3D Models

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3D Models

- A 3D model is a computational representation of a real-world object. This is typically:
  - C0
  - Closed (not always!)
  - Discretized

3D Models

- Two main representations:
  - **Boundary representations** (b-rep): a 3D object is represented as a collection of connected surface elements; i.e., the boundary between solid and non-solid
  - **Volume representations**: a 3D object is represented by its interior volume. For example, 3D volumes or volume mesh (FEM)

Our focus is on

boundary representations

Polygonal Meshes

Surfaces

- A 2-dimensional region of 3-dimensional space
- A portion of space having length and breadth but no thickness

3D Representation: Polygonal Meshes

- Discretize the surface in a set of simple primitives:
  - Many points
  - **Triangles**
  - Quads
  - Polygons
  - Our focus is on:
    - simplicial complexes, e.g., triangles!

Why triangular meshes?

- Two main practical reasons:
  - Data-structures are straightforward
  - Graphics hardware (e.g., a GPU) uses triangles;

Why triangular meshes?

- Two main theoretical reasons:
  - Nice theory, i.e., simplicial complexes
  - Less limiting cases:
    - a triangle is always planar!
    - if we remove a vertex, we get another simplicial!

Simplex

- A k-simplex, $\boldsymbol{\sigma}$, is convex combination of $k+1$ points, $\mathbf{p}_i$, that are **linearly independent** in the k-dimensional Euclidian space, $\mathbb{R}^k$:

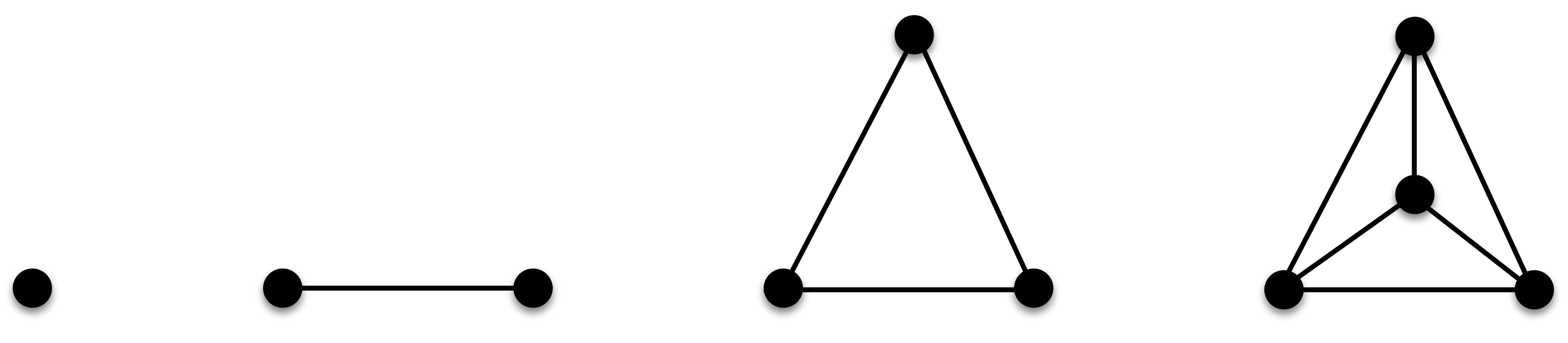
$$\mathbf{x} = \sum_{\mathbf{p}_i \in \sigma} \alpha_i \cdot \mathbf{p}_i$$

$$\sum_i \alpha_i = 1 \ \wedge \ \alpha_i \geq 0 \ \forall i$$

- A point $\mathbf{p}_i$ is called a vertex.

- k is the order of the simplex.

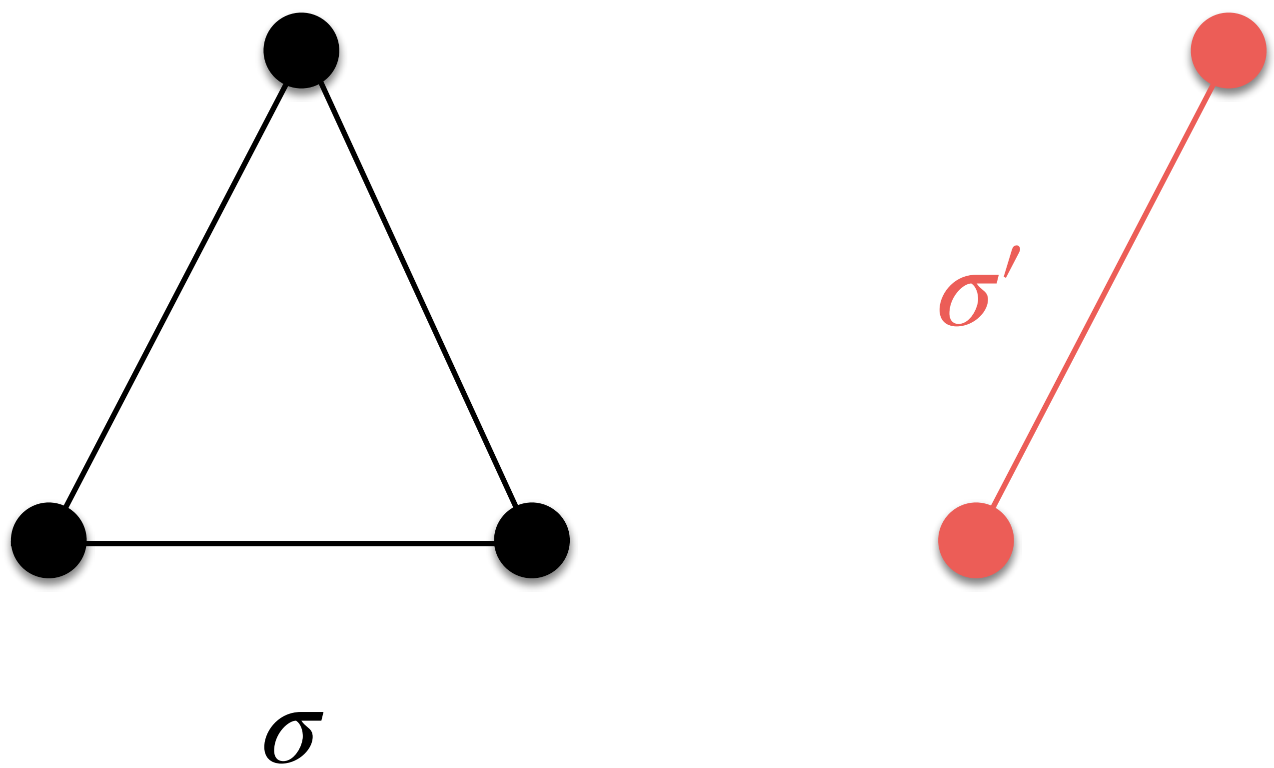
Simplices Example



$k = 0$ $k = 1$ $k = 2$ $k = 3$

Sub-Simplex

- A sub-simplex σ' is called a **face** of a simplex $\boldsymbol{\sigma}$ if it is a sub-set of vertices of $\boldsymbol{\sigma}$.



A Simplicial k-Complex

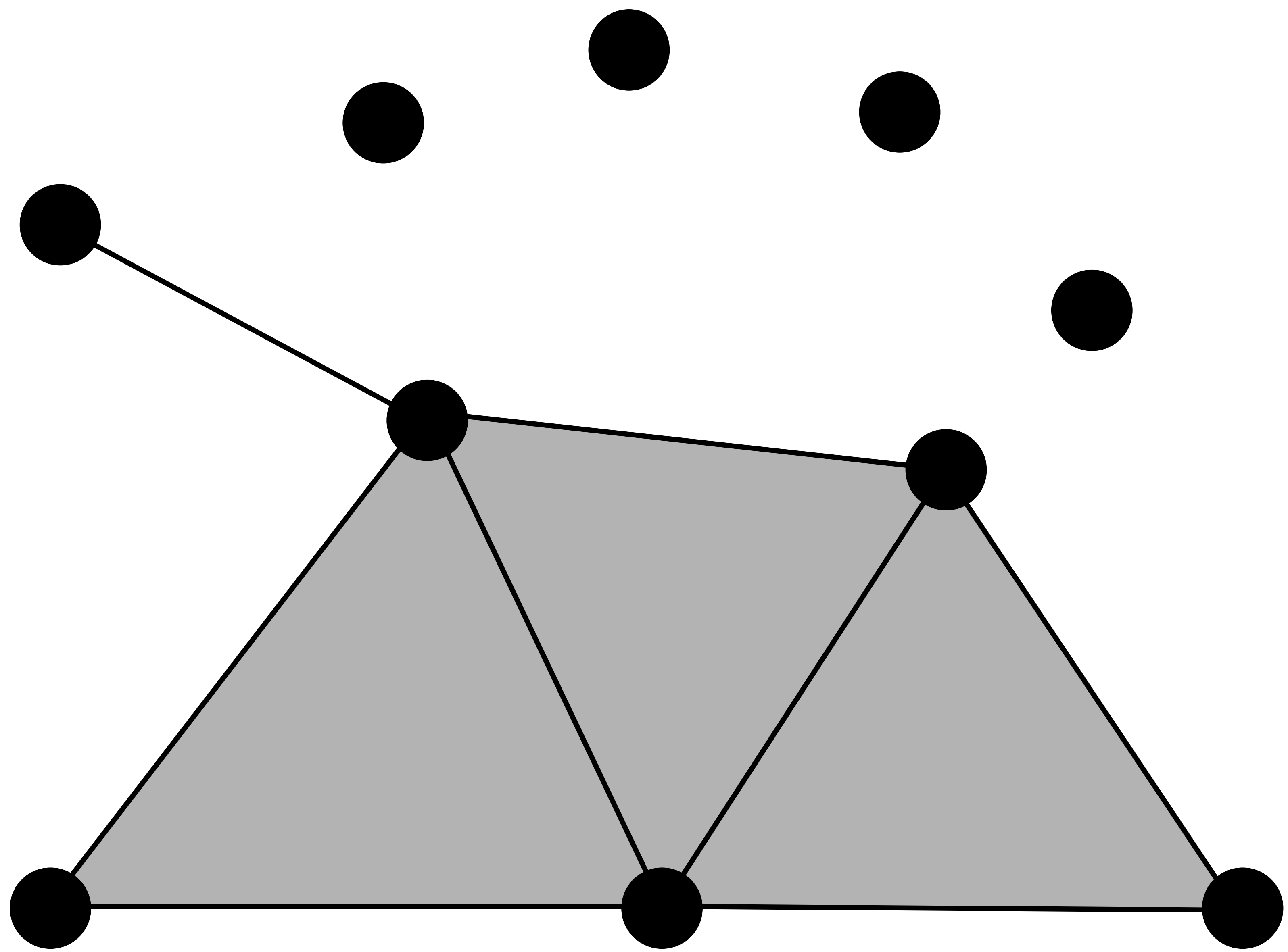
- A simplicial k-complex, Σ, is a finite collection of n simplices such that:

  (i) The intersection of any two simplices of Σ is a face of each of them

  (ii) Every face of a simplex, σ, of is in Σ

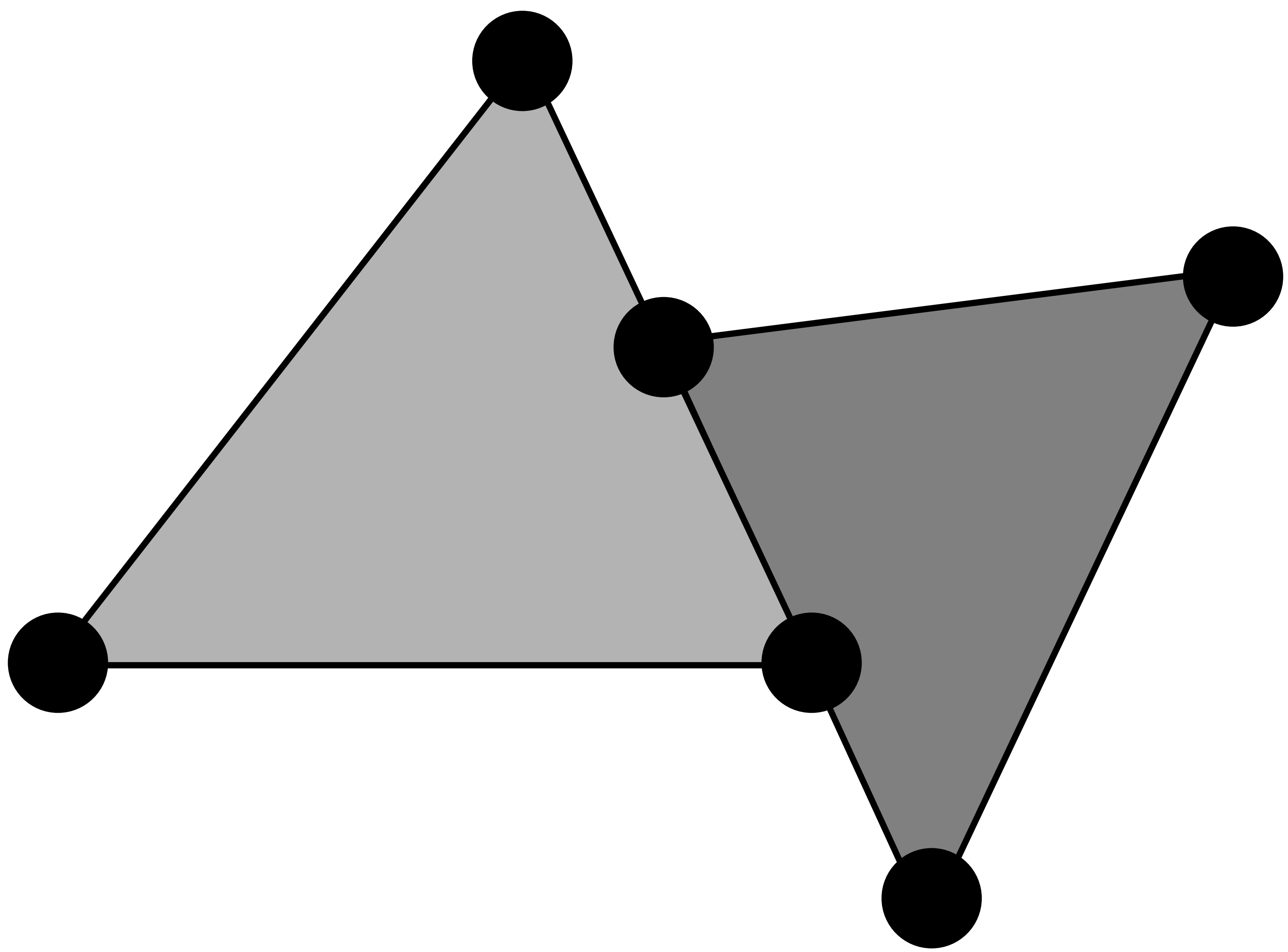
  NOTE: k is the maximum order of all σ in Σ

Simplicial Complexes Example



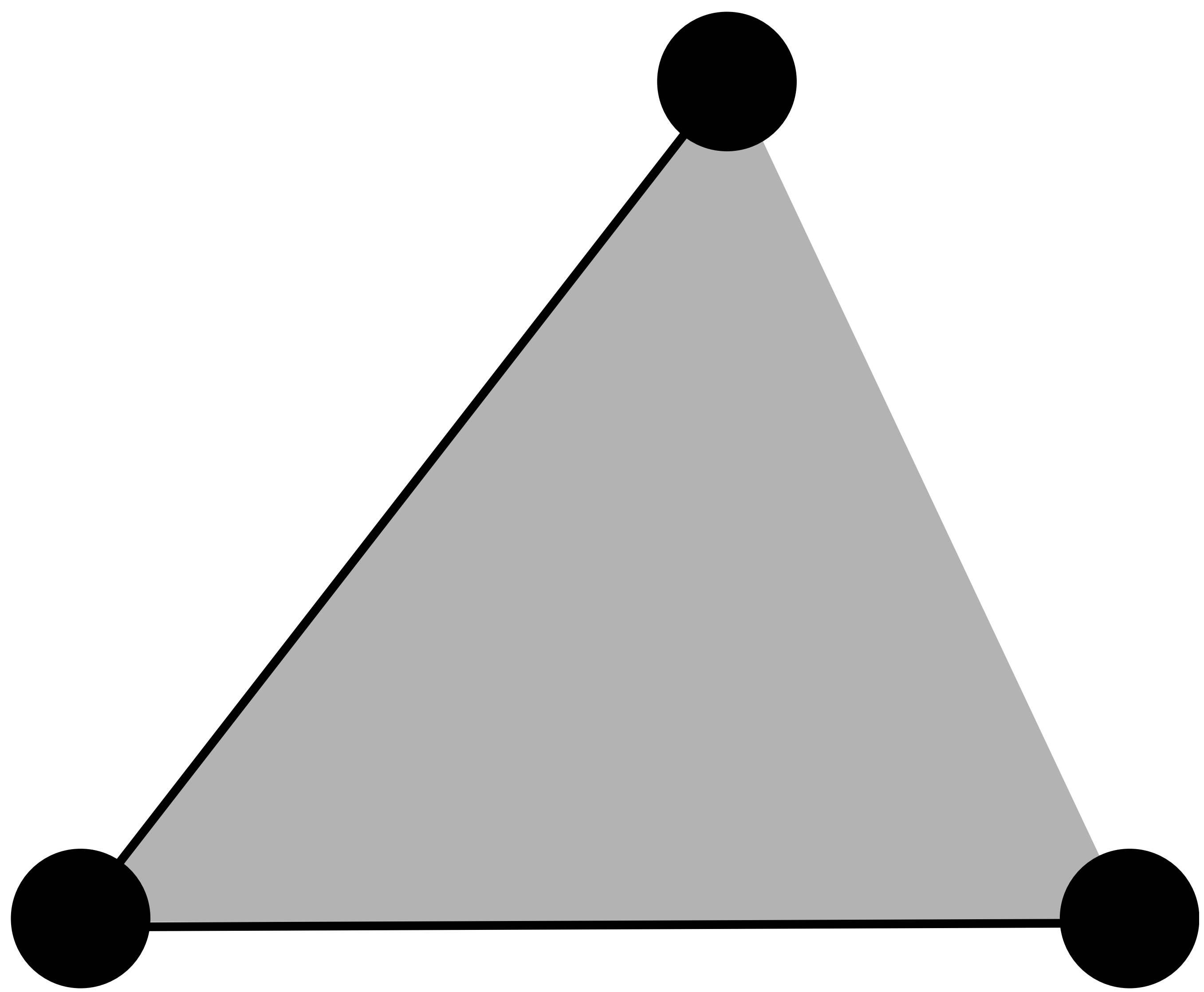
GOOD!

Simplicial Complexes Example



BAD: is not valid! —> Condition (i)

Simplicial Complexes Example

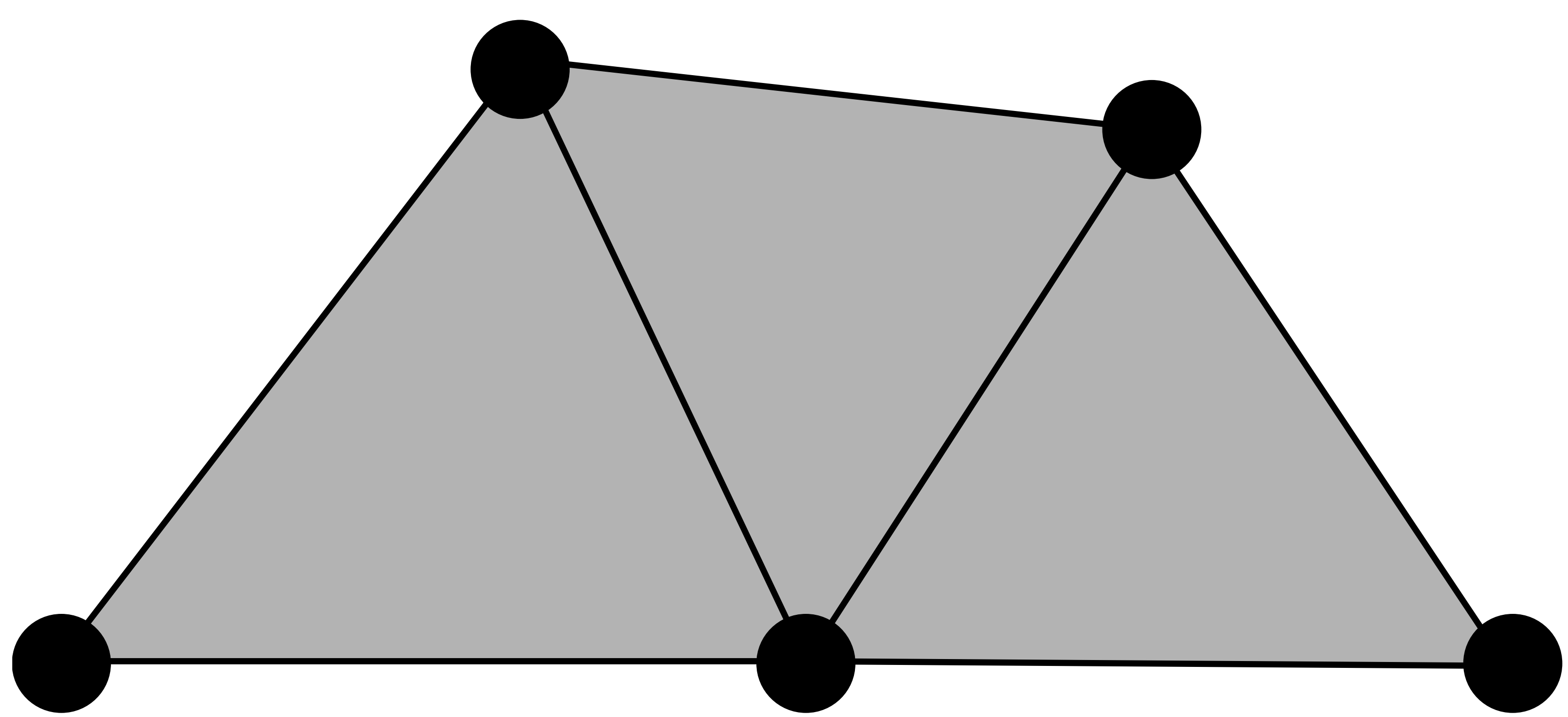


BAD: is not valid! —> Condition (ii)

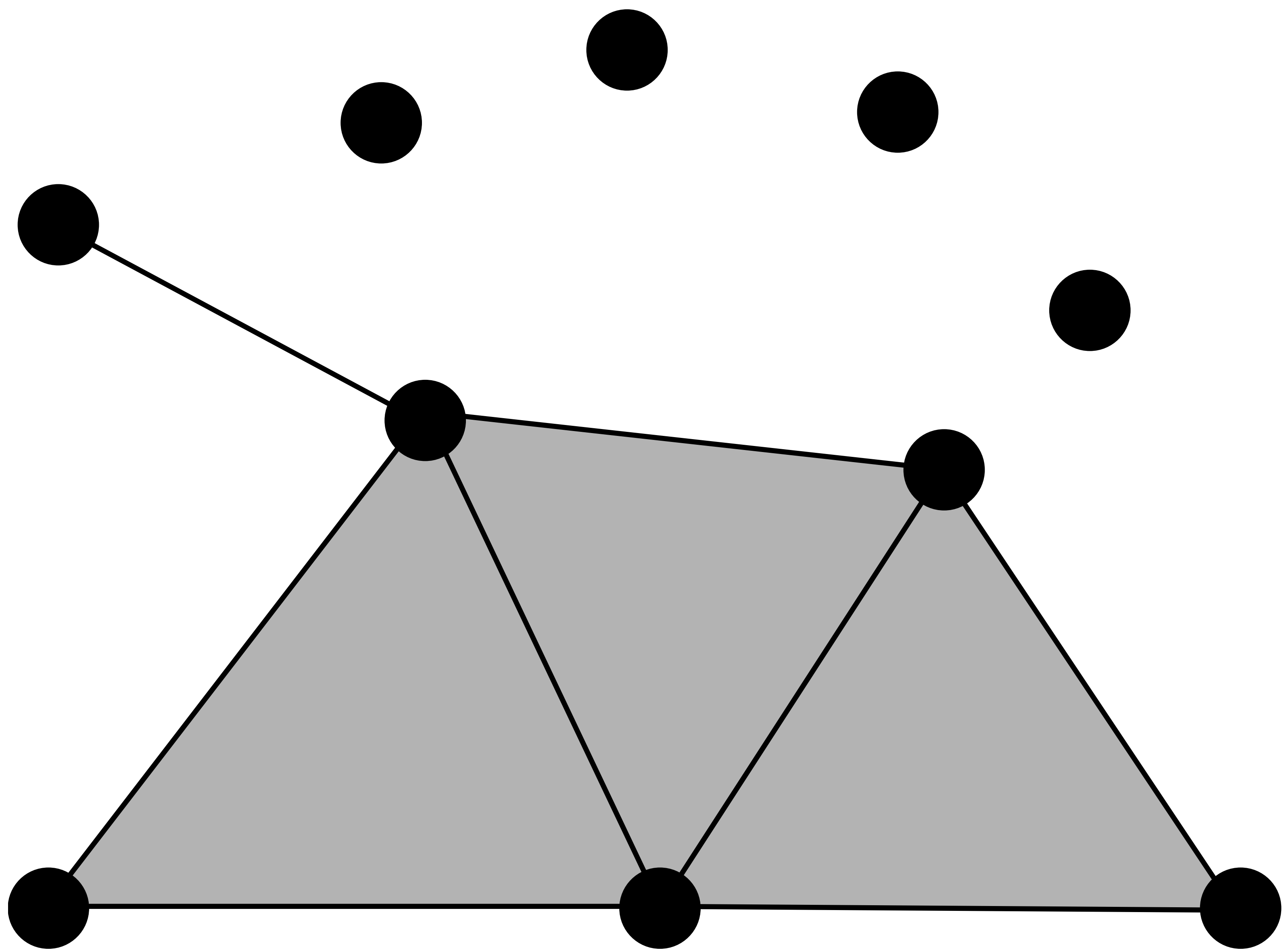
Simplicial Complexes

- A simplex, σ, is maximal in a simplicial complex, Σ, if it does not belong to any other simplex σ_2 of Σ.
- A *k*-simplicial complex, Σ, is maximal if all maximal simplices have order k.

A Maximal Simplicial Complex Example



A Non-Maximal Simplicial Complex Example



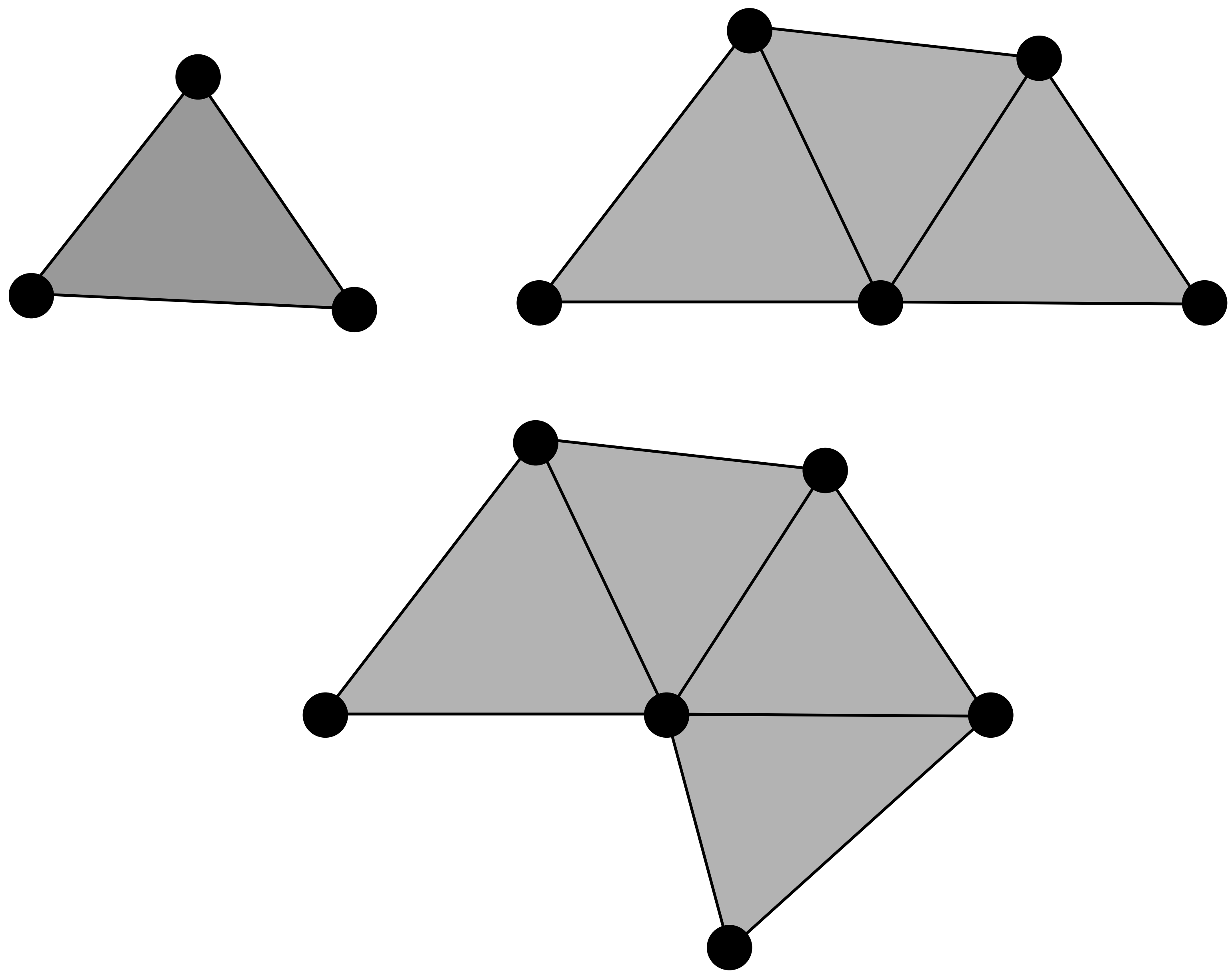
Manifoldness

- A surface, $S \in \mathbb{R}^3$ is manifold if and only if:
    - The neighborhood of each point is homeomorphic to an Euclidean space in two dimension or in other words:
        - The neighborhood of each point is homeomorphic to a disk or a semi-disk if the surface has boundaries!

Manifoldness

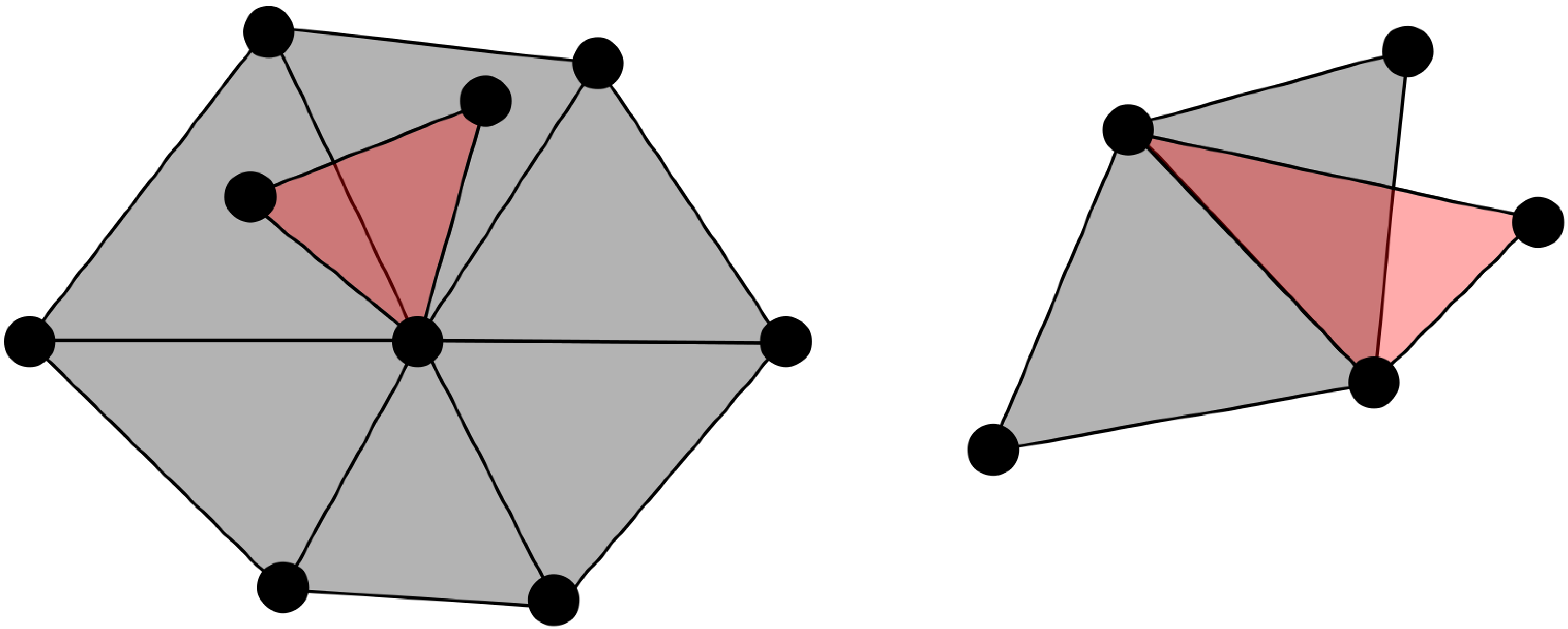
- In other words:
  - Each edge, E, is incident to only one or two faces!
  - The faces that are incident to a vertex form a closed or an open fan

Manifoldness Example



GOOD!

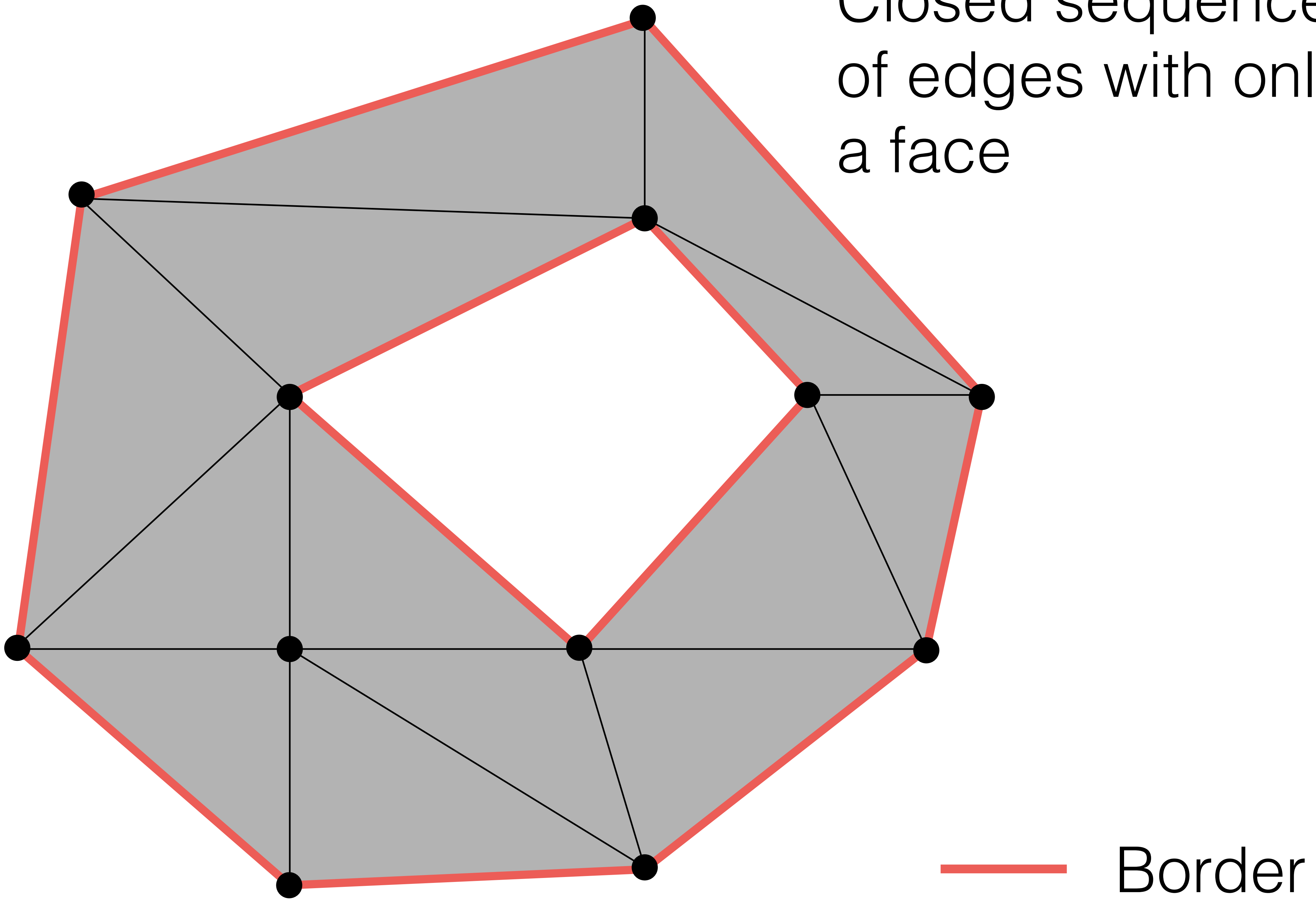
Manifoldness Example



BAD!

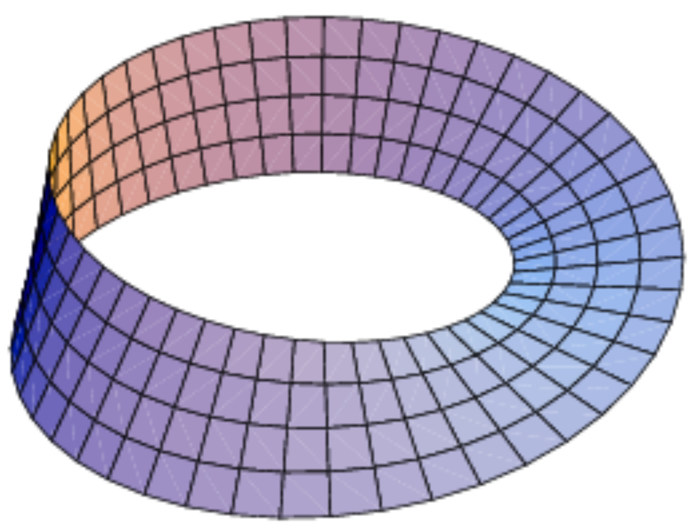
Borders

Closed sequence of edges with only a face

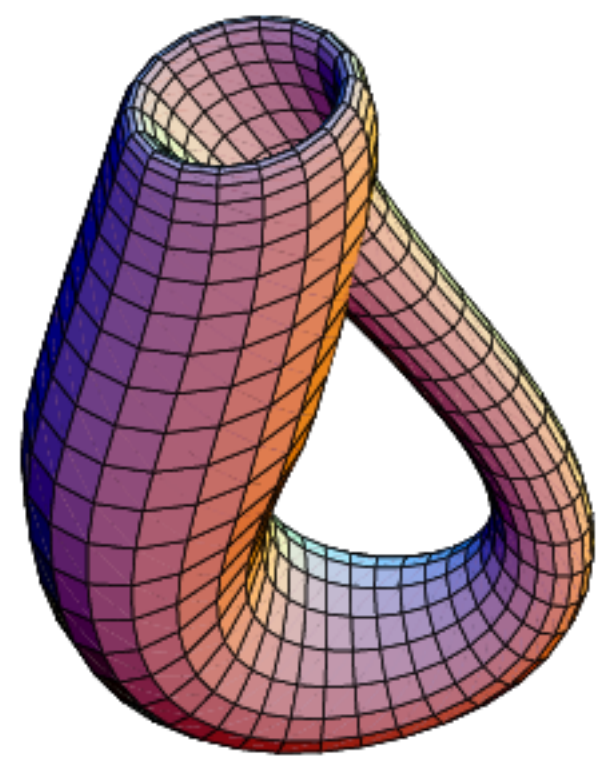


Orientability

- A surface, S, is orientable if it is possible to set a coherent normal to each point of the surface
- **NOTE**: Möbius strip and Klein bottle and non-manifold surfaces are not orientable:

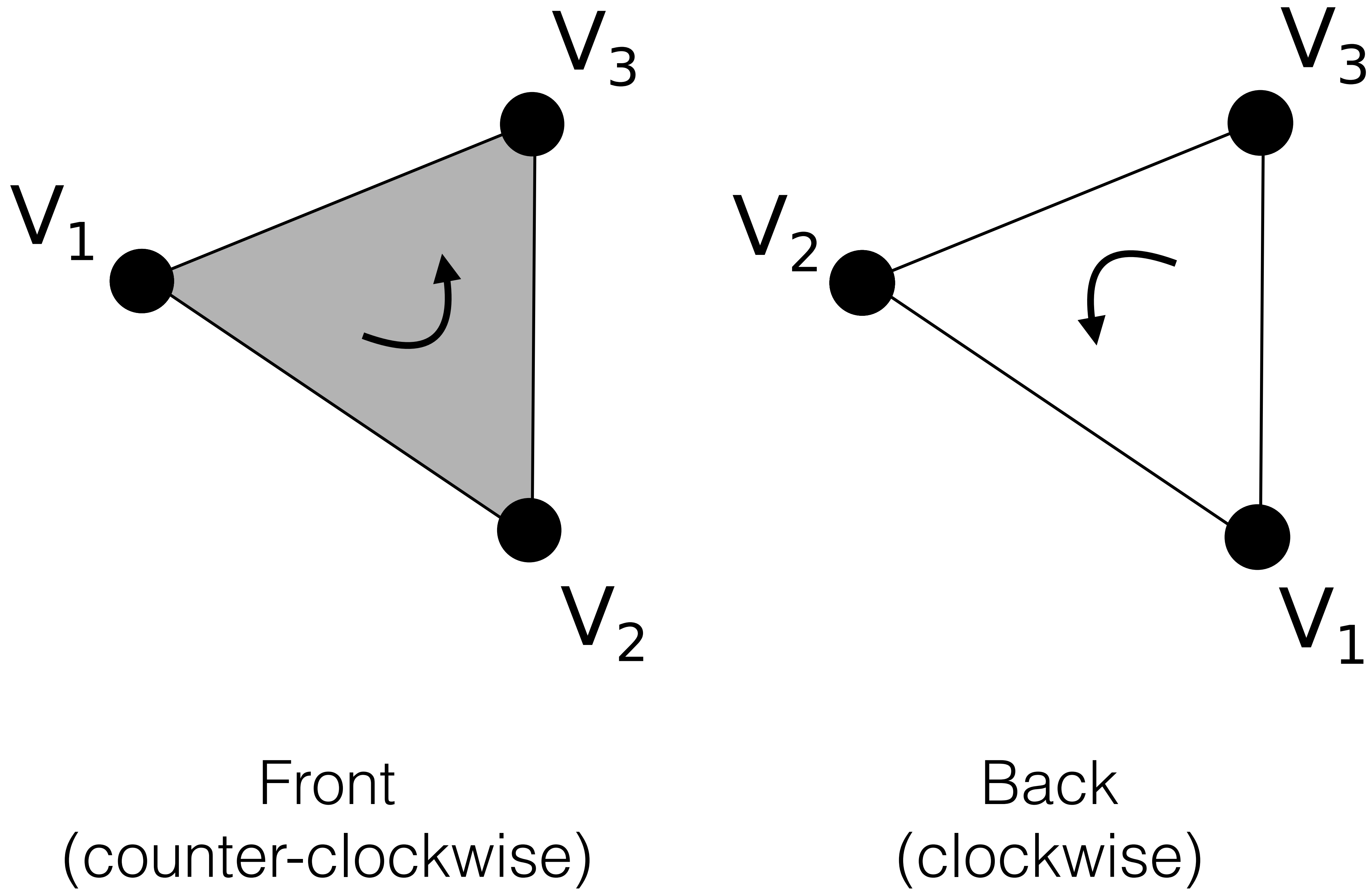


Möbius strip



Klein bottle

Orientability

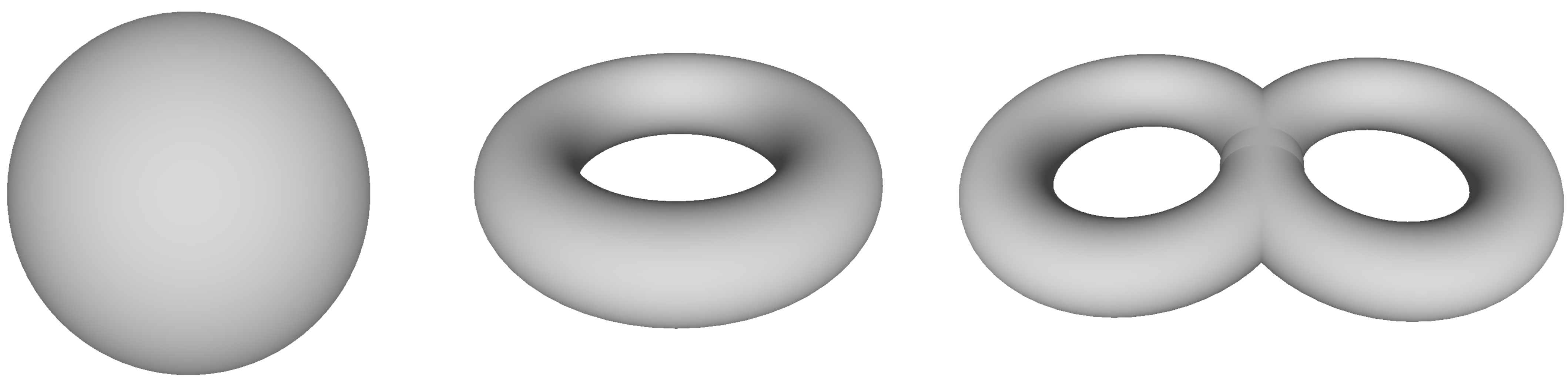


Mesh

- A mesh is maximal 2-simplicial complexes that is a 2-manifold orientable surface.
  - We can have non 2-manifold meshes
  - We assume that they are maximal

Genus

- The genus, G , is the maximum number of cuttings along non-intersecting closed simple curves without rendering the resultant manifold disconnected



0 1 2

- Genus —> “the number of handles”

Euler Characteristic

- Given V vertices, E edges, and F faces of a polygonal closed and orientable surface with genus G, we have:

$$2 - 2G = V - E + F$$

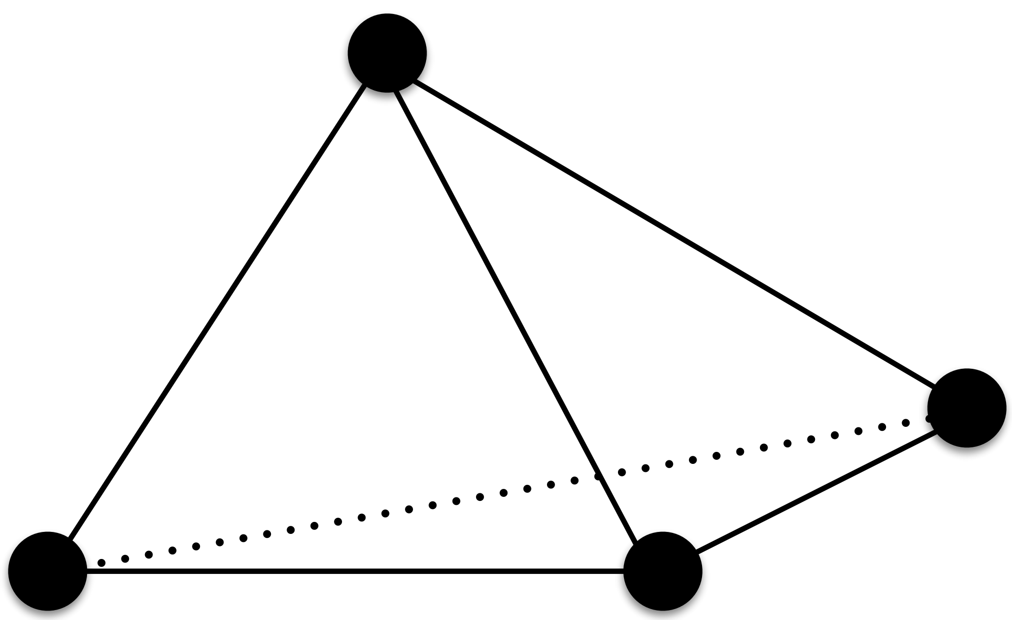
$$\chi = V - E + F$$

- More in general for a 2-manifold orientable polygonal mesh (with S connected components and B borders):

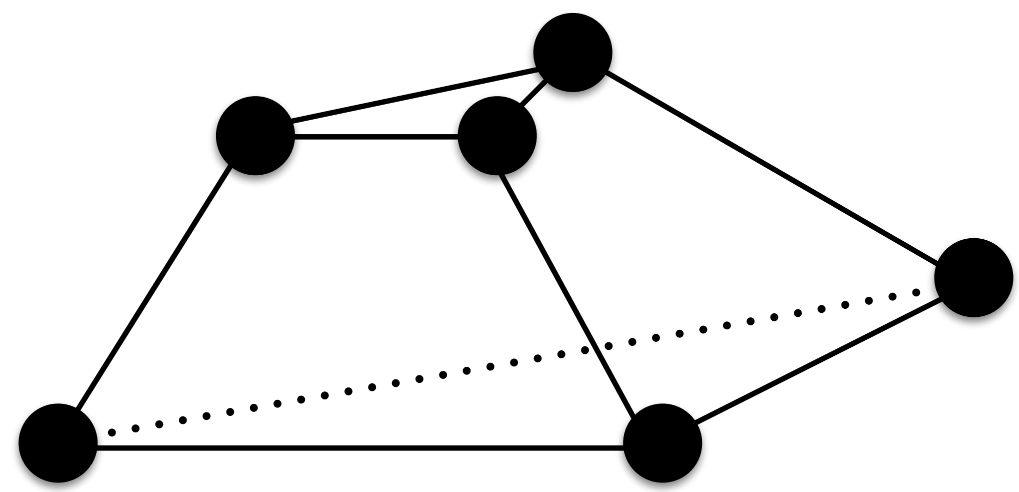
$$V - L + F = 2(S - G) - B$$

Euler Characteristic Example

The Euler characteristic is 2 for any simply connected polyhedron

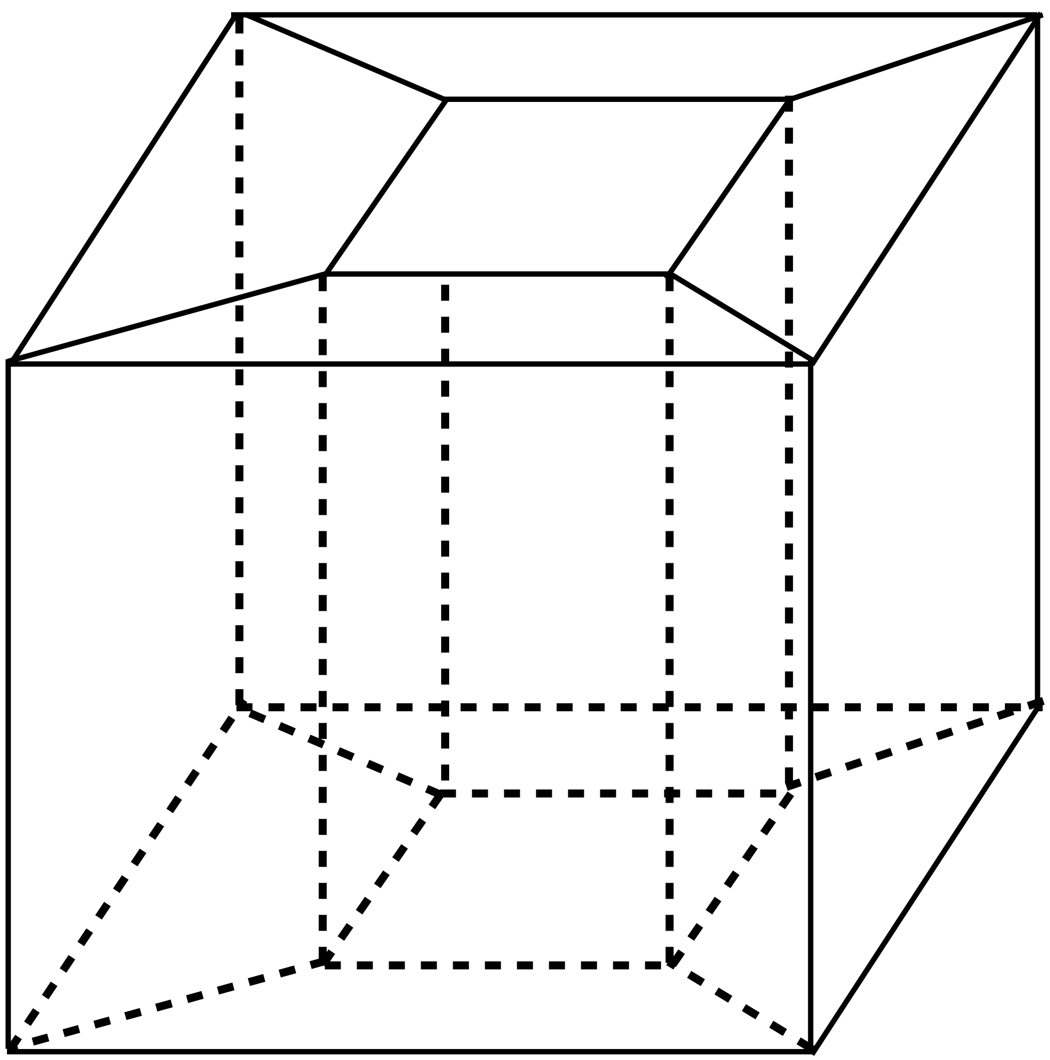


$$\chi = V - E + F$$
$$\chi = 4 - 6 + 4 = 2$$



$$\chi = V - E + F$$
$$\chi = 6 - 9 + 5 = 2$$

Euler Characteristic Example

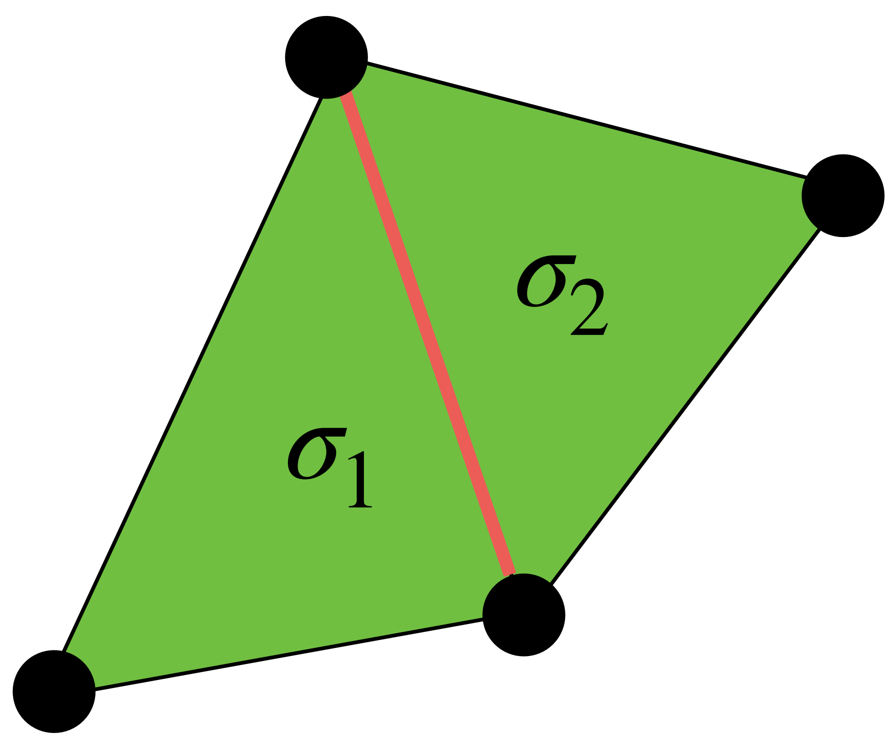


$$\chi = V - E + F$$

$$\chi = 16 - 32 + 16 = 0 = 2 - 2g$$

Adjacency Relations

- Given two simplices, σ_1 and σ_2, they are incident if σ_1 is a face of σ_2 or vice-versa:



Adjacency Relations

- Two k-simplices are m-adjacent ($k > m$) if a m-simplex exists such that it is a face of both.
- For example:
    - Two triangles sharing an edge are 1-adjacent
    - Two triangle sharing a vertex are 0-adjacent

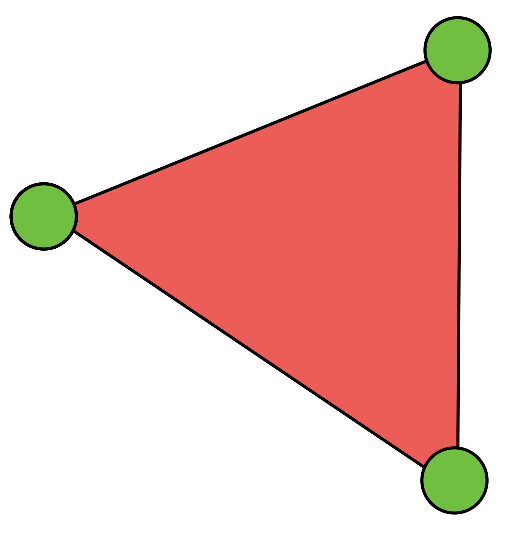
Adjacency Relations

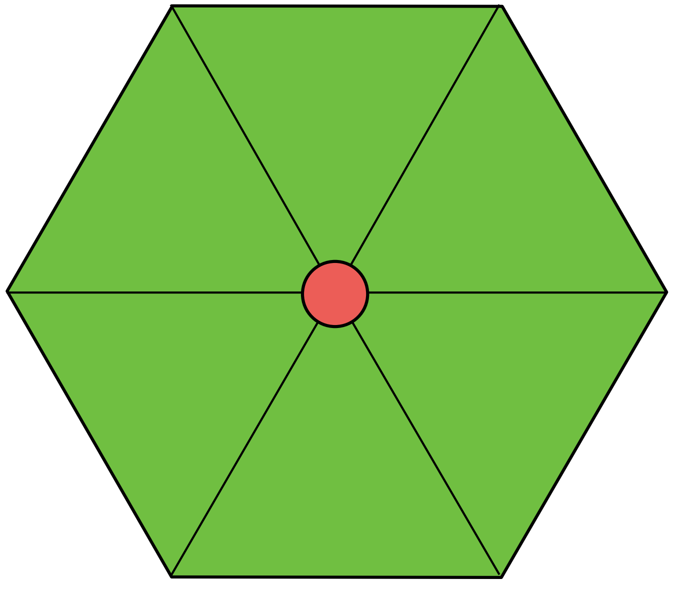
- An adjacency relations is an ordered couple of the following elements:
  - E —> edge
  - F —> Face
  - V —> Vertex
- For example: (E,E), (V,V), (F,F), (E,F), (F,E), (E,V), (V,E), (F,V), (V,F), (E,V), and (V,E).

Adjacency Relations Example

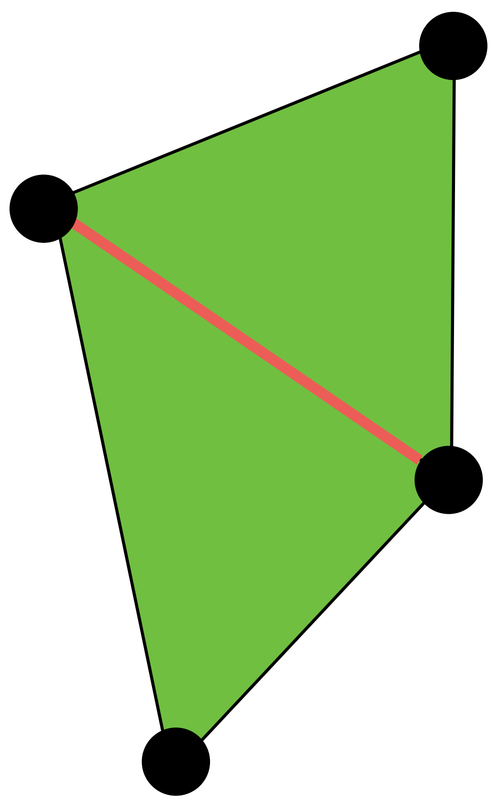
- Meaning of some relations:
  - FF —> adjacency between triangles
  - FV —> vertices of a triangle
  - VF —> triangles sharing a vertex

Adjacency Relations Example

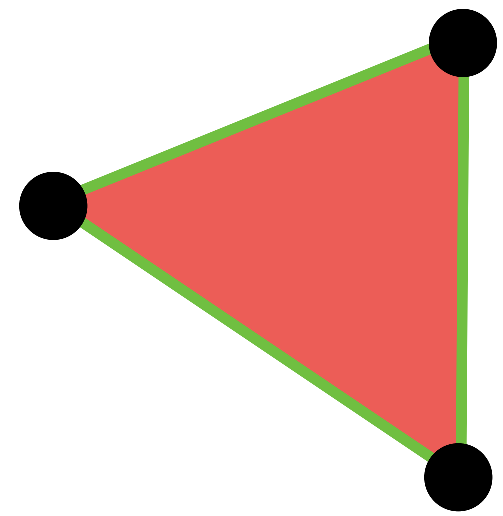




Adjacency Relations Example



EF

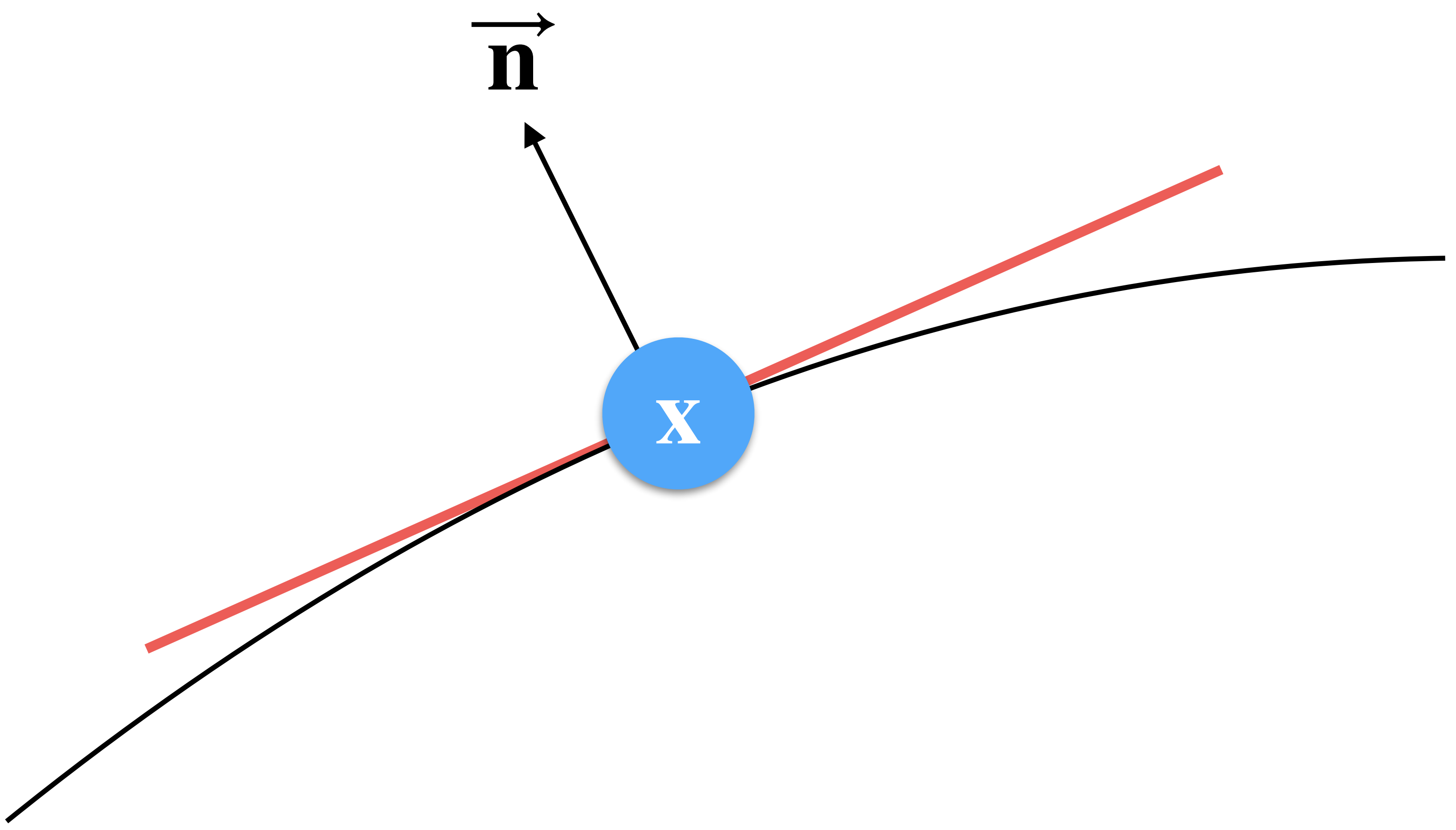


FE

Normals

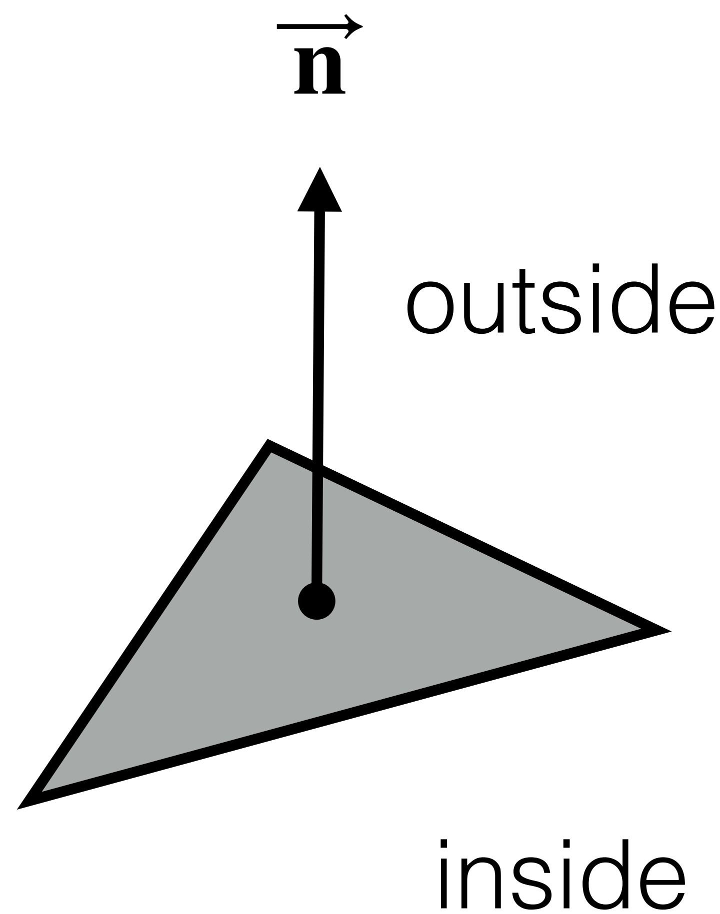
The Unit Normal

- The unit normal, $\vec{\mathbf{n}}$, to a point, $\mathbf{x}$, is the unit vector perpendicular to the tangent plane



The Unit Normal

- A normal is an important attribute for a vertex:
  - It defines the direction of the object boundary

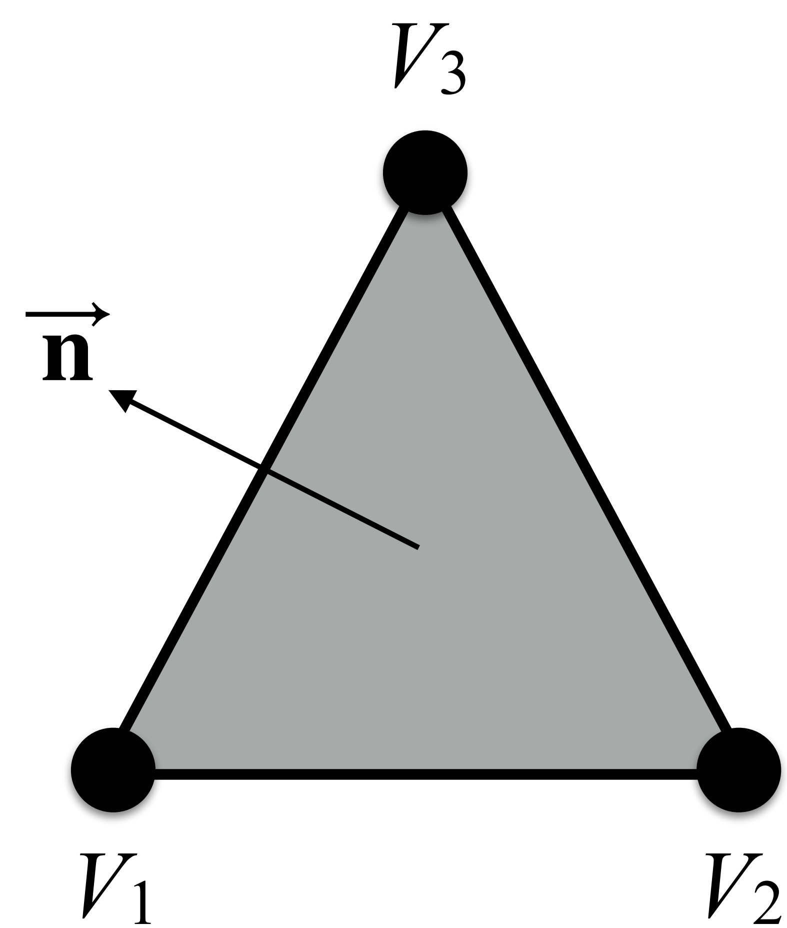


How to compute triangle normal?

- Given a triangle (V_1, V_2, and V_3), its normal (outer-pointing normal):

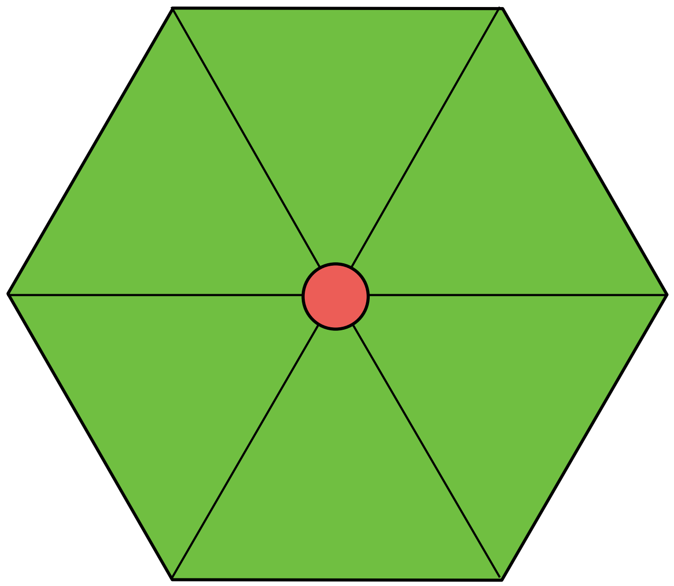
$$\overrightarrow{\mathbf{n}} = (V_3 - V_2) \times (V_1 - V_2)$$

$$\overrightarrow{\mathbf{n}} = \frac{\overrightarrow{\mathbf{n}}}{\|\overrightarrow{\mathbf{n}}\|}$$



- This means that vertices order is important! Typically is counter-clockwise

How to compute per vertex normal?



- We compute normals for each triangle
- For each vertex:
  - We compute the sum of normals of all triangles sharing that vertex:

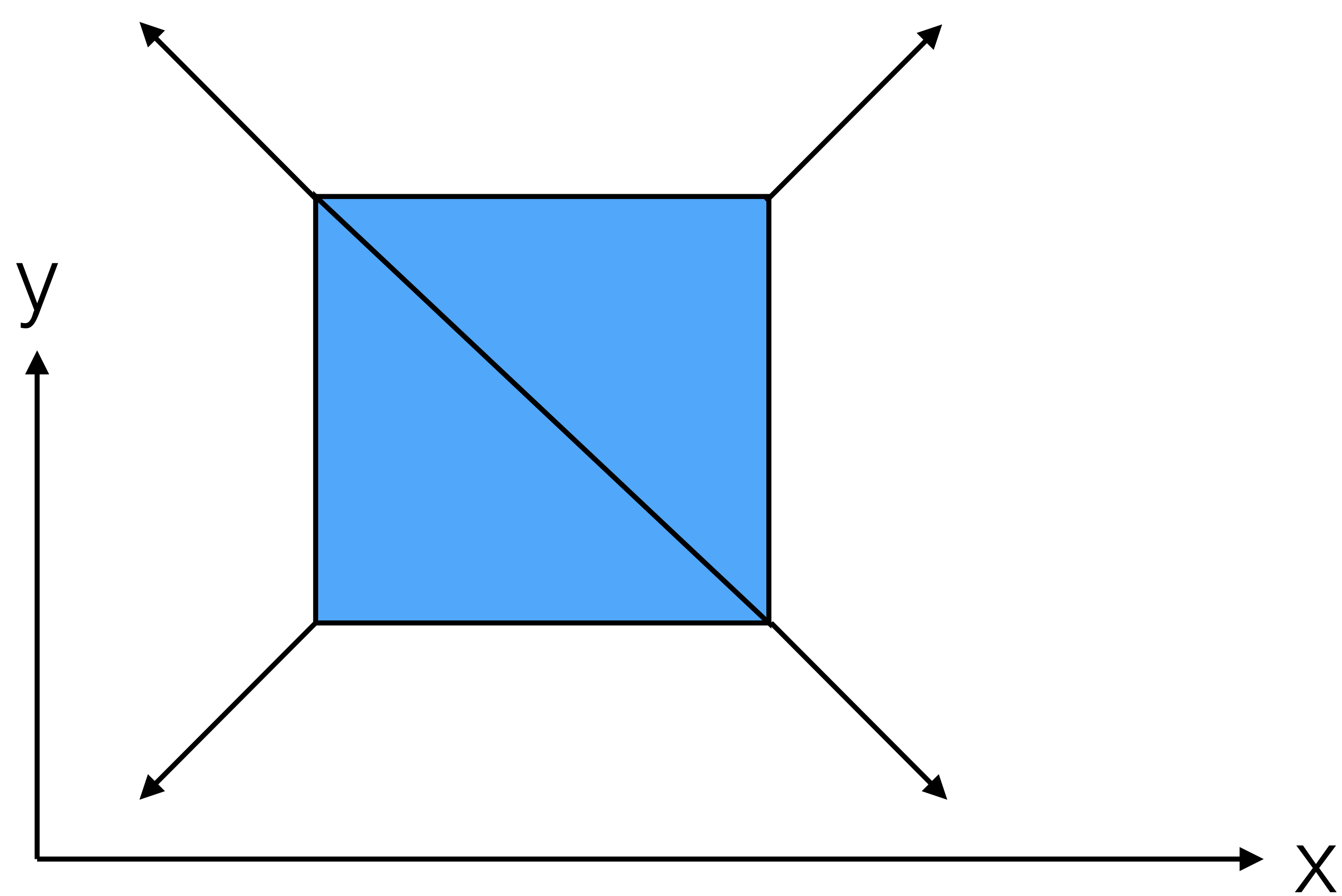
$$\vec{n}_s(V) = \sum_{\{i|V \in T_i\}} \vec{n}_{T_i}$$

  - We normalize this sum
- **Note**: per-vertex normals are useful but not correct!

VF

How to compute per vertex normal?

- Problems:
  - We may end up with a null vector $\mathbf{n}_i = [0,0,0]^\mathsf{T}$.
  - If the model does not have too many triangles we may have a poor result. For example, for this cube (top view):

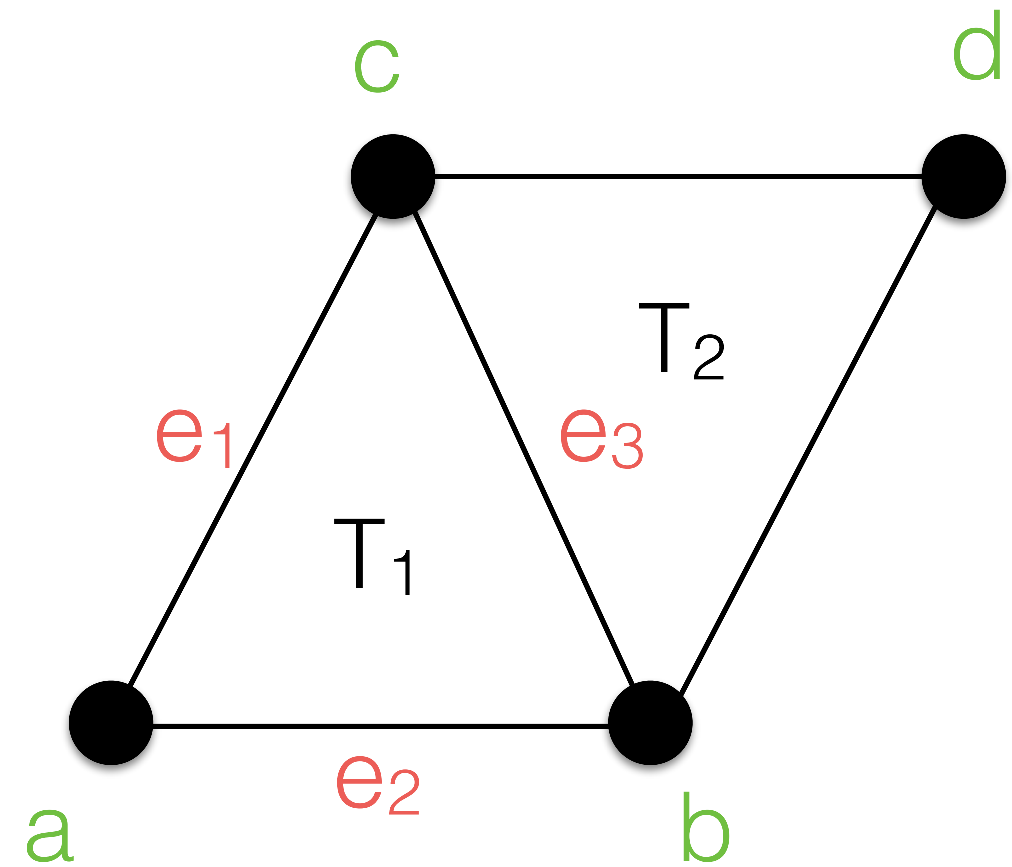


Data Structures
for
3D Meshes

List of Triangles

- For each triangle of the 3D model, we store its coordinates.

- For example:



Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)
Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)
Triangle 3: (1,2,-2); (3,-2,5); (-6,2,4)
....
Triangle *n*: (-8,2,7); (-2,3,9); (1,2,-7)

What's ***very wrong*** with this??

Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)
Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)
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....
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What's ***very wrong*** with this??

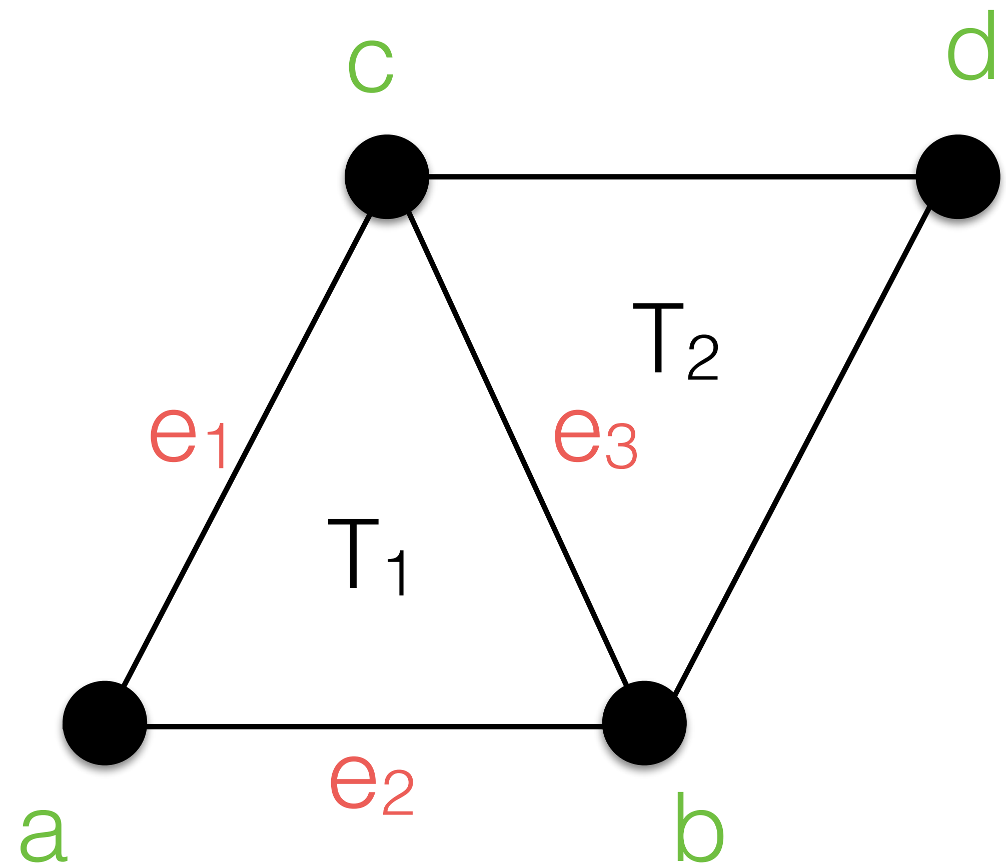
Triangle 1: (**3,-2,5**); (**2,2,4**); (**-6,-2,4**)
Triangle 2: (**2,2,4**) ; (0,-1,-2); (9,4,0)
Triangle 3: (1,2,-2); (**3,-2,5**); (**-6,-2,4**)
....
Triangle *n*: (-8,2,7); (-2,3,9); (1,2,-7)

List of Triangles

- Disadvantages:
  - Wasted disk and memory space:
    - Vertices are duplicated!
    - Memory: $|V| \times |T|$
  - Difficult to manage:
    - if we modify a vertex of a triangle, we will need to find and update its clones!
  - How do we query neighbors?

List of Unique Vertices

- We store vertices in a list
- For each triangle of the 3D model, we store indices to the vertices' list



Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)
5. (1.0, -1.0, -1.0)
6. (1.0, -1.0, 1.0)
7. (1.0, 1.0, -1.0)
8. (1.0, 1.0, 1.0)

Faces:

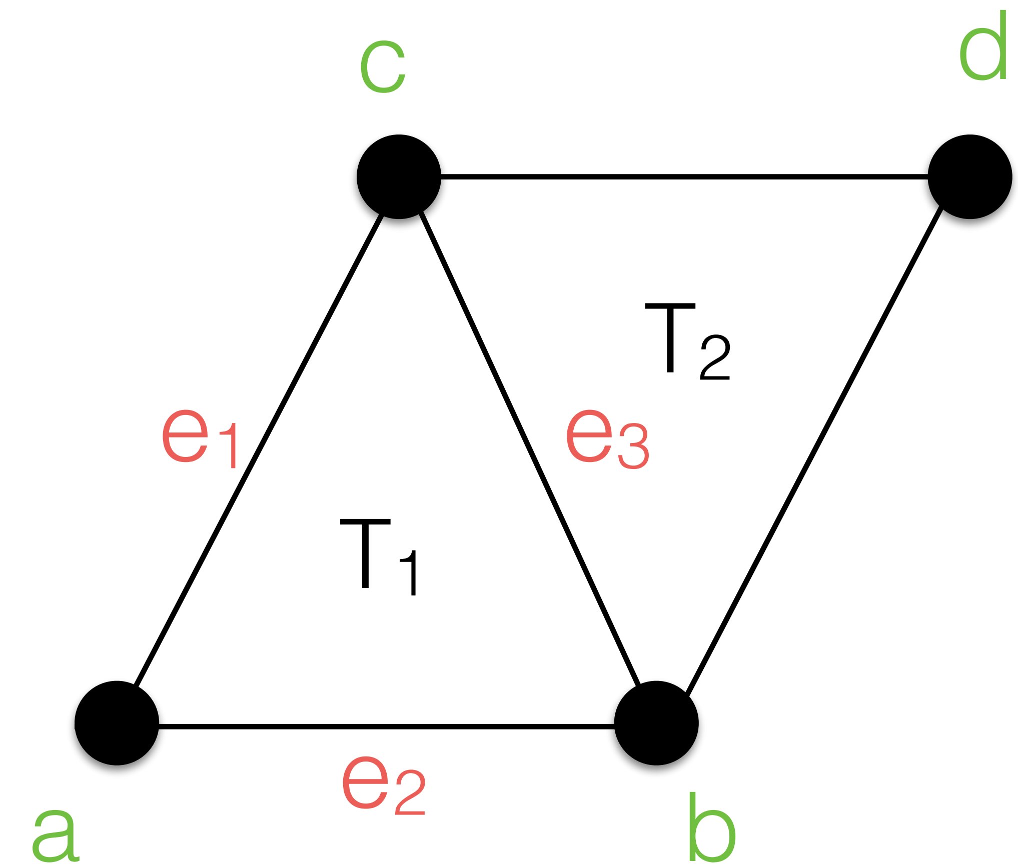
1. 1 2 4
2. 5 7 6
3. 1 5 2
4. 3 4 7
5. 1 7 5

List of Unique Vertices

- Wasted disk and memory space:
  - Common edges between two triangles are stored two times in the list of faces!
  - Memory: $|V| + |T|$
- Better management:
  - Easy to edit a vertex's attribute (e.g., its position)!
- How do we query neighbors?

List of Unique Edges

- We store vertices in a list
- For each edge, we store indices to the vertices' list
- For each triangle of the 3D model, we store indices to edges's list



Vertices:
1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)

Edges:
1. 1 2
2. 2 3
3. 4 2
4. 3 4
5. 1 3

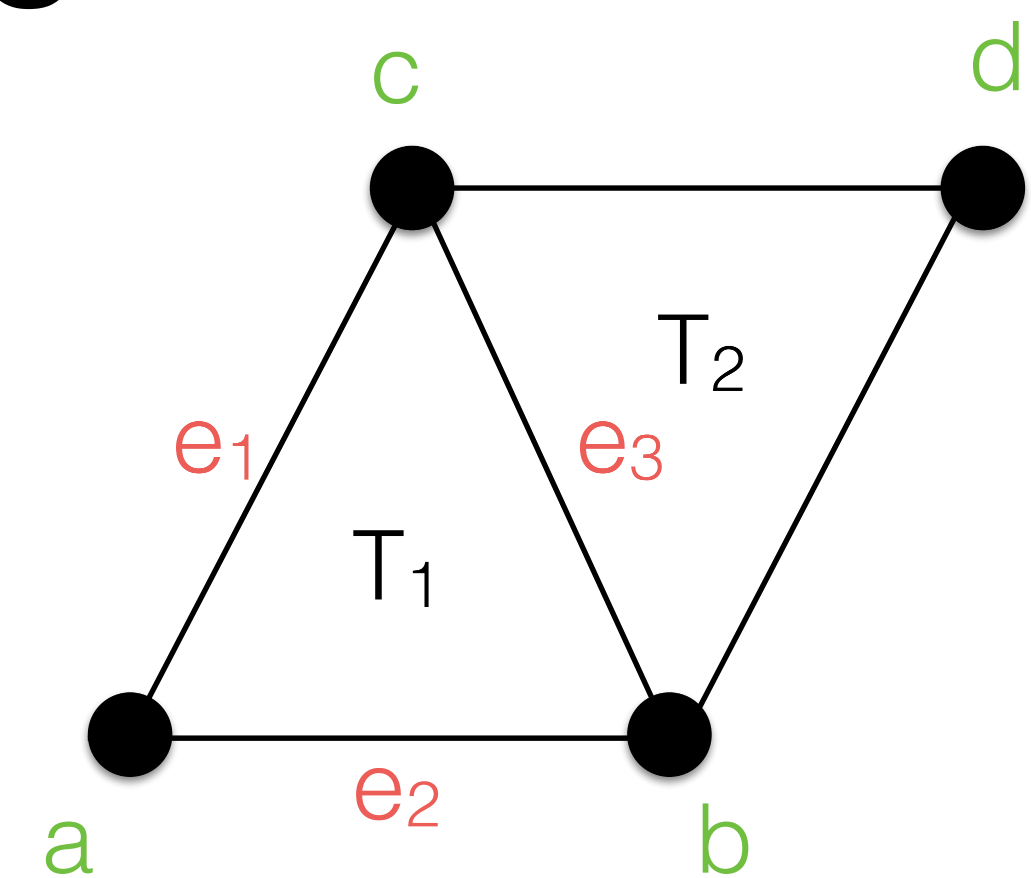
Faces:
1. 1 2 5
2. 2 4 3

List of Unique Edges

- Better management:
  - Easy to edit an edge’s attribute (e.g., its color)!
- We can do some queries, but not all of them!

Extended List of Unique Edges

- We add to an edge the indices of its left and right triangle
- This simplifies edge-face queries!



Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)

Edges:

1. 1 2
2. 2 3
3. 4 2
4. 3 4
5. 1 3

Faces:

1. -1 1
2. 1 2
3. -1 2
4. -1 2
5. 1 -1

Faces:

1. 1 2 5
2. 2 4 3

File Formats

File Formats

- There are many 3D file formats. The most used, and de-facto standard:
  - STL
  - PLY
  - OBJ
- Standards:
  - COLLADA: https://www.khronos.org/collada/
  - X3D: http://www.web3d.org/x3d/

STL File Format

- Standard Triangle Language (STL) created by 3D Systems
- This format represents only the 3D geometry:
  - No color/texture
  - No other attributes
- The format specifies both ASCII and binary representations

STL File Format

- Data structure: list of triangles
- Vertices are ordered using the right-hand rule
- 3D coordinates must be positive
- No scale metadata; i.e., units are arbitrary

STL File Format

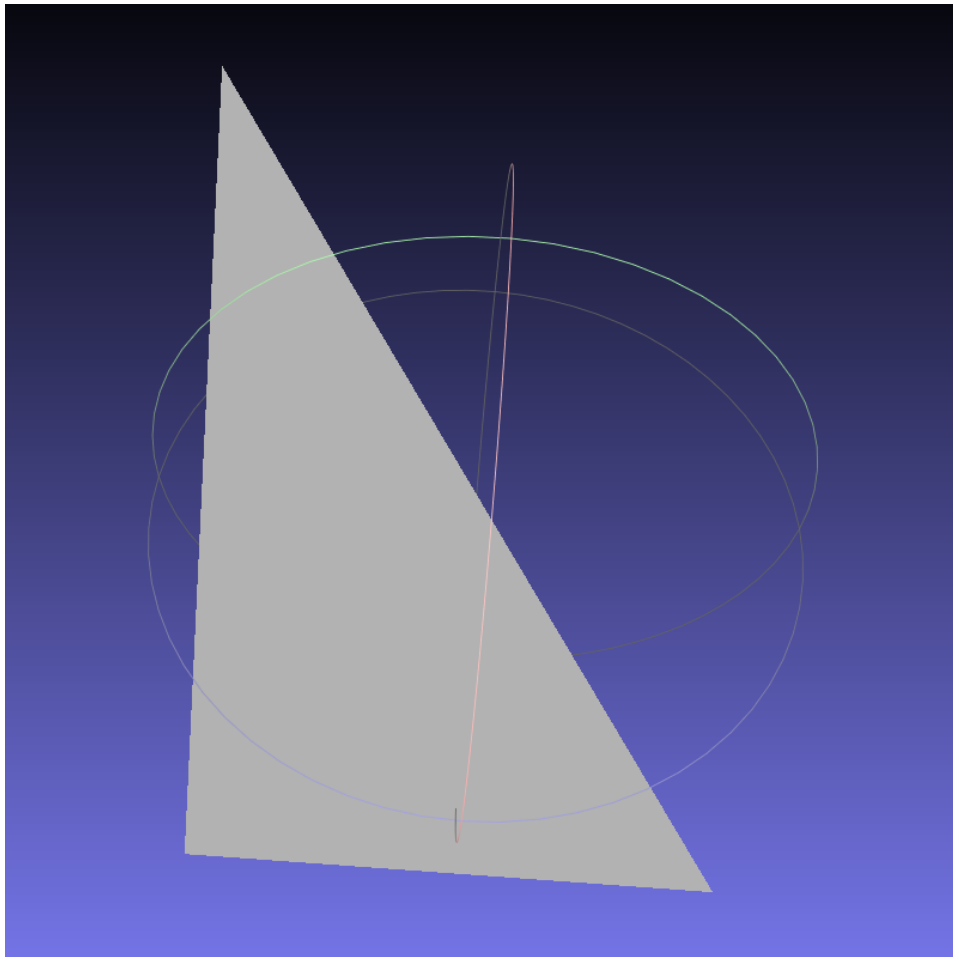
- The file begins as

```
solid name
```

- A face is defined as

```
facet normal nx ny nz
    outer loop
        vertex v1x v1y v1z
        vertex v2x v2y v2z
        vertex v3x v3y v3z
    endloop
endfacet
```

STL File Format: An Example



```
solid triangle
facet normal 0 1 0
        outer loop
                vertex 0.0 0.0 0.0
                vertex 1.0 0.0 0.0
                vertex 0.0 1.0 1.0
        endloop
endfacet
endsolid triangle
```

PLY File Format

- Polygon File Format (PLY) is a popular format created by Stanford University (Greg Turk)
- The format is very flexible:
  - we can add many attributes
  - we can define triangular and polygonal meshes
- The format specifies both ASCII and binary representations

PLY File Format

- Data structure: list of unique vertices
- No scale metadata; i.e., units are arbitrary
- The file is divided into two parts:
    - **Header** that specifies vertices and faces
    - **Body** that specifies the concrete data

PLY File Format: Header

- The file begins as

```
ply
format ascii 1.0
```

- Vertex specification is defined as

```
element vertex num_vertices
property float x
property float y
property float z
```

properties can be: char, uchar, short, ushort, int, uint float, double, etc.

PLY File Format: Header

- Faces are defined as

```
element face num_faces
property list uchar int vertex_indices

end_header
```

PLY File Format: Body

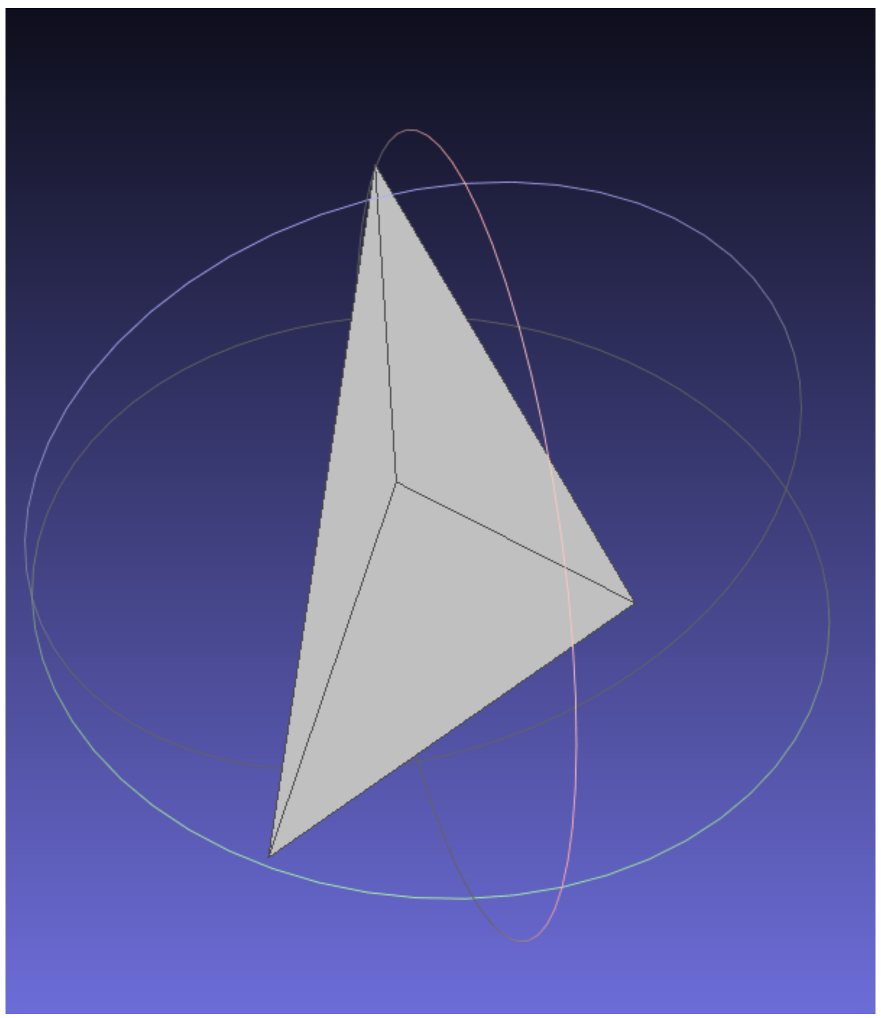
- Each i-th vertex is specified as

```
vix viy viz
```

- Each face is specified as

```
3 index_v1 index_v2 index_v2
```

PLY File Format: An Example



```
ply
format ascii 1.0
element vertex 4
property float x
property float y
property float z
element face 4
property list uchar int vertex_indices
end_header
-0.60 -0.97 0.37
-0.34 0.98 0.76
0.037 0.65 -1.06
0.88 -0.75 -0.25
3 1 3 2
3 0 1 2
3 0 3 1
3 3 0 2
```

Acknowledgements

- Some images and text are based on work by:
  - Dr. Paolo Cignoni:
    - http://vcg.isti.cnr.it/~cignoni/