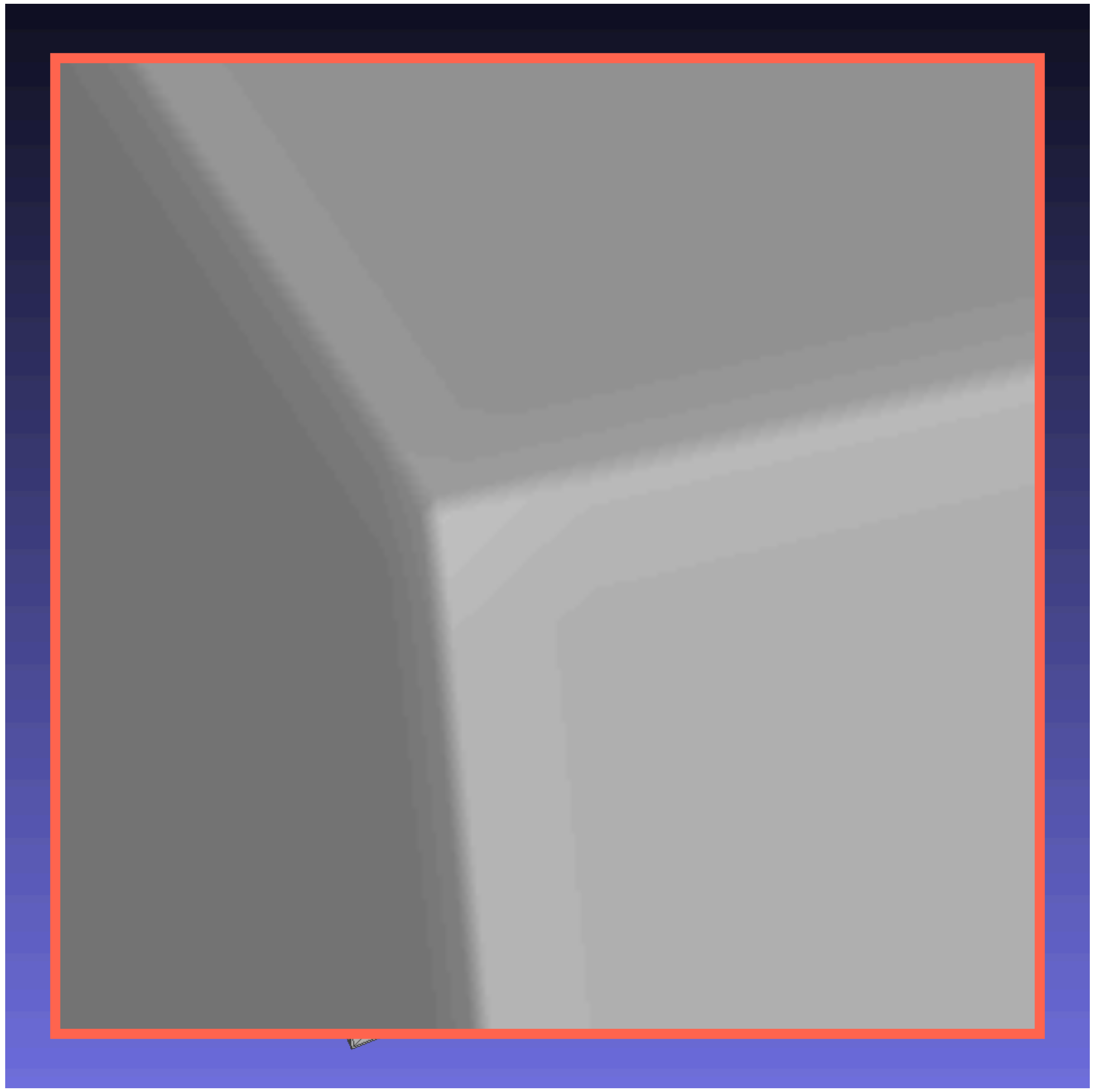


Wireframe

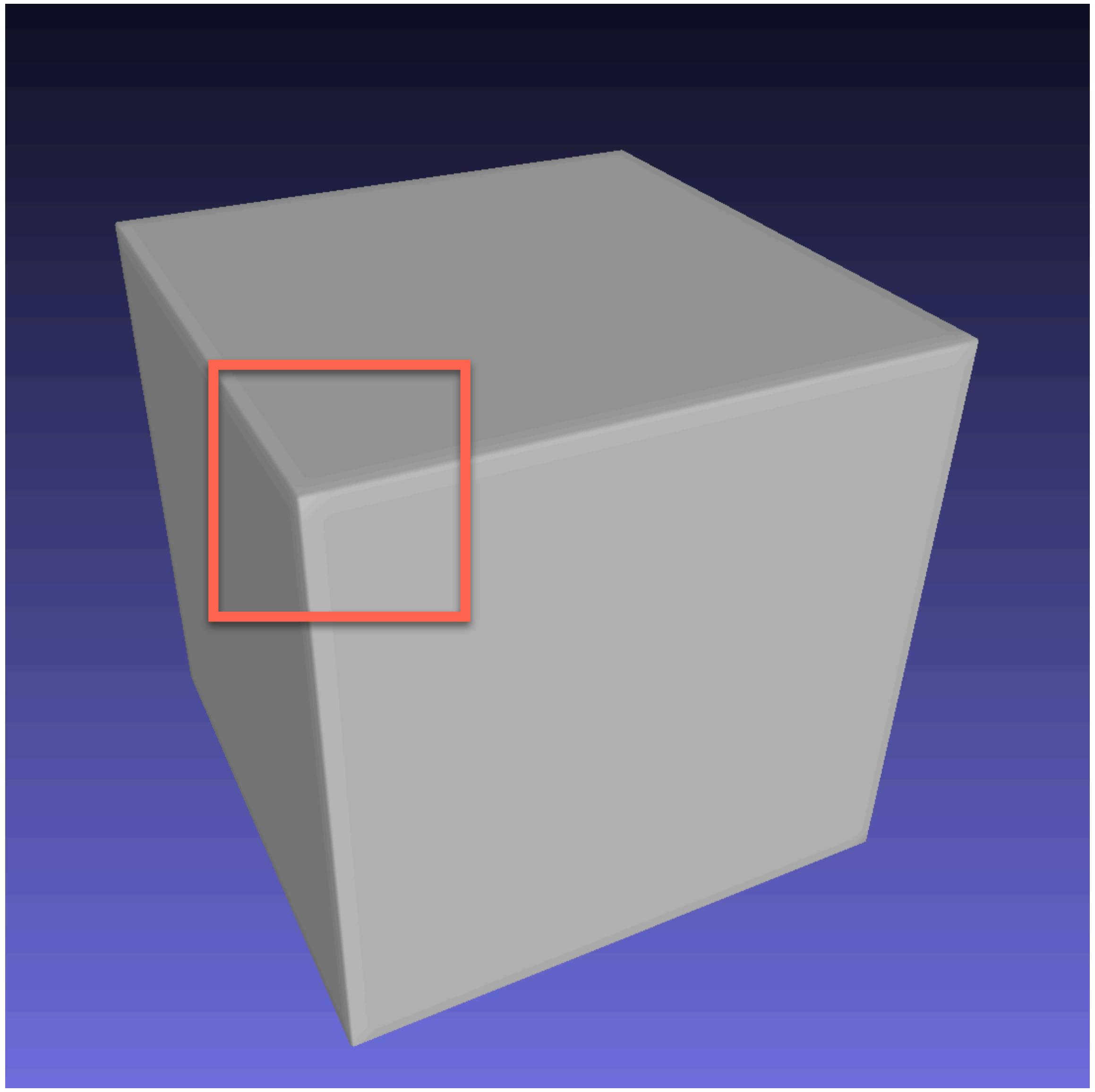


Normal per Vertex

How to Compute per Vertex Normals?



Wireframe



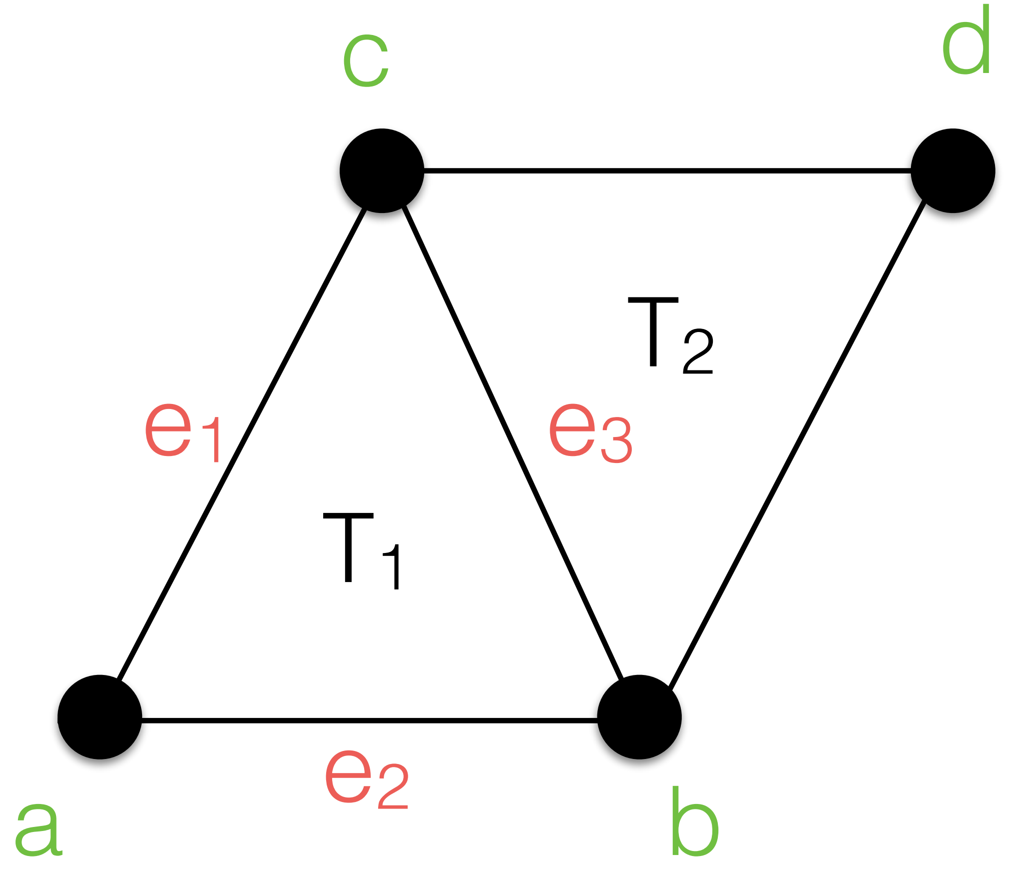
Normal per Vertex

Data Structures for 3D Meshes

List of Triangles

- For each triangle of the 3D model, we store its coordinates.

- For example:



Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)
Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)
Triangle 3: (1,2,-2); (3,-2,5); (-6,2,4)
....
Triangle *n*: (-8,2,7); (-2,3,9); (1,2,-7)

What's ***very wrong*** with this??

Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)
Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)
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What’s ***very wrong*** with this??

Triangle 1: (**3,-2,5**); (**2,2,4**);   (**-6,-2,4**)
Triangle 2: (**2,2,4**) ; (0,-1,-2); (9,4,0)
Triangle 3: (1,2,-2); (**3,-2,5**); (**-6,-2,4**)

….

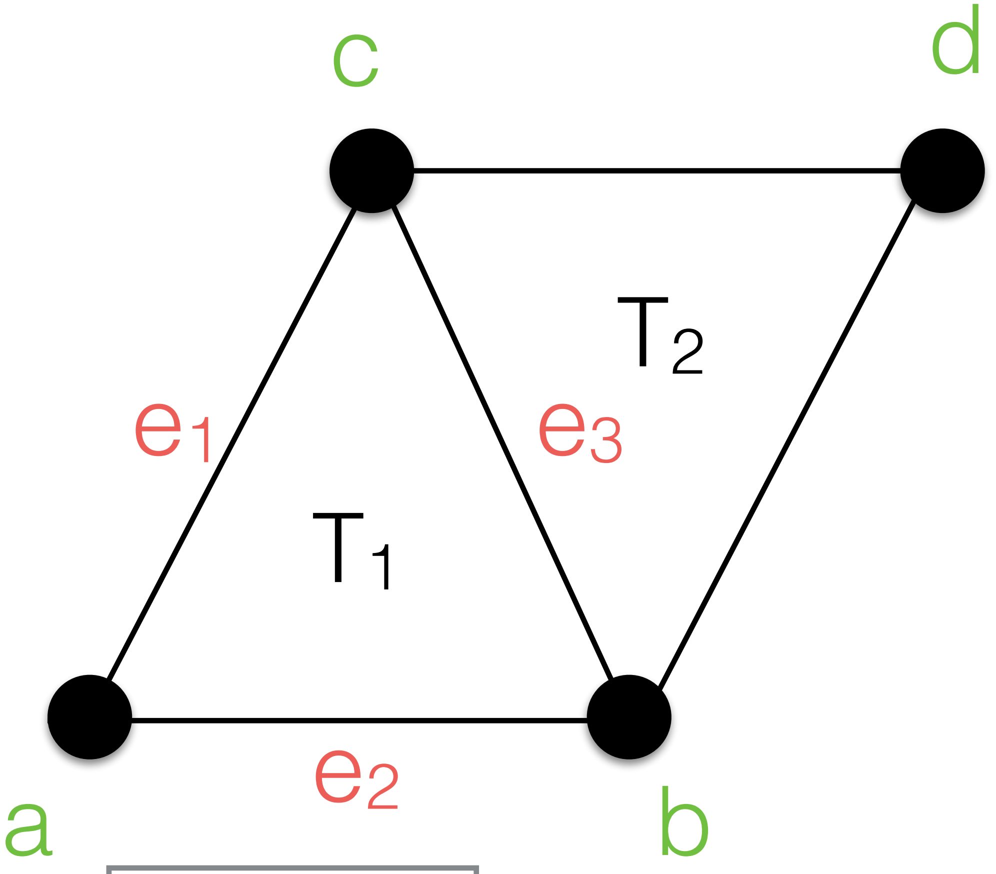
Triangle *n*: (-8,2,7); (-2,3,9); (1,2,-7)

List of Triangles

- Disadvantages:
  - Wasted disk and memory space:
    - Vertices are duplicated!
    - Memory: $|V| \times |T|$
  - Difficult to manage:
    - if we modify a vertex of a triangle, we will need to find and update its clones!
  - How do we query neighbors?

List of Unique Vertices

- We store vertices in a list
- For each triangle of the 3D model, we store indices to the vertices' list



Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)
5. (1.0, -1.0, -1.0)
6. (1.0, -1.0, 1.0)
7. (1.0, 1.0, -1.0)
8. (1.0, 1.0, 1.0)

Faces:

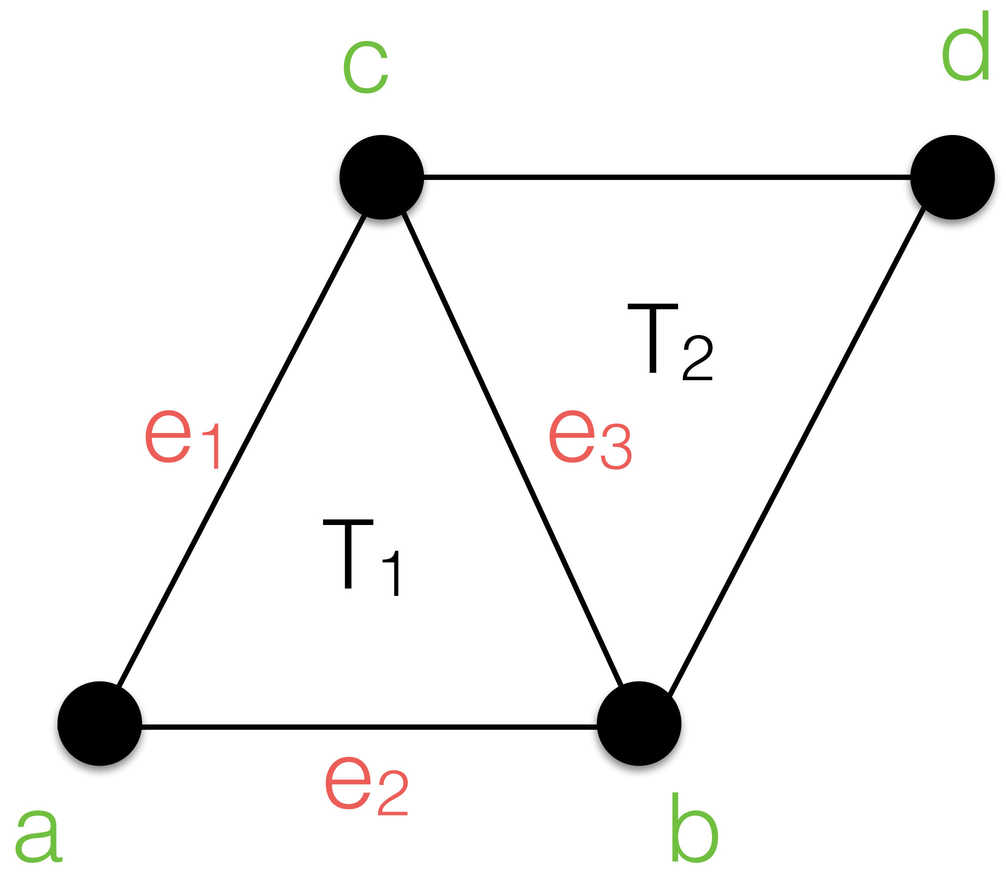
1. 1 2 4
2. 5 7 6
3. 1 5 2
4. 3 4 7
5. 1 7 5

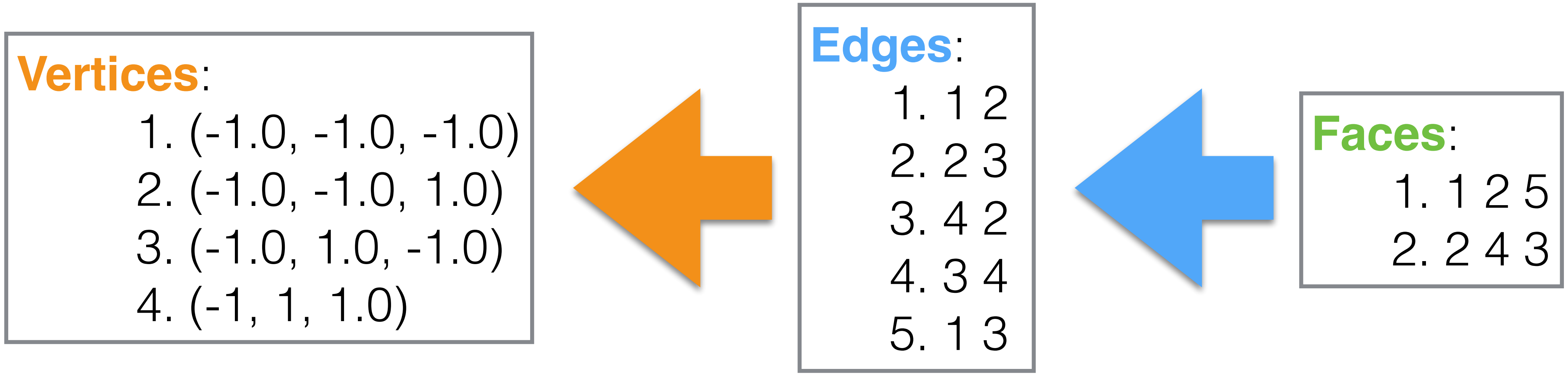
List of Unique Vertices

- Wasted disk and memory space:
  - Common edges between two triangles are stored two times in the list of faces!
  - Memory: $|V| + |T|$
- Better management:
  - Easy to edit a vertex’s attribute (e.g., its position)!
- How do we query neighbors?

List of Unique Edges

- We store vertices in a list
- For each edge, we store indices to the vertices' list
- For each triangle of the 3D model, we store indices to edges's list





Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)

Edges:

1. 1 2
2. 2 3
3. 4 2
4. 3 4
5. 1 3

Faces:

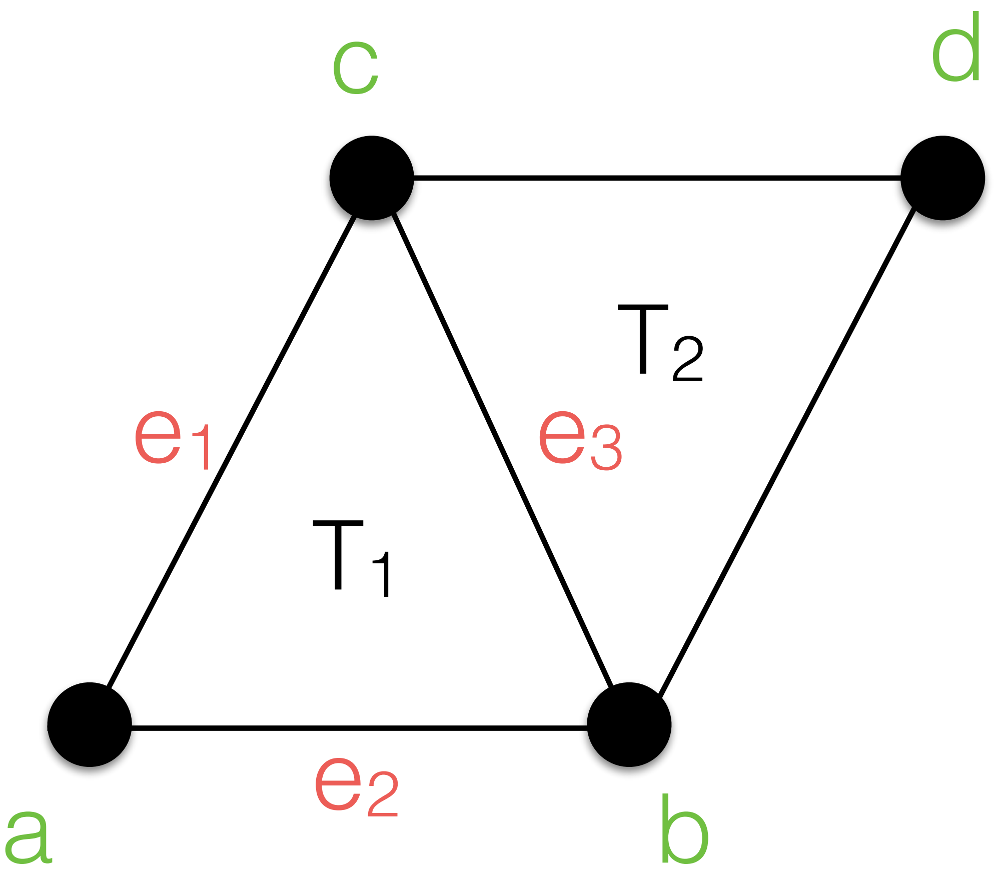
1. 1 2 5
2. 2 4 3

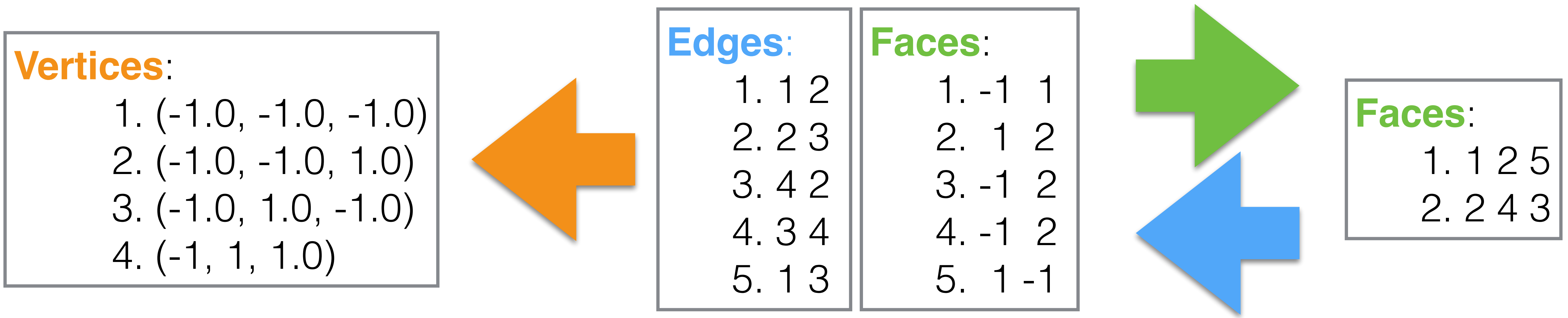
List of Unique Edges

- Better management:
  - Easy to edit an edge’s attribute (e.g., its color)!
- We can do some queries, but not all of them!

Extended List of Unique Edges

- We add to an edge the indices of its left and right triangle
- This simplifies edge-face queries!





Vertices:

1.	(-1.0, -1.0, -1.0)
2.	(-1.0, -1.0, 1.0)
3.	(-1.0, 1.0, -1.0)
4.	(-1, 1, 1.0)

Edges:

1.	1	2
2.	2	3
3.	4	2
4.	3	4
5.	1	3

Faces:

1.	-1	1
2.	1	2
3.	-1	2
4.	-1	2
5.	1	-1

Faces:

1.	1	2	5
2.	2	4	3

File Formats

File Formats

- There are many 3D file formats. The most used, and de-facto standard:
    - STL
    - PLY
    - OBJ
- Standards:
    - COLLADA: https://www.khronos.org/collada/
    - X3D: http://www.web3d.org/x3d/

STL File Format

- Standard Triangle Language (STL) created by 3D Systems
- This format represents only the 3D geometry:
    - No color/texture
    - No other attributes
- The format specifies both ASCII and binary representations

STL File Format

- Data structure: list of triangles
- Vertices are ordered using the right-hand rule
- 3D coordinates must be positive
- No scale metadata; i.e., units are arbitrary

STL File Format

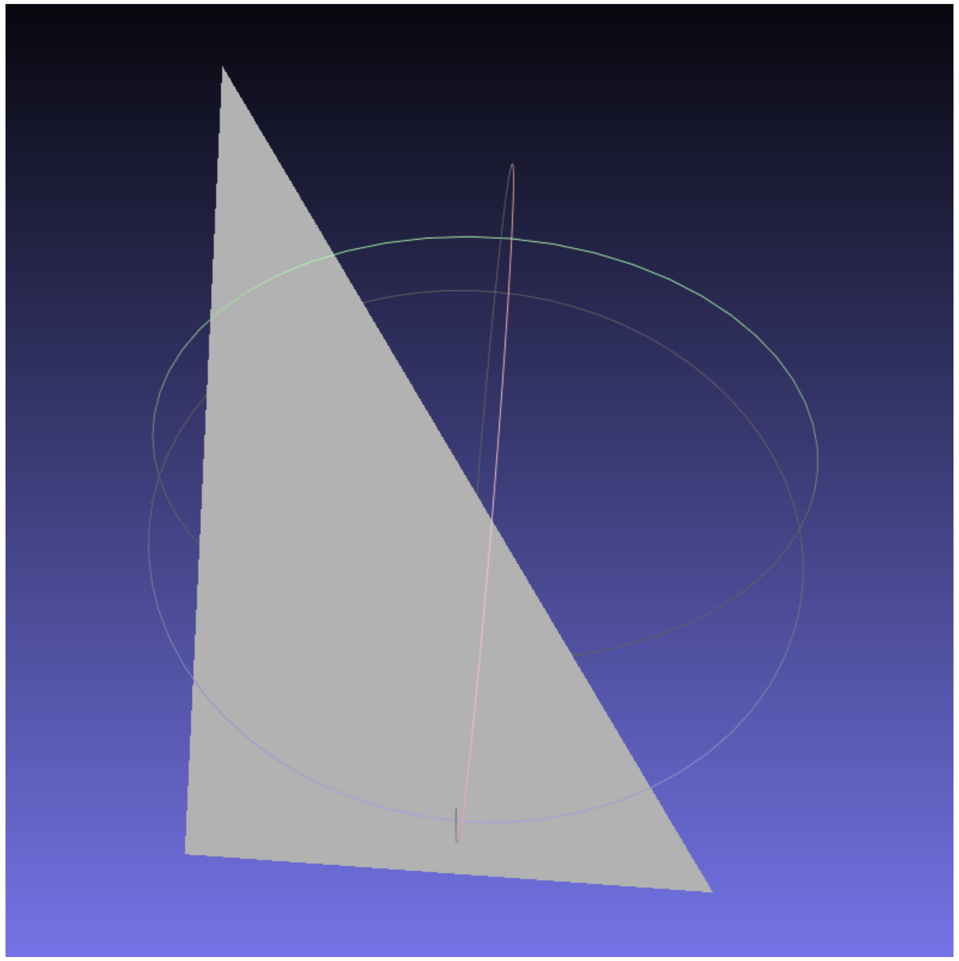
- The file begins as

```
solid name
```

- A face is defined as

```
facet normal nx ny nz
    outer loop
        vertex v1x v1y v1z
        vertex v2x v2y v2z
        vertex v3x v3y v3z
    endloop
endfacet
```

STL File Format: An Example



```
solid triangle
facet normal 0 1 0
        outer loop
                vertex 0.0 0.0 0.0
                vertex 1.0 0.0 0.0
                vertex 0.0 1.0 1.0
        endloop
endfacet
endsolid triangle
```

PLY File Format

- Polygon File Format (PLY) is a popular format created by Stanford University (Greg Turk)
- The format is very flexible:
  - we can add many attributes
  - we can define triangular and polygonal meshes
- The format specifies both ASCII and binary representations

PLY File Format

- Data structure: list of unique vertices
- No scale metadata; i.e., units are arbitrary
- The file is divided into two parts:
  - **Header** that specifies vertices and faces
  - **Body** that specifies the concrete data

PLY File Format: Header

- The file begins as

```
ply
format ascii 1.0
```

- Vertex specification is defined as

```
element vertex num_vertices
property float x
property float y
property float z
```

properties can be: char, uchar, short, ushort, int, uint float, double, etc.

PLY File Format: Header

- Faces are defined as

```
element face num_faces
property list uchar int vertex_indices

end_header
```

PLY File Format: Body

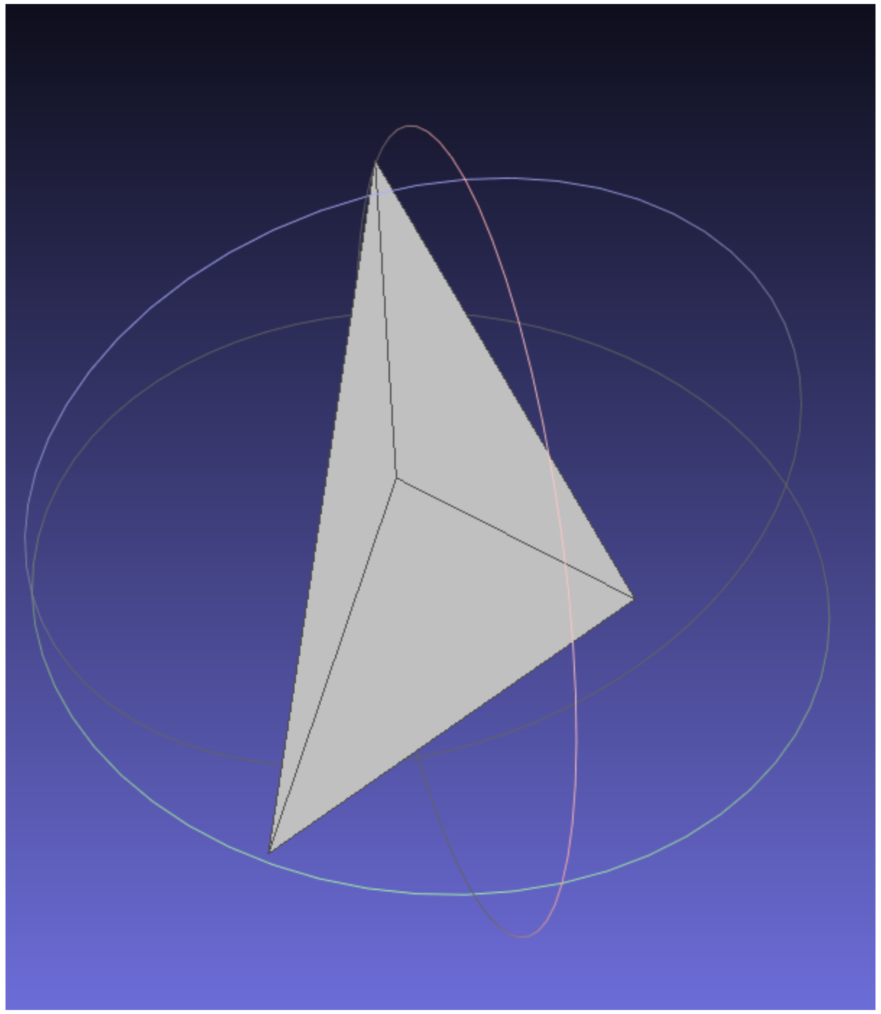
- Each i-th vertex is specified as

```
vix viy viz
```

- Each face is specified as

```
3 index_v1 index_v2 index_v2
```

PLY File Format: An Example



```
ply
format ascii 1.0
element vertex 4
property float x
property float y
property float z
element face 4
property list uchar int vertex_indices
end_header
-0.60 -0.97 0.37
-0.34 0.98 0.76
0.037 0.65 -1.06
0.88 -0.75 -0.25
3 1 3 2
3 0 1 2
3 0 3 1
3 3 0 2
```

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    - http://vcg.isti.cnr.it/~cignoni/