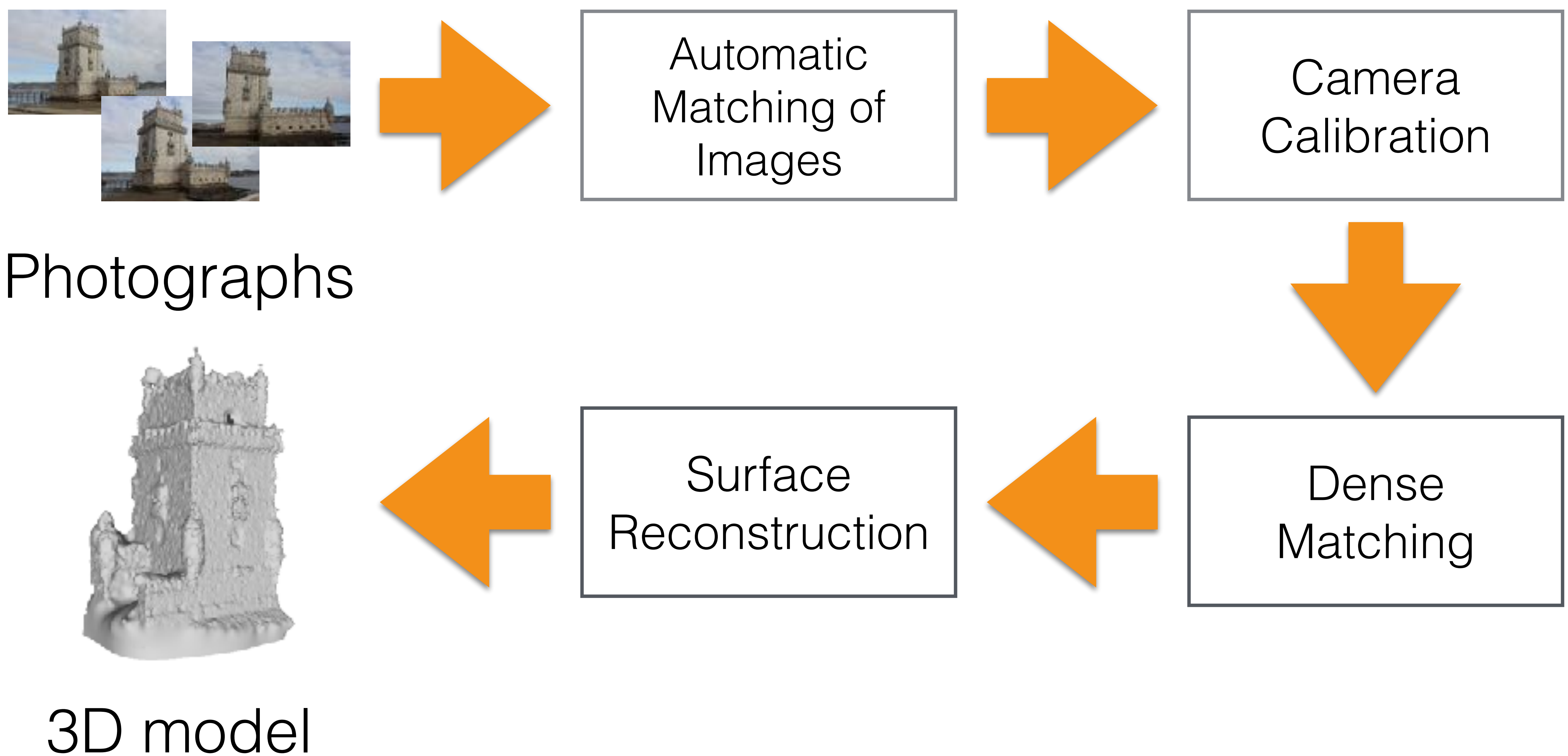
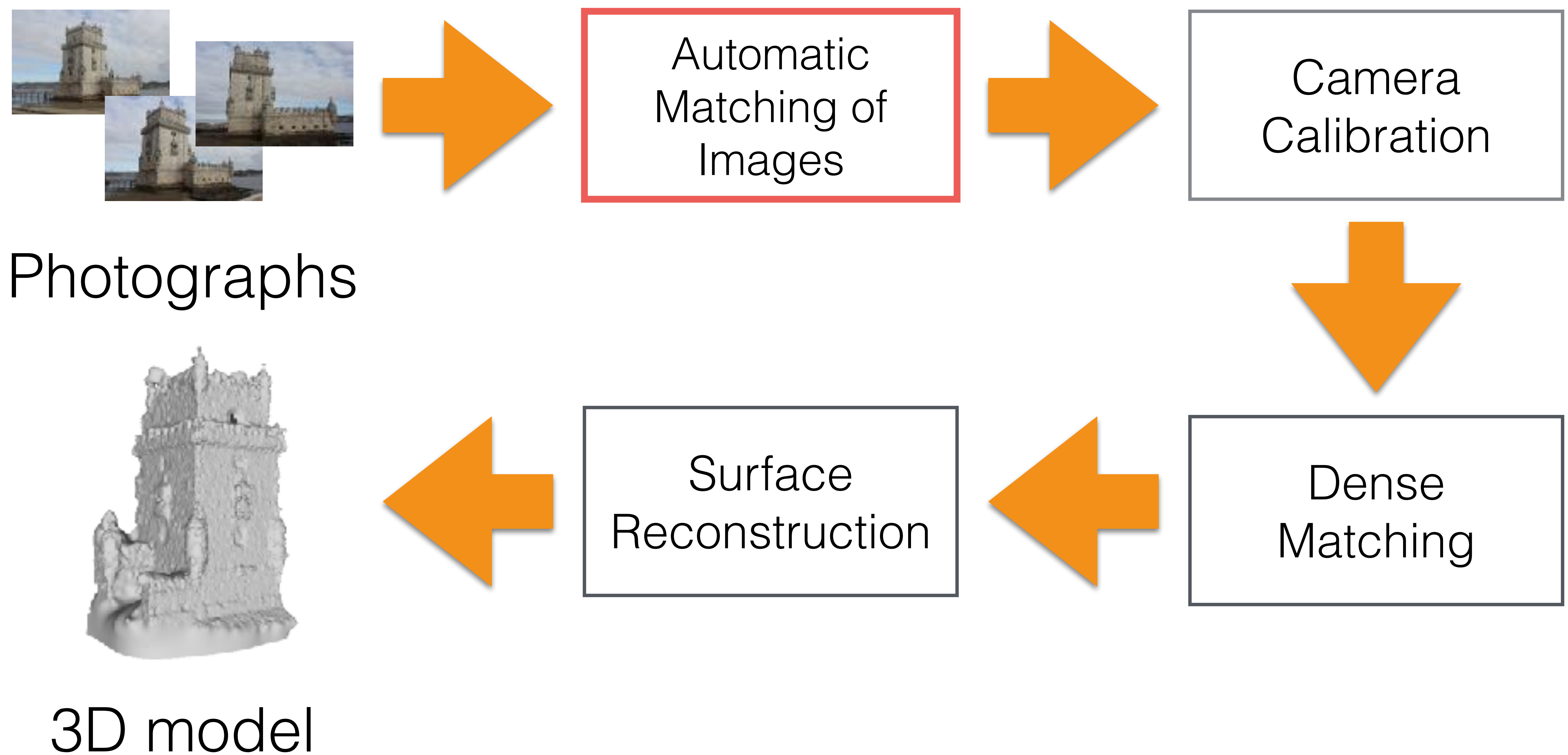
3D from Photographs: Automatic Matching of Images

Francesco Banterle, Ph.D.
francesco.banterle@isti.cnr.it

3D from Photographs

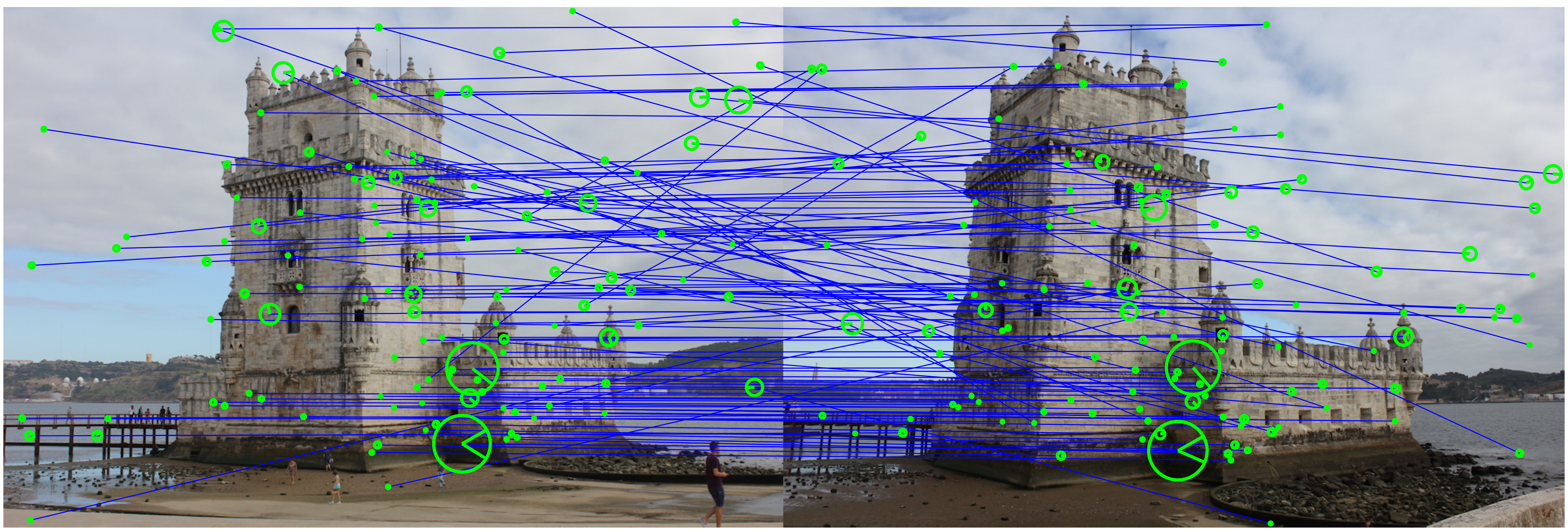


3D from Photographs



The Matching Problem

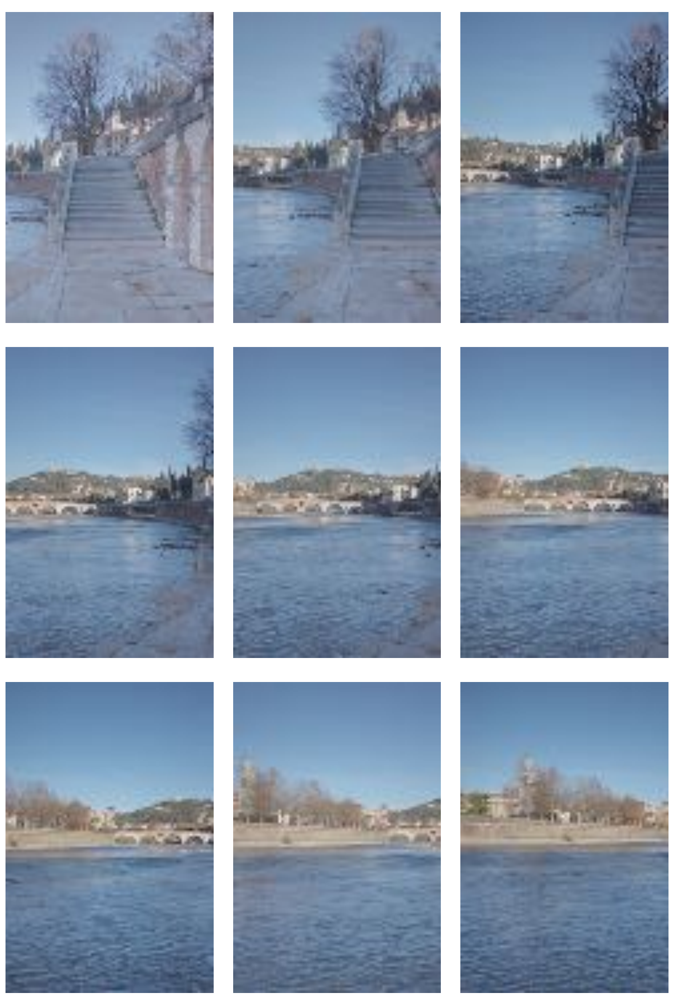
- We need to find corresponding feature across two or more views:



The Matching Problem

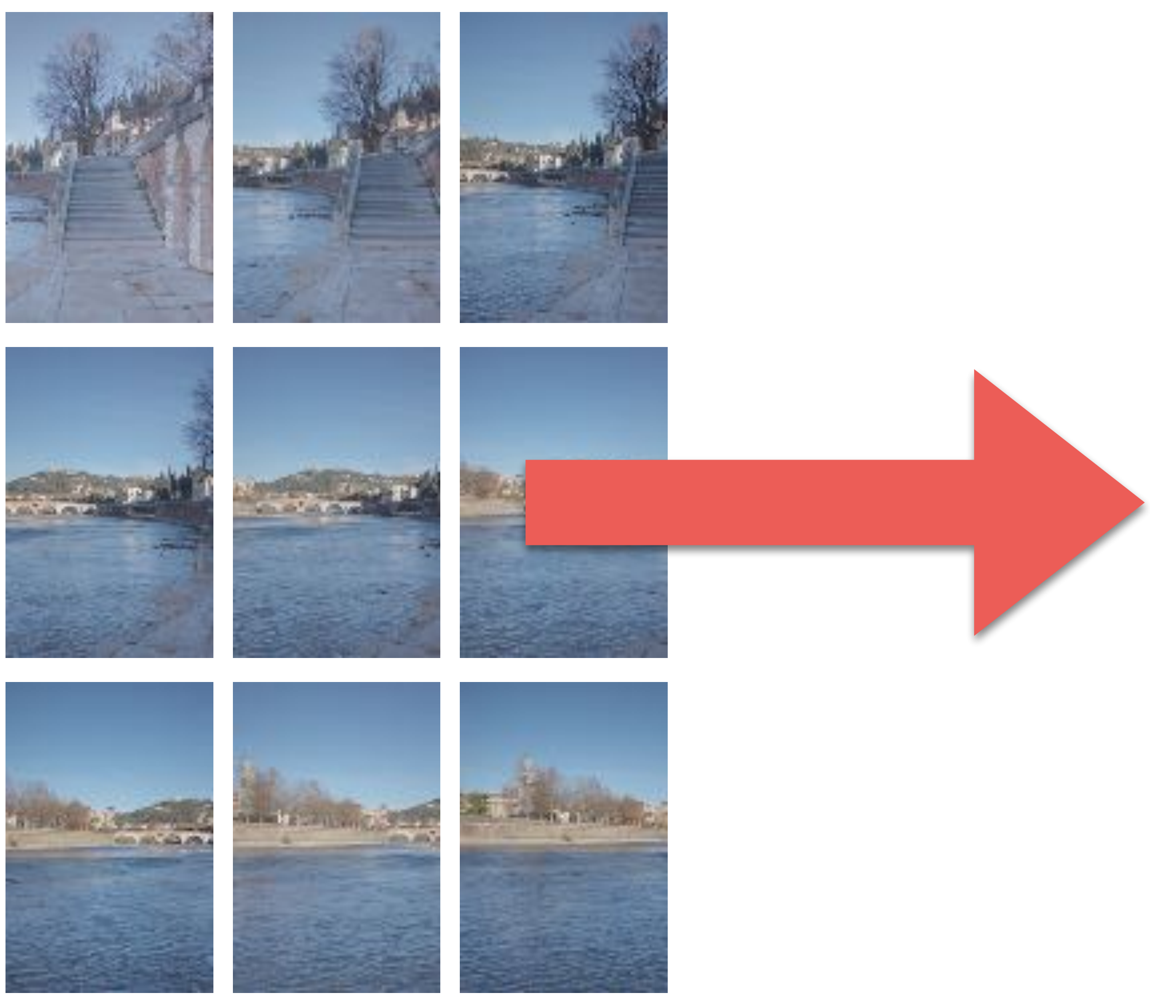
- Why?
  - 3D Reconstruction.
  - Image Registration.
  - Visual Tracking.
  - Object Recognition.
  - etc.

The Matching Problem: Automatic Panorama Generation



Input
Photographs

The Matching Problem: Automatic Panorama Generation



Input
Photographs

The Matching Problem: Automatic Panorama Generation

Input Photographs

Panorama

Extraction of Features

Features

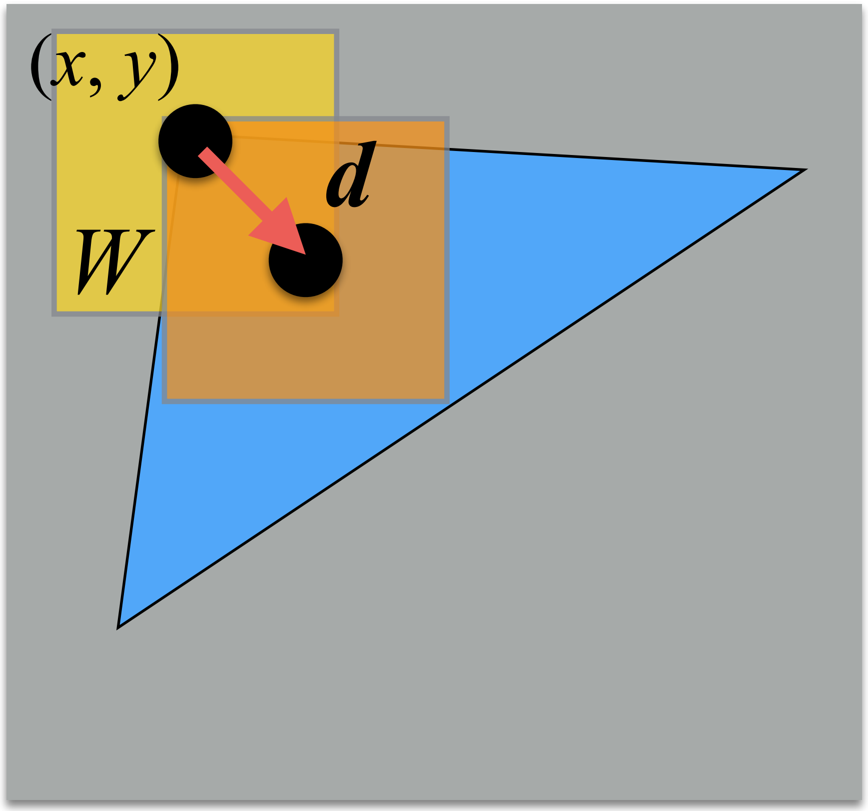
- A feature is a piece of the input image that is relevant for solving a given task.
- Features can be global or local.
- We will focus on local features that are more robust to occlusions and variations.

Extraction of Local Features

- We can extract different kind of features:
    - Flat regions or Blobs
    - Edges
    - Corners

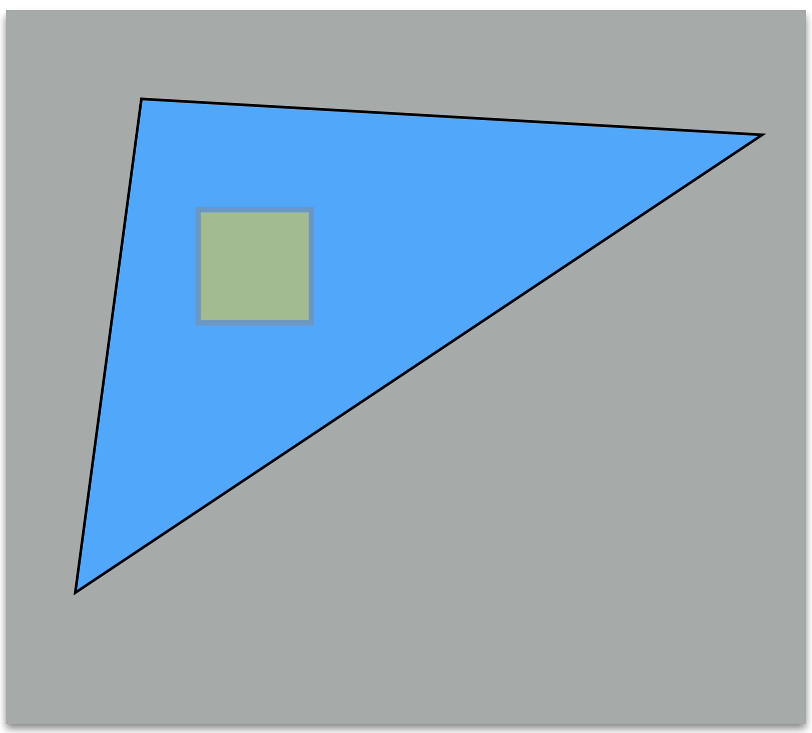
Harris Corner Detector

- Let's consider a window W centered in (x, y):
  - how do pixels change from a window in (x, y) to another one with a shift $\mathbf{d} = (u, v)$?
  - Let's compare each pixel before and after moving W by $\mathbf{d} = (u, v)$ using the sum of squared differenced (SSD).

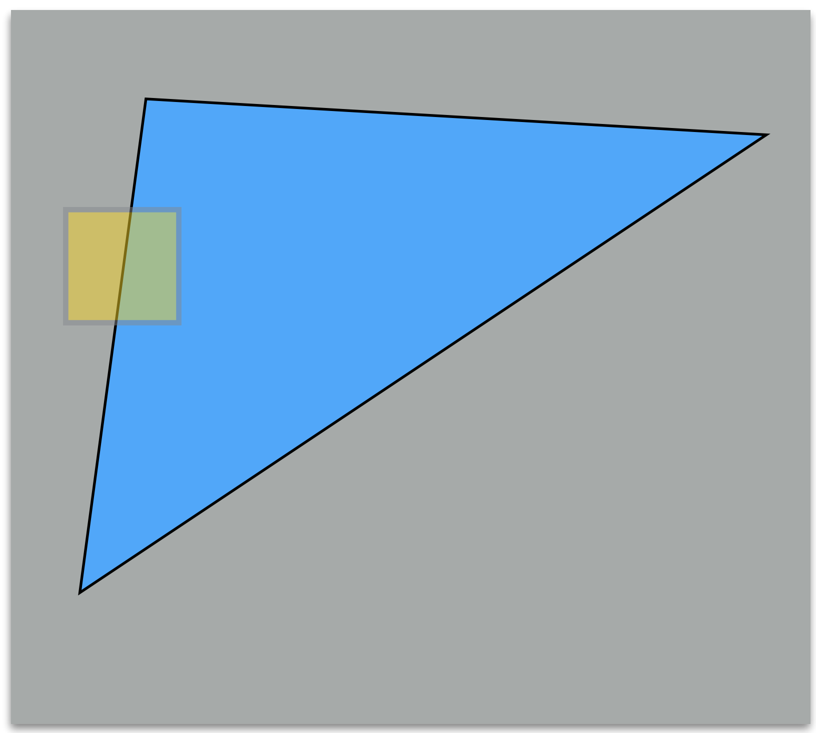


$$E(x, y) = \sum_{x_k, y_k \in W(x,y)} \Big(I(x_k + u, y_k + v) - I(x_k, y_k) \Big)^2$$

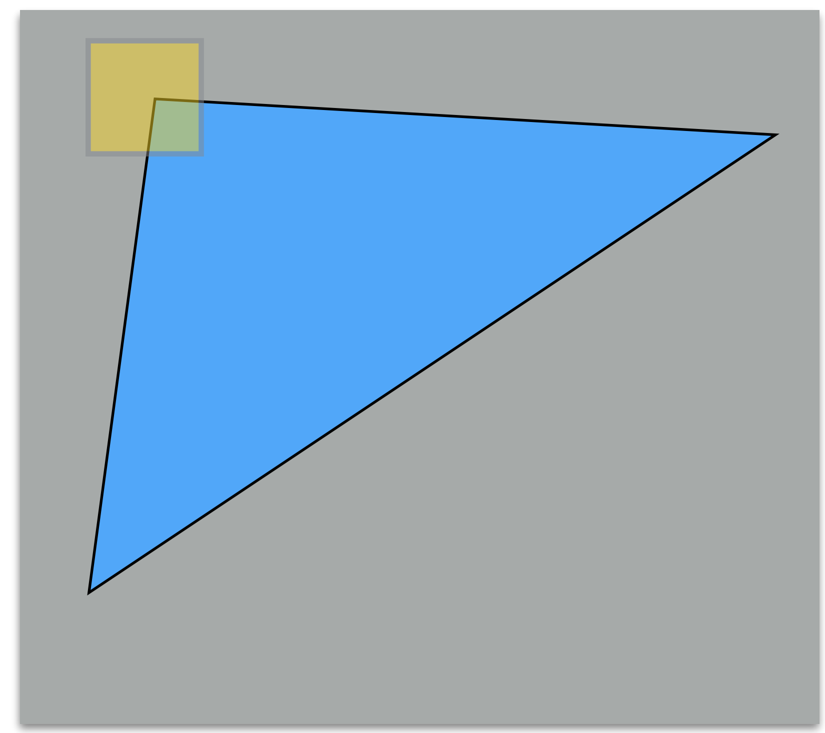
What a Corners is



Flat Region:
no change
in all directions.



Edge:
no change
along the edge.



Corner:
significant change
in all directions.

Harris Corner Detector: Small Motion Assumption

- Let's apply a first-order approximation, which provides good results for small motions:

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

Harris Corner Detector: Small Motion Assumption

$$
\begin{aligned}
E(x,y) &= \sum_{x_k, y_k \in W(x,y)} \Big(I(x_k + u, y_k + v) - I(x_k, y_k) \Big)^2 \\
&\approx \sum_{x_k, y_k \in W(x,y)} \Big(I(x_k, y_k) + I_x(x_k, y_k)u + I_y(x_k, y_k)v - I(x_k, y_k) \Big)^2 \\
&= \sum_{x_k, y_k \in W(x,y)} \Big(I_x(x_k, y_k)u + I_y(x_k, y_k)v \Big)^2 \\
&= \sum_{x_k, y_k \in W(x,y)} \Big(I_x(x_k, y_k)^2 u^2 + 2 I_x(x_k, y_k) I_y(x_k, y_k) + I_y(x_k, y_k)^2 v^2 \Big)
\end{aligned}
$$

Harris Corner Detector: Small Motion Assumption

$$E(x,y) \approx \sum_{x_k, y_k \in W(x,y)} \left(I_x(x_k, y_k)^2 u^2 + 2I_x(x_k, y_k)^2 I_y(x_k, y_k) uv + I_y(x_k, y_k)^2 v^2 + \right) = Au^2 + 2Buv + Cv^2$$

$$A = \sum_{x_k, y_k \in W(x,y)} I_x(y_k, x_k)^2$$

$$B = \sum_{x_k, y_k \in W(x,y)} I_x(y_k, x_k) I_y(y_k, x_k)$$

$$C = \sum_{x_k, y_k \in W(x,y)} I_y(y_k, x_k)^2$$

Harris Corner Detector: Small Motion Assumption

- The surface at (x, y) can be locally approximate by a quadratic form:

$$E(x, y) \approx Au^2 + 2Buv + Cv^2 \approx [u \quad v] \cdot \begin{bmatrix} A & B \\ B & C \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x_k, y_k \in W(x,y)} I_x(y_k, x_k)^2$$

$$B = \sum_{x_k, y_k \in W(x,y)} I_x(y_k, x_k) I_y(y_k, x_k)$$

$$C = \sum_{x_k, y_k \in W(x,y)} I_y(y_k, x_k)^2$$

Harris Corner Detector: Small Motion Assumption

- $E(x,y)$ can be rewritten as:

$$E(x,y) \approx \sum_{x_k, y_k \in W(x,y)} [u \quad v] \cdot \begin{bmatrix} I_x^2(x_k, y_k) & I_x(x_k, y_k) I_y(x_k, y_k) \\ I_x(x_k, y_k) I_y(x_k, y_k) & I_y^2(x_k, y_k) \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} =$$

$$= [u \quad v] \cdot \mathbf{M} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\mathbf{M} = \sum_{x_k, y_k \in W(x,y)} \begin{bmatrix} I_x^2(x_k, y_k) & I_x(x_k, y_k) I_y(x_k, y_k) \\ I_x(x_k, y_k) I_y(x_k, y_k) & I_y^2(x_k, y_k) \end{bmatrix}$$

Harris Corner Detector: Small Motion Assumption

- $E(x,y)$ can be rewritten as:

$$E(x,y) \approx \sum_{x_k,y_k \in W(x,y)} [u \quad v] \cdot \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} =$$

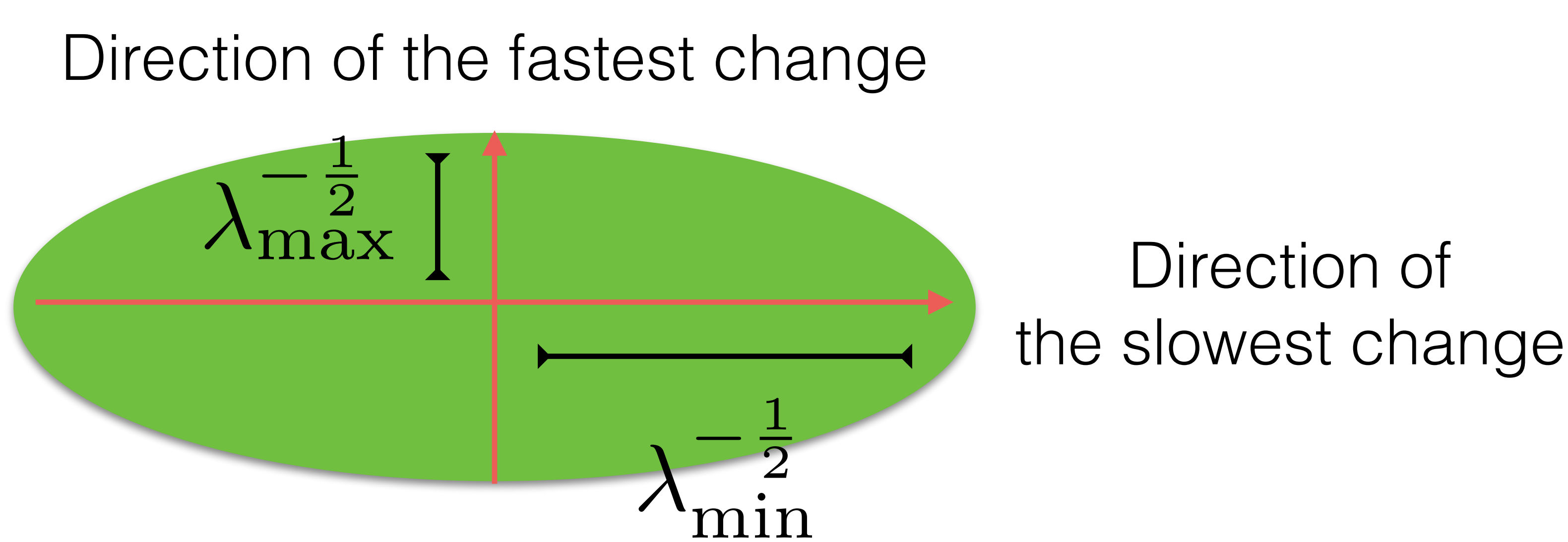
$$= [u \quad v] \cdot \mathbf{M} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

Ellipse Equation:
$E(u,v) = k$

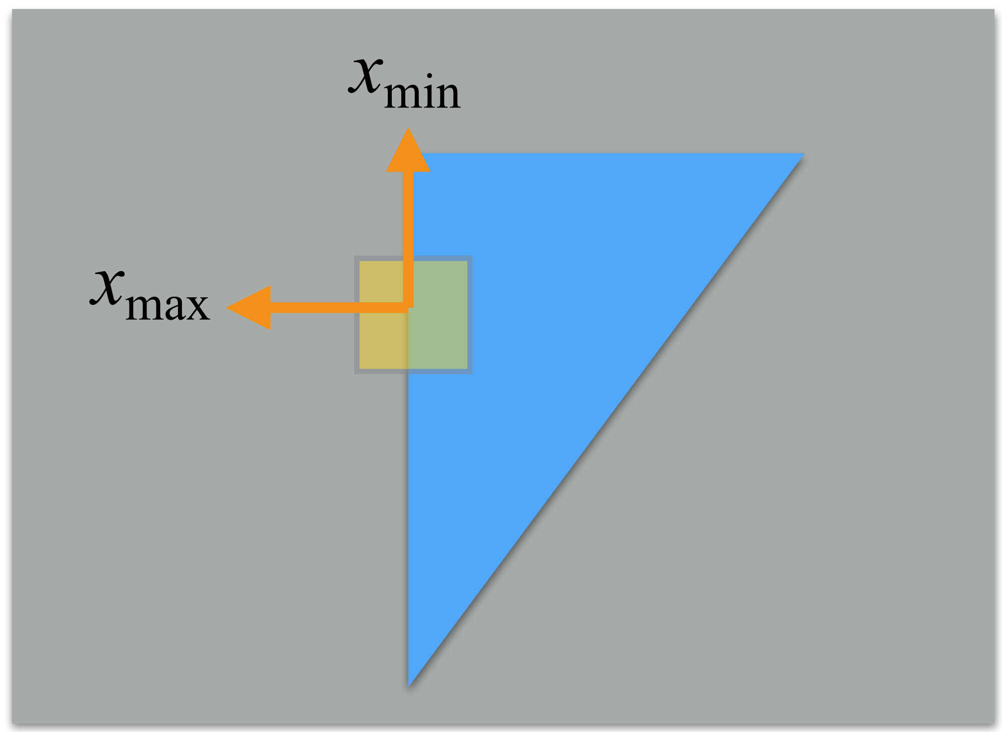
$$\mathbf{M} = \sum_{x_k,y_k \in W(x,y)} \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix}$$

Harris Corner Detector: Second Moment Matrix

- $\mathbf{M}$ reveals information about the distribution of gradients around a pixel.
- The eigenvectors of $\mathbf{M}$ identify the directions of fastest and slowest change.



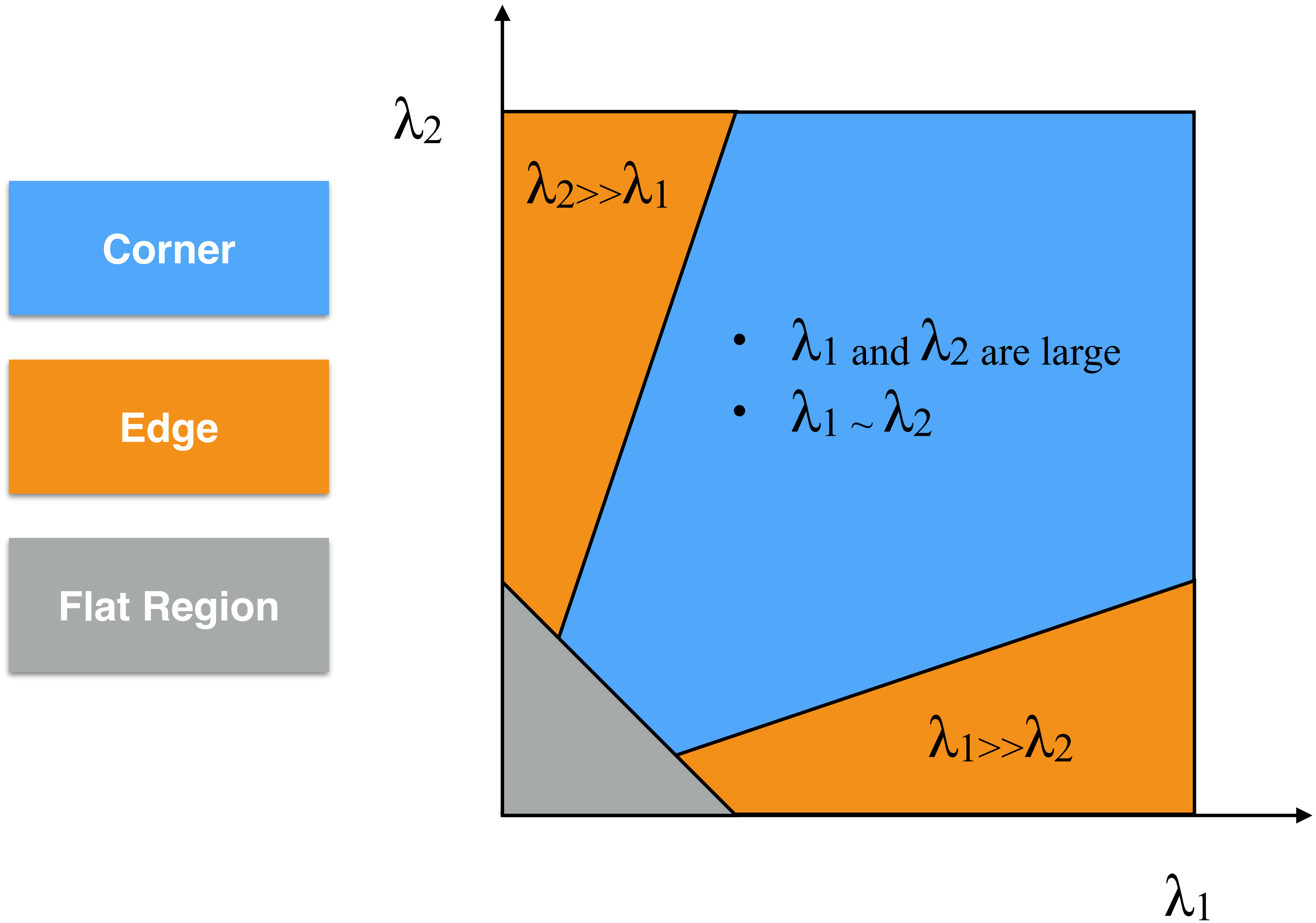
Harris Corner Detector: Second Moment Matrix



Eigenvalues and eigenvectors of $\mathbf{M}$ define shift directions with the smallest and largest change in E:

- x_{max} = direction of largest increase in E
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in E
- λ_{min} = amount of increase in direction x_{min}

Classification



Harris Corner Detector: Cornerness Measure

- Instead of directly computing the eigenvalues, we use a measure that determines the "***cornerness***" of a pixel (i.e., how close to be a corner is):

$$R = \mathrm{Det}(\mathbf{M}) - k\mathrm{Tr}(\mathbf{M})^2 \quad \text{or} \quad R = \frac{Det(\mathbf{M})}{\mathrm{Tr}(\mathbf{M})}$$

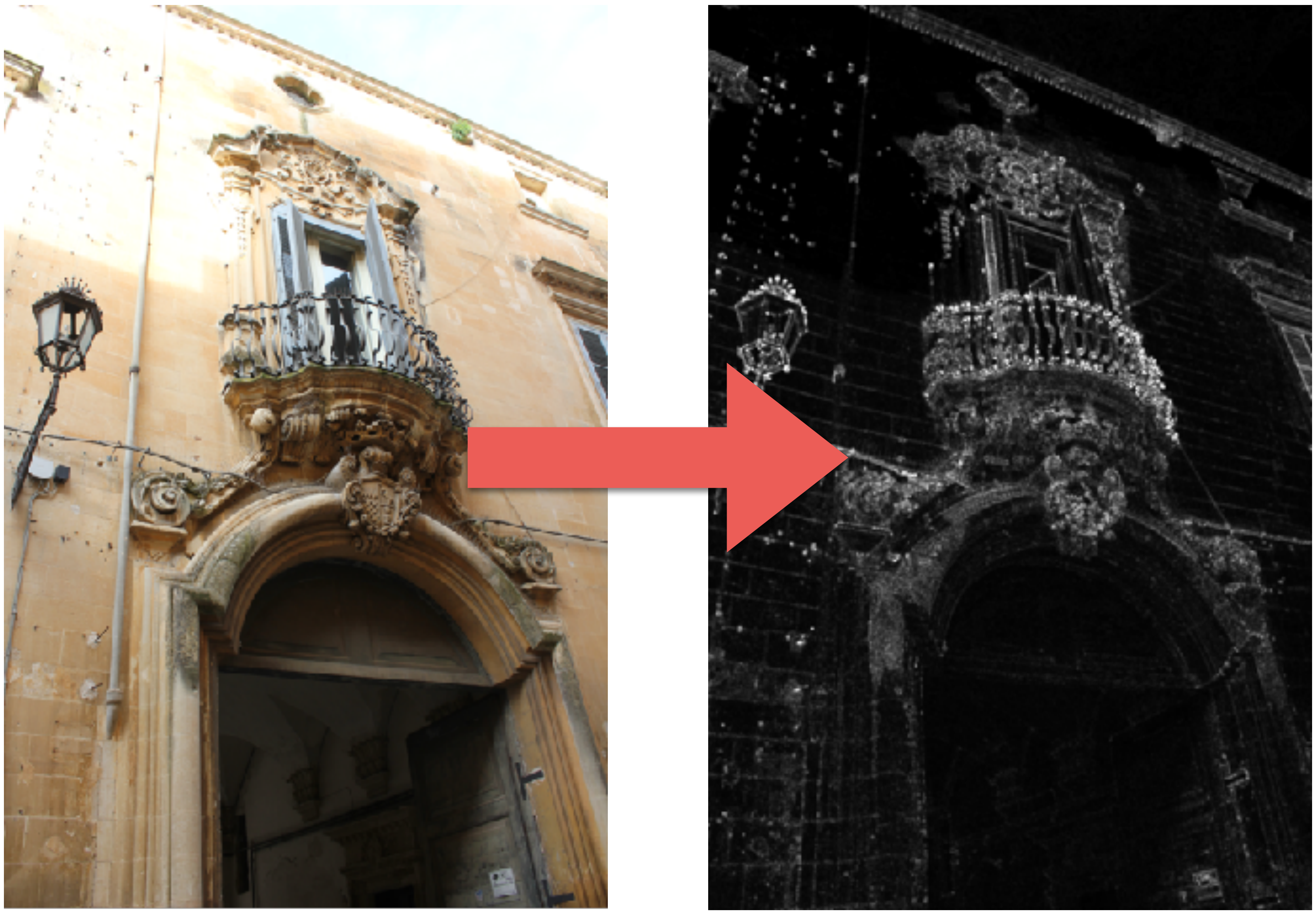
where:

- $\mathrm{Det}(\mathbf{M}) = \lambda_1\lambda_2$
- $\mathrm{Tr}(\mathbf{M}) = \lambda_1 + \lambda_2$
- $k \in [0.04, 0.06]$

Harris Corner Detector: Cornerness Measure

- Note that for 2×2 matrix $\mathbf{M}$, we can compute the trace and the determinant as:
  - Tr$(\mathbf{M}) = \lambda_1 + \lambda_2 = m_{11} + m_{22}$
  - Det$(\mathbf{M}) = \lambda_1 \lambda_2 = m_{11} m_{22} - m_{12} m_{12}$

Harris Corner Detector: Cornerness Measure

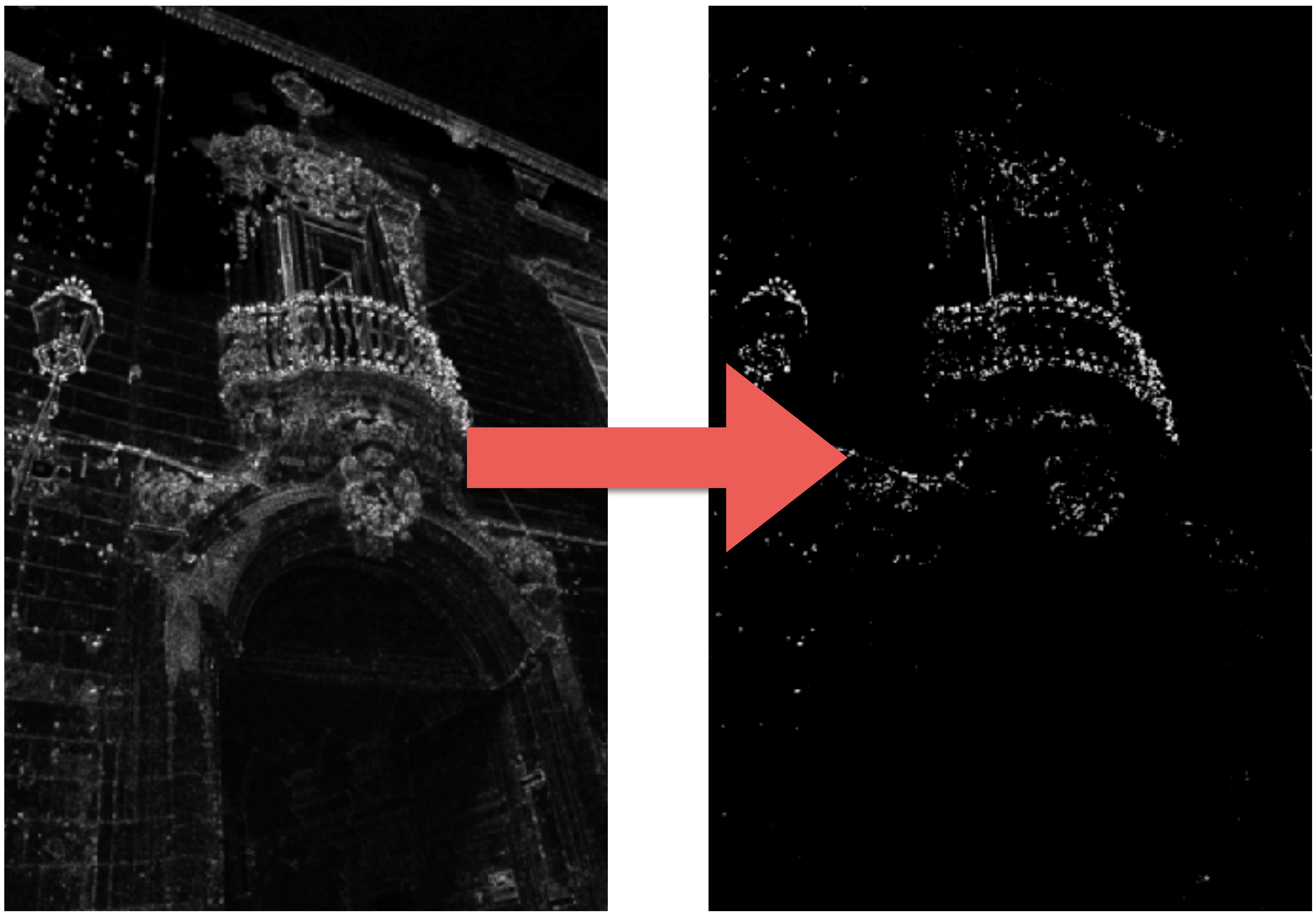


Input Image $\quad\quad$ R

Harris Corner Detector: Pruning Corners

- We have to find pixels with large corner response, R, i.e., $R > T_0$.
    - Typically, T_0 in $[0,1]$ depends on the number of points we want to extract; a default value is 0.01.

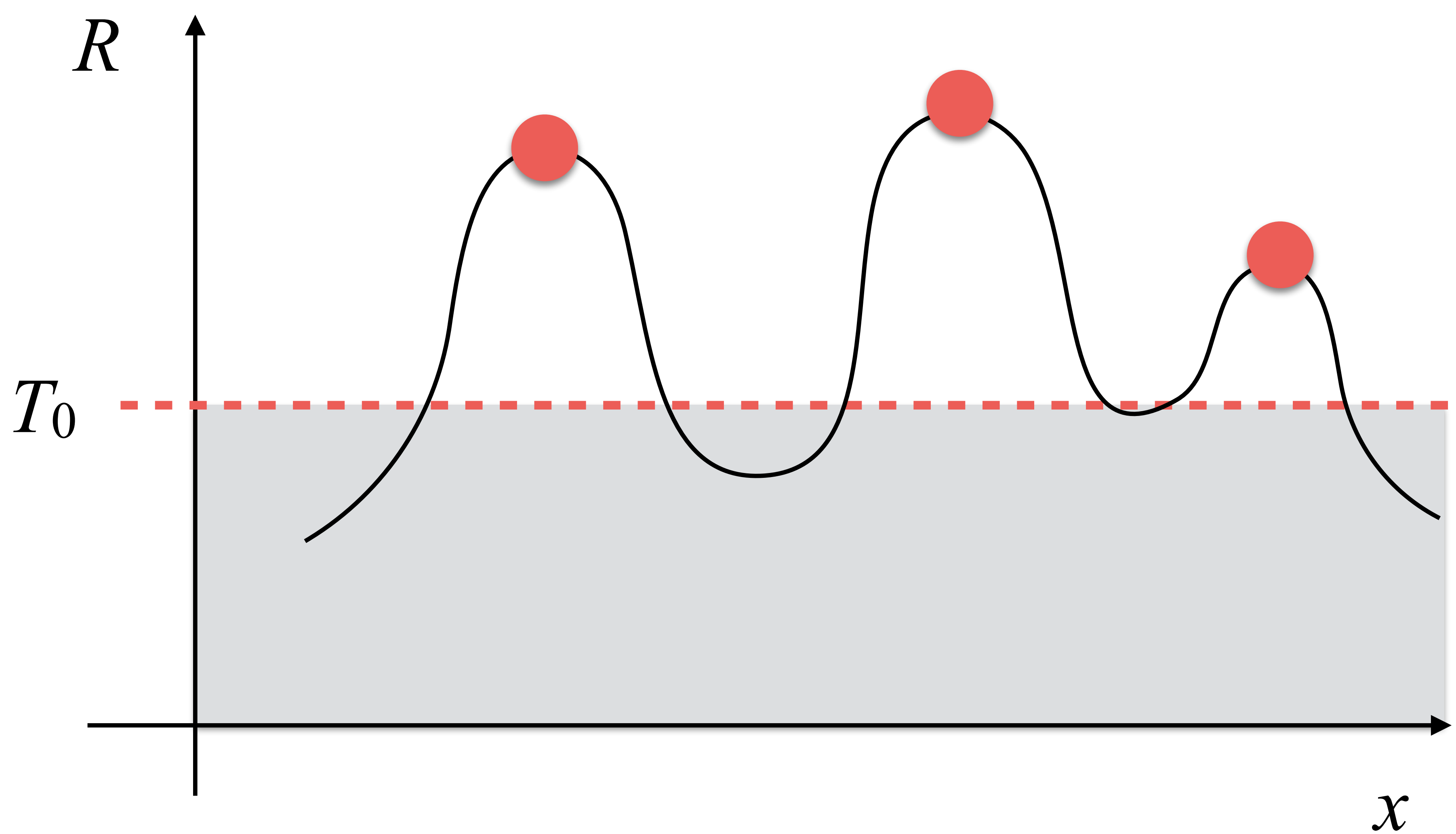
Harris Corner Detector: Thresholding



R $\qquad$ R after thresholding

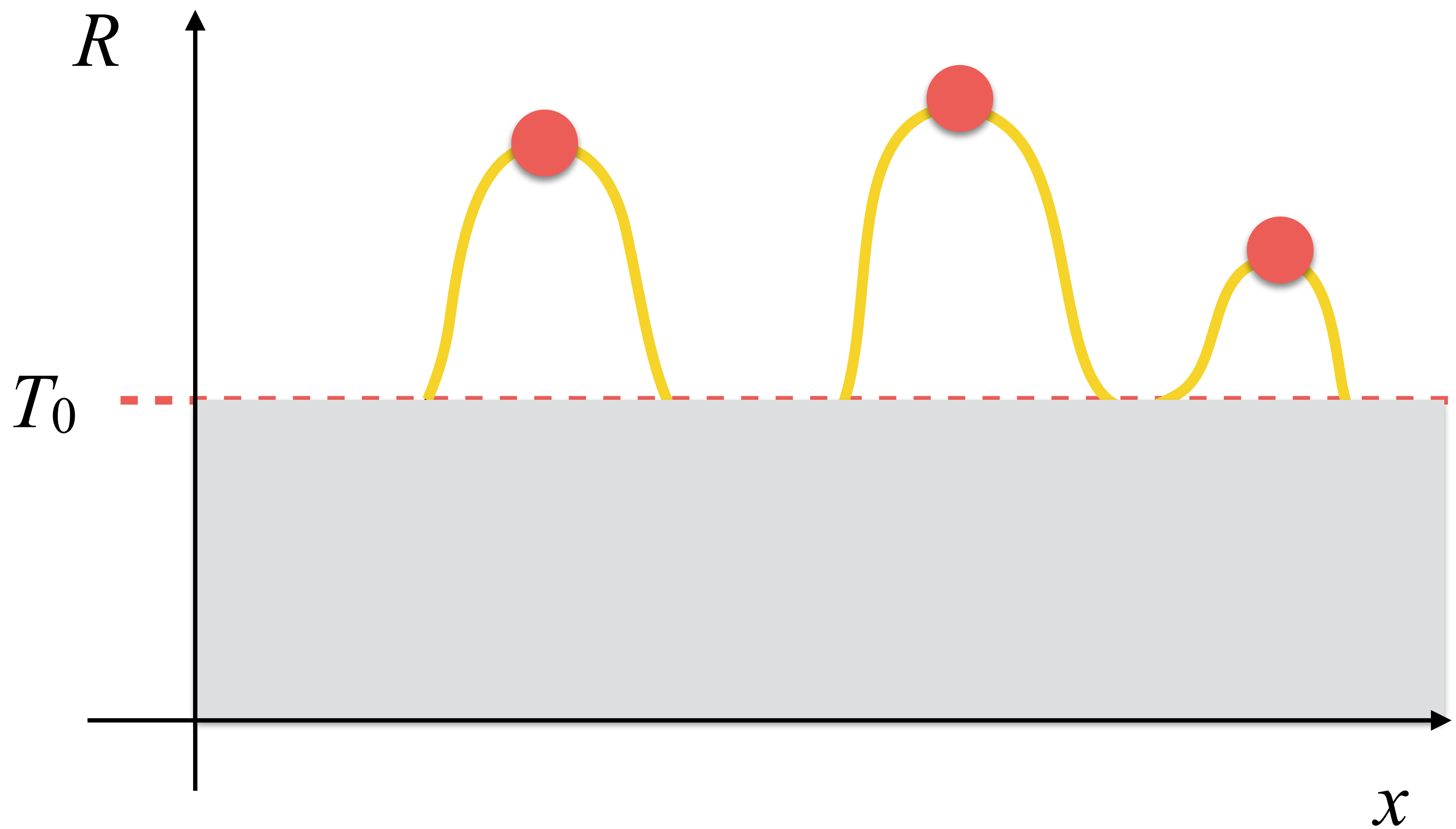
Harris Corner Detector: Pruning Corners

- At this point, we need to suppress/remove values that are not maxima.



Harris Corner Detector: Pruning Corners

- At this point, we need to suppress/remove values that are not maxima, but they are over the threshold (yellow pixels).



Harris Corner Detector: Pruning Corners

- We set a radius (in pixel) for suppressing non-maxima; e.g., 3-5.

- We apply to R a maximum filter; it is similar to a median filter, but it computes the maximum instead of the median. After this we obtain a filtered image called $R_{\max}$.

- A pixel at position (x, y) is a local maximum if and only if:

$$R_{\max}(x, y) = R(x, y) \quad \wedge \quad R(x, y) > T_0$$

Harris Corner Detector: Pruning Corners Example 1

100	0	30
40	**20**	20
0	0	0

The current pixel that we are evaluating is the central one!

$$T_0 = 5$$

Harris Corner Detector: Pruning Corners Example 1

100	0	30
40	**20**	20
0	0	0

The maximum is **100**!

Harris Corner Detector: Pruning Corners Example 1

100	0	30
40	**0**	20
0	0	0

20 < 100 so it has to be suppressed; i.e., set to **0**!

Harris Corner Detector: Pruning Corners Example 2

20	0	30
40	**100**	20
0	0	0

The current pixel that we are evaluating is the central one!

$$T_0 = 5$$

Harris Corner Detector: Pruning Corners Example 2

20	0	30
40	**100**	20
0	0	0

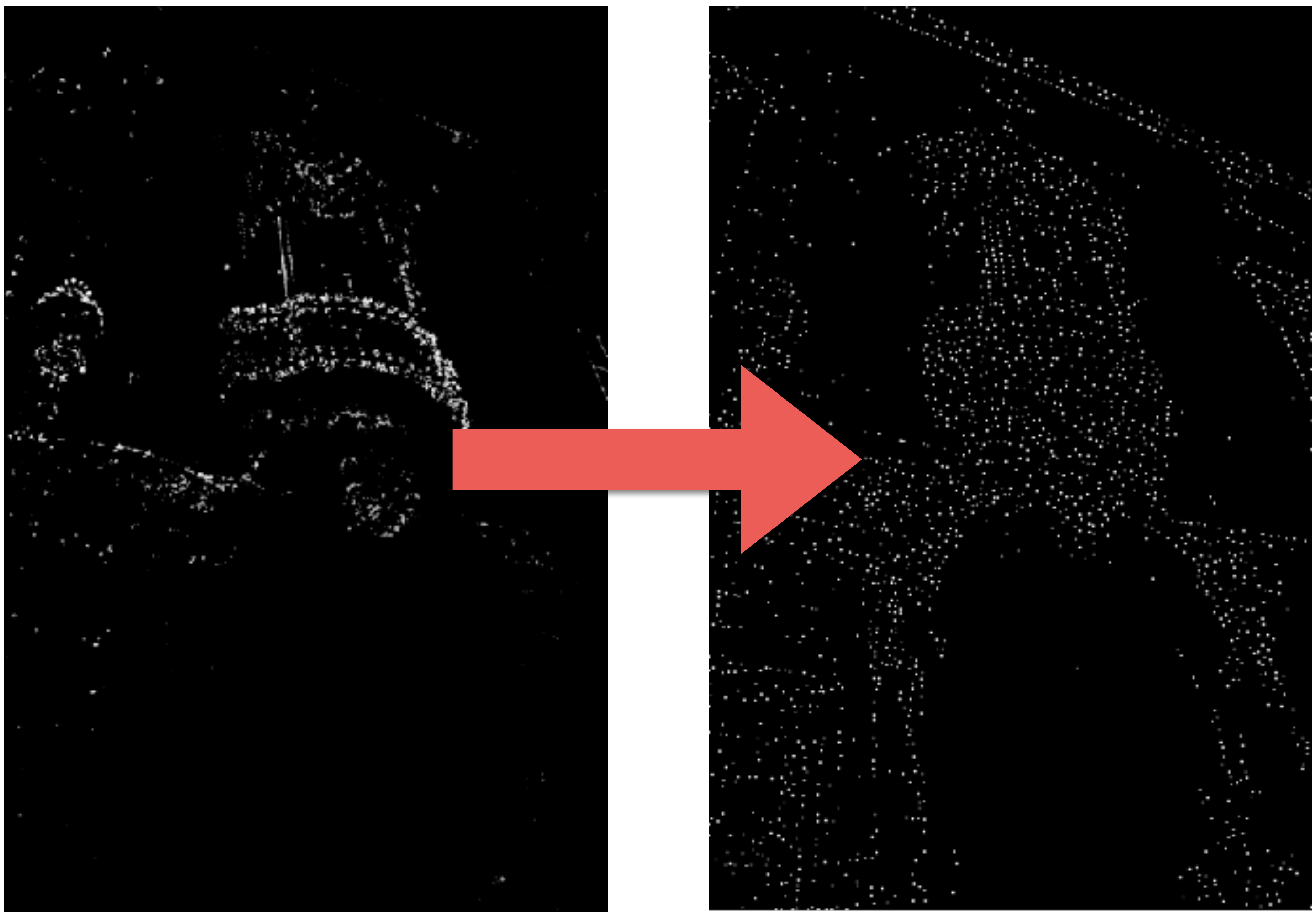
The maximum is **100**!

Harris Corner Detector: Pruning Corners Example 2

20	0	30
40	**100**	20
0	0	0

100 == 100 so it has to be kept!

Harris Corner Detector: Non-Maximal Suppression



R after thresholding Non-Maximal Suppression

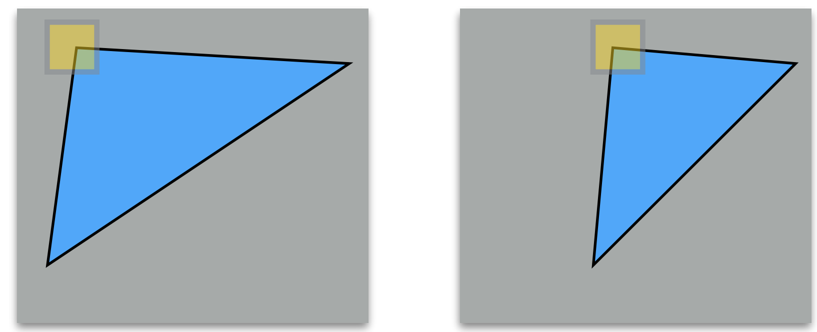
Harris Corner Detector: Non-Maximal Suppression

Harris Corner Detector: Non-Maximal Suppression

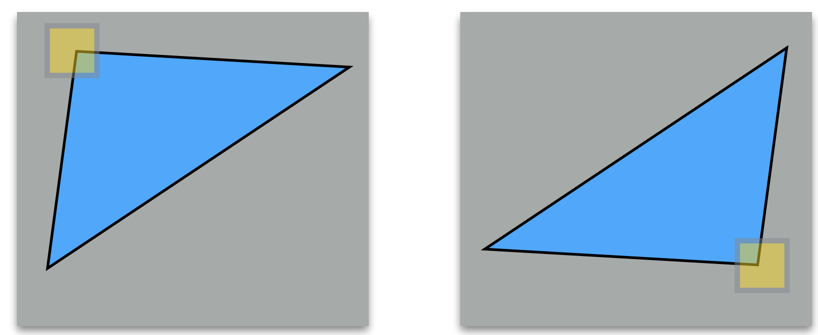
Harris Corner Detector: Non-Maximal Suppression

Harris Corner: Advantages

- Translational invariance:



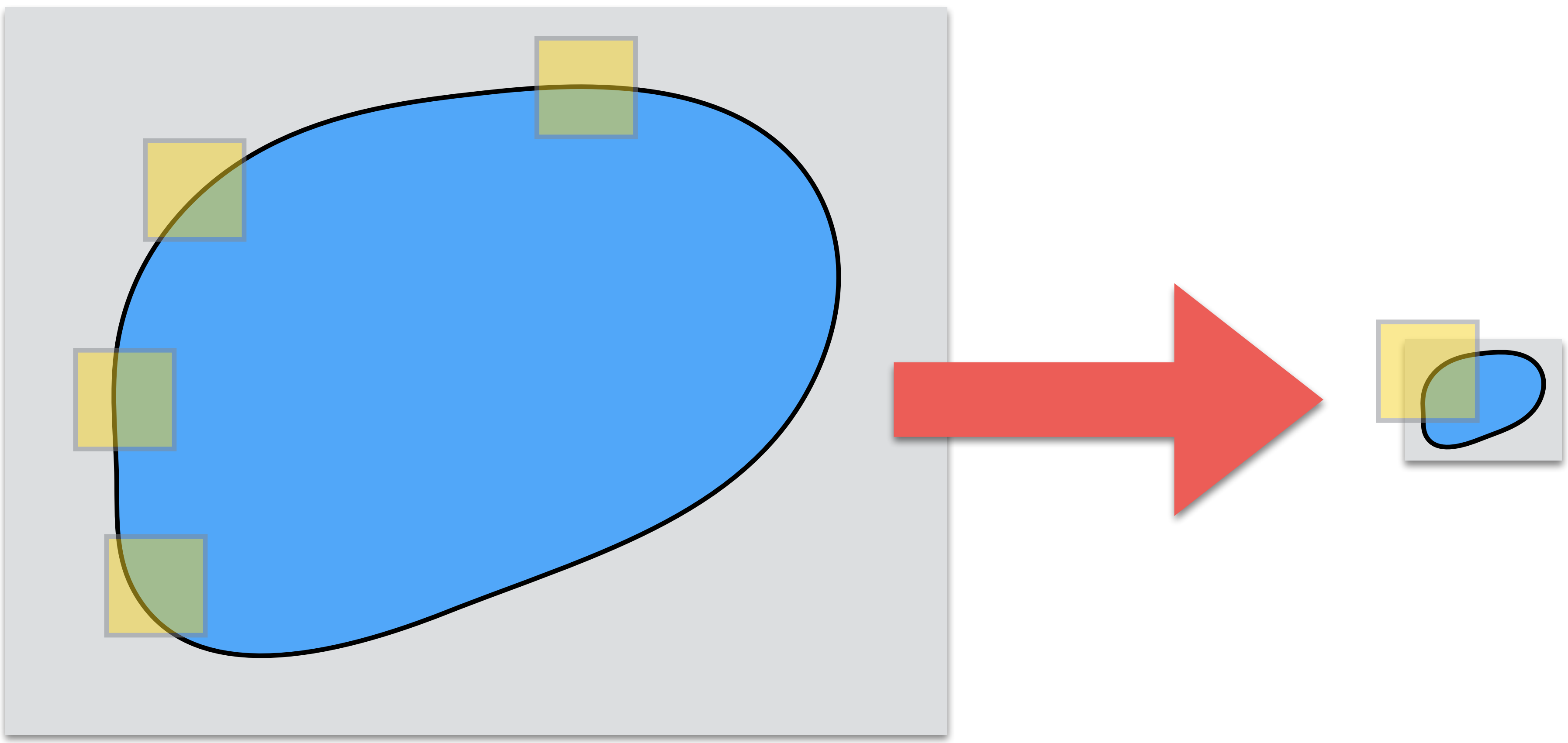
- Rotation invariance:



- Only derivatives are employed:
  - Intensity shift invariance: $I' = I + b$
  - Intensity scale invariance: $I' = I\,a$

Harris Corner: Disadvantage

- Not scale invariant!



All points are
classified as edges

It is now
a corner!

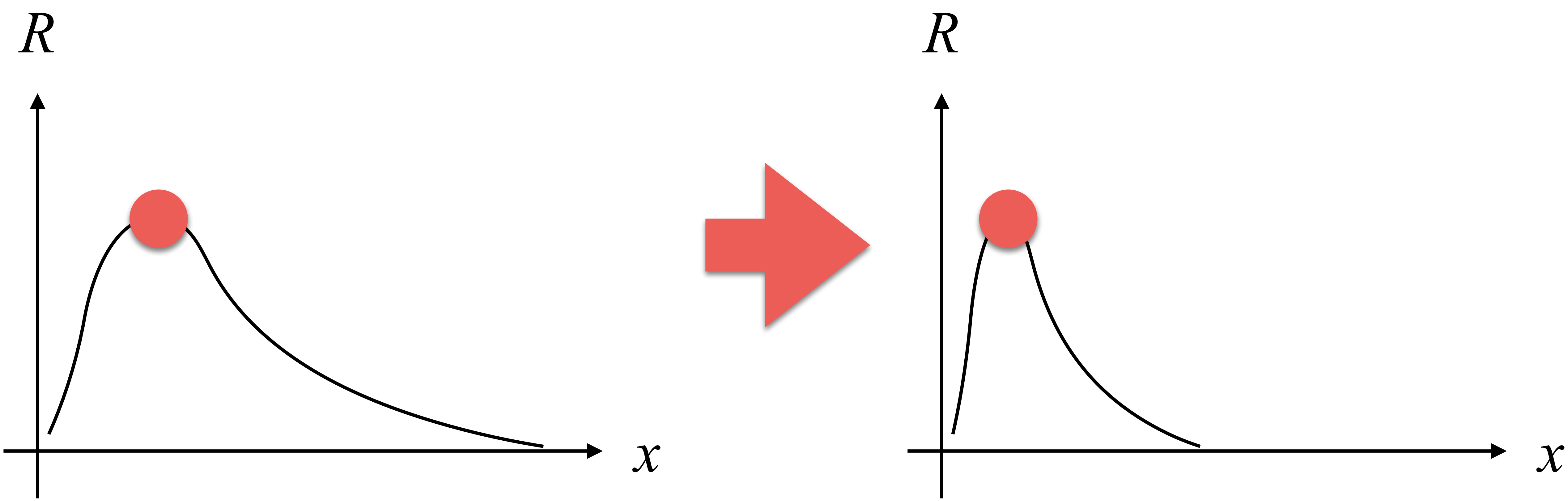
The same feature in different images can have different size!

The Scale Problem

Near Object

Far Object

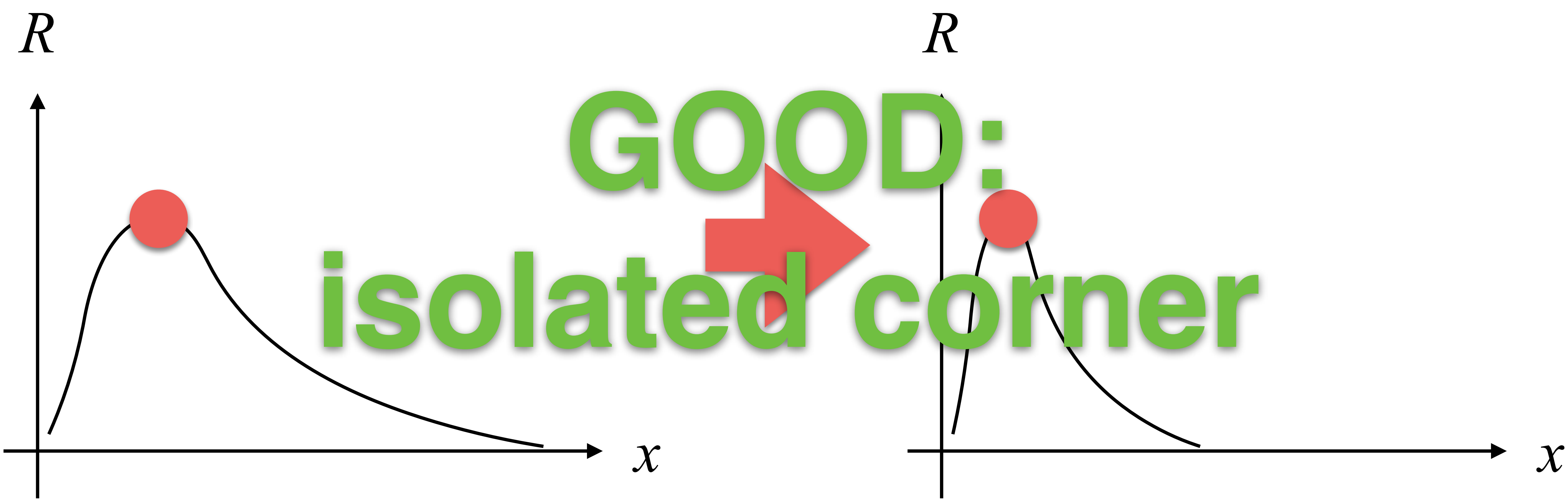
Scale Invariant:
Stable Corners



Original

1/2 scale

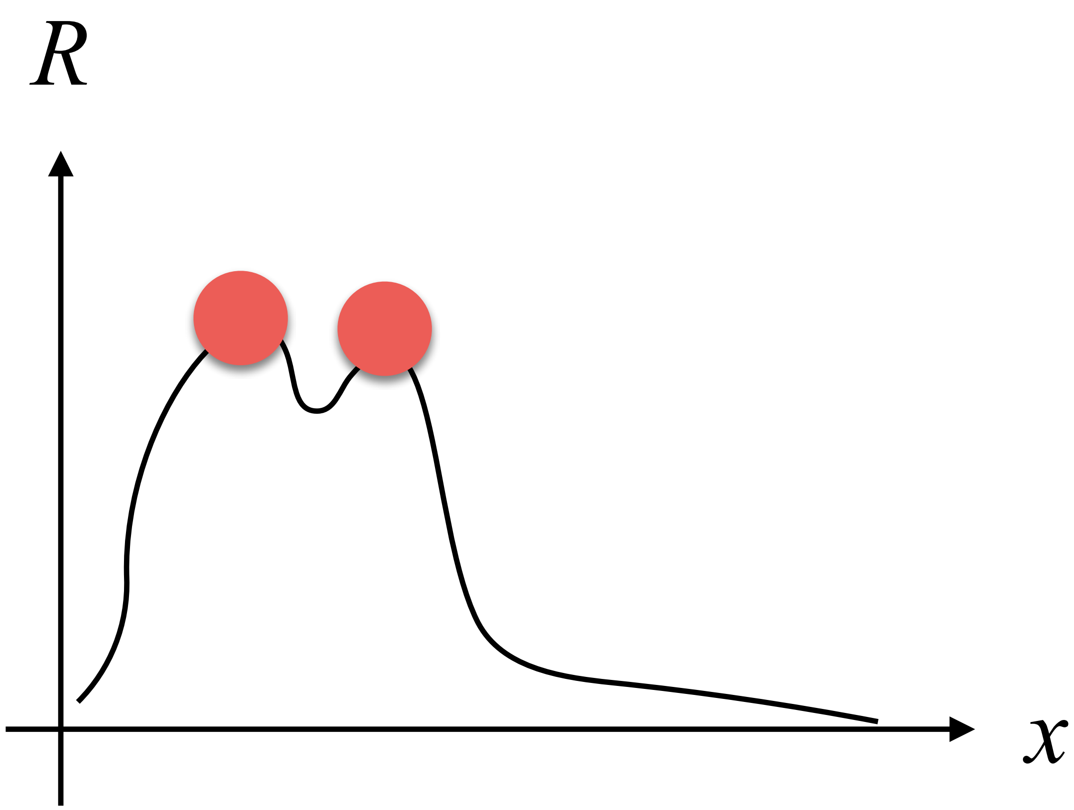
Scale Invariant: Stable Corners



Original

1/2 scale

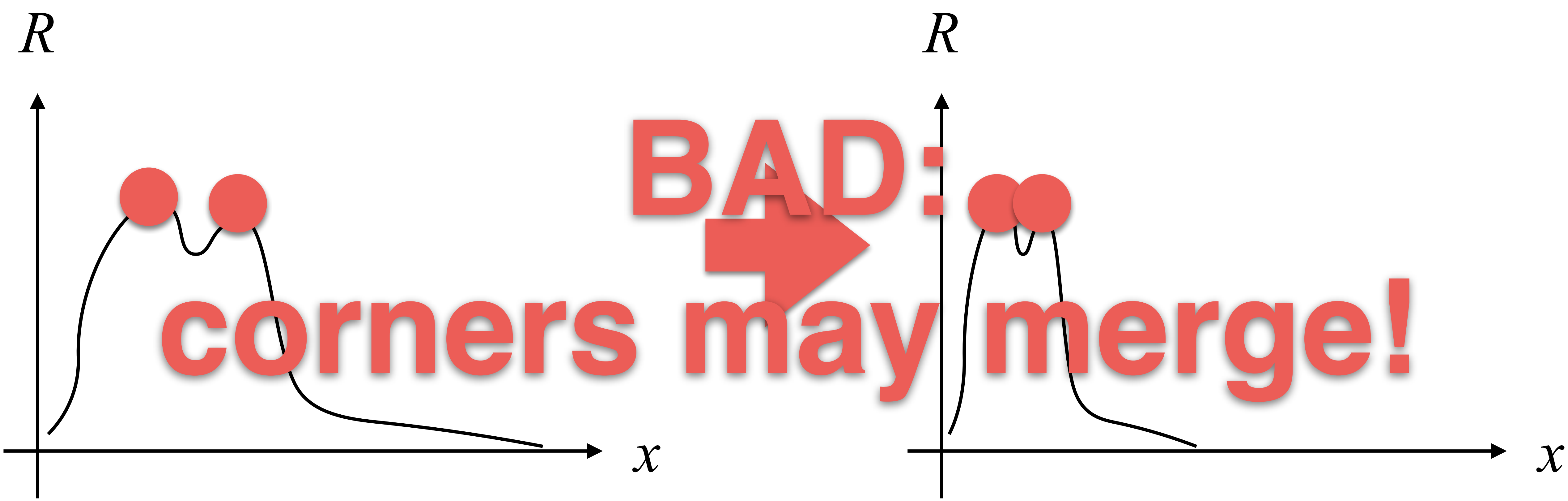
Scale Invariant: Unstable Corners



Original

1/2 scale

Scale Invariant: Unstable Corners



Original

1/2 scale

Scale Invariant: A Multi-Scale Approach

- Depending on the content of the image:
  - We need to detect the scale of corner.
  - We need to use its scale to vary the size of the window W for computing corners!

Scale Invariant: The Signature Function

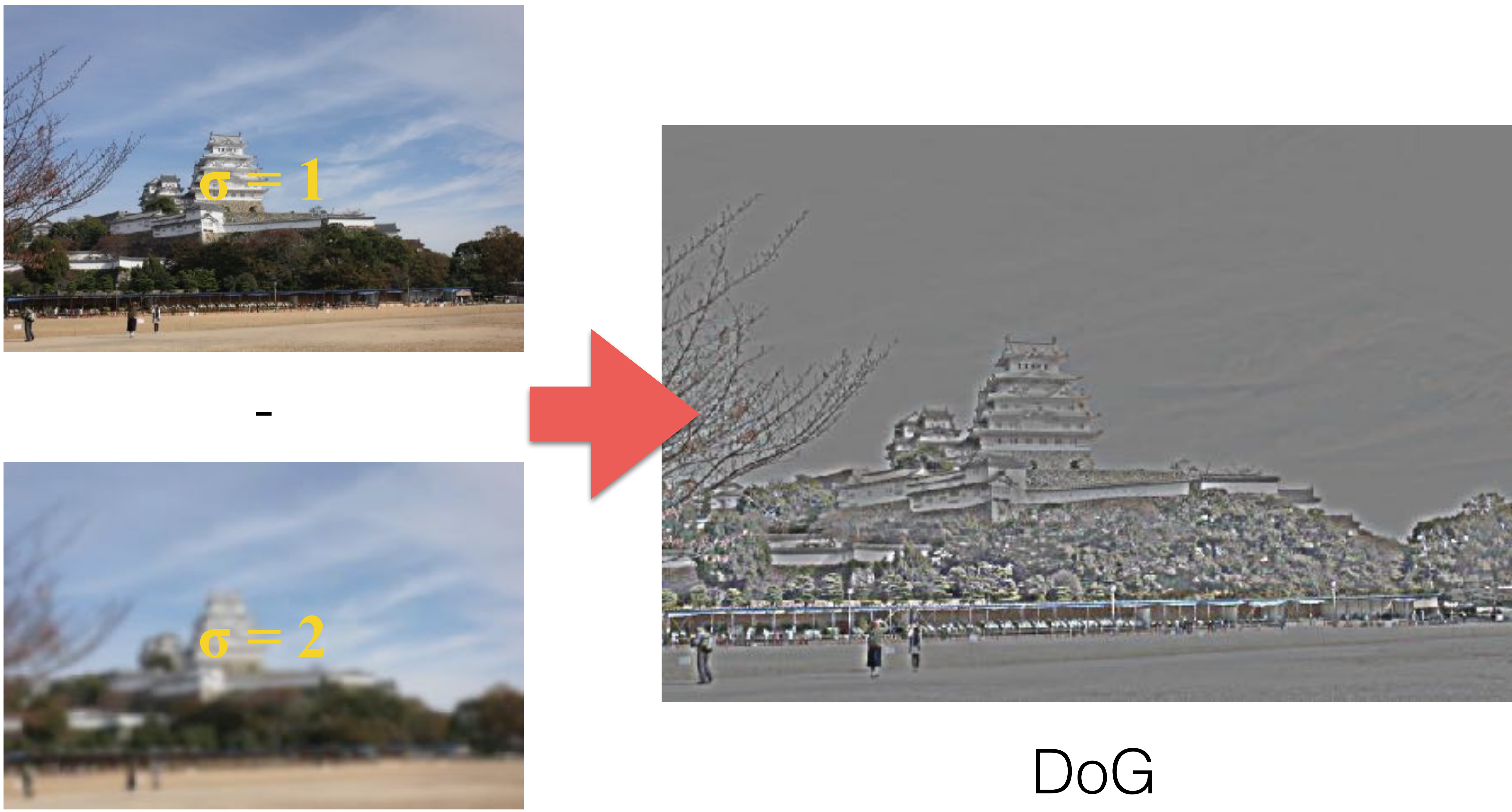
- A signature function, s, is a function giving us an idea of the local content of the image, I, around a point with coordinates (x, y) at a given scale σ.

- An example of signature function is the Difference of Gaussians (DoG):

$$s(I, x, y, \sigma) = [I \otimes G(\sigma)](x, y) - [I \otimes G(\sigma \cdot 2)](x, y)$$

where G is a Gaussian kernel.

Scale Invariant:
The Signature Function



Scale Invariant: The Approach

We need to find the right scale for resizing W for each image!

Scale Invariant: The Approach

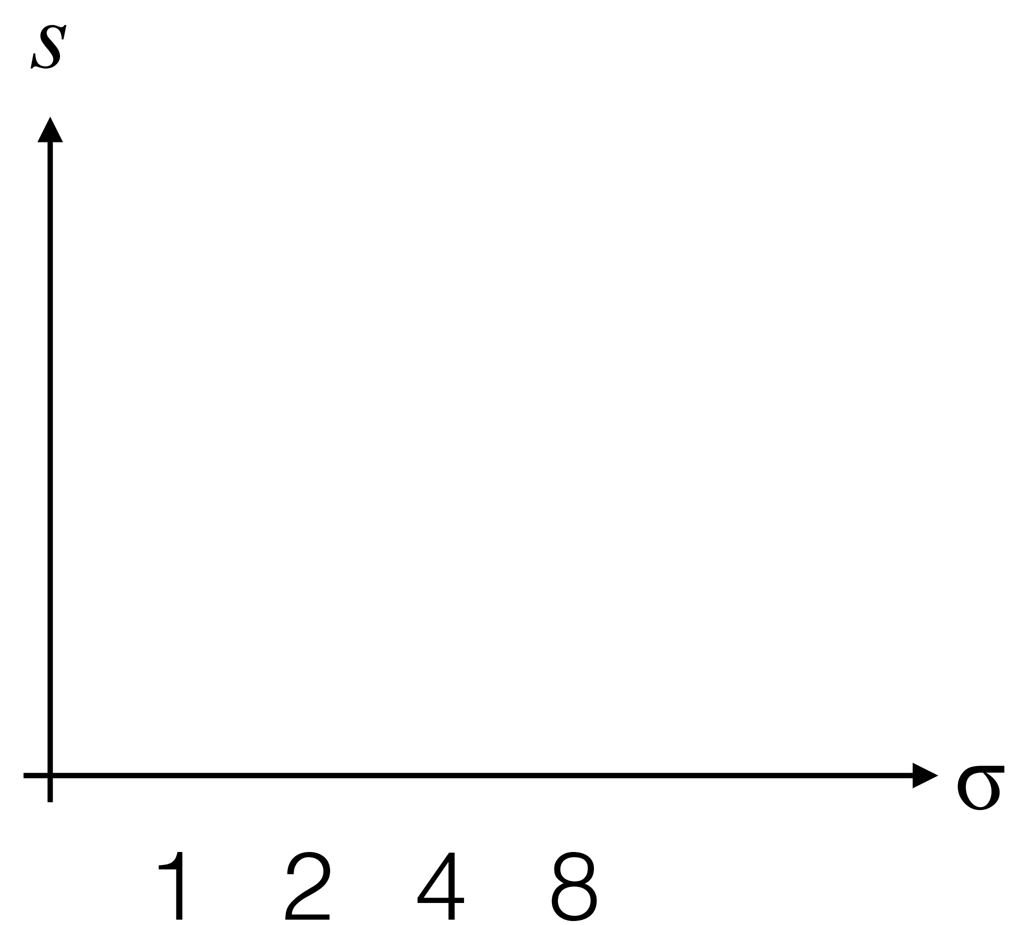
- The signature function, s, can give us an idea of the content of the image.
- Therefore, we need to find a maximum point of s for pixel of an input image!

Scale Invariant: The Approach

Let's build s at the red point!

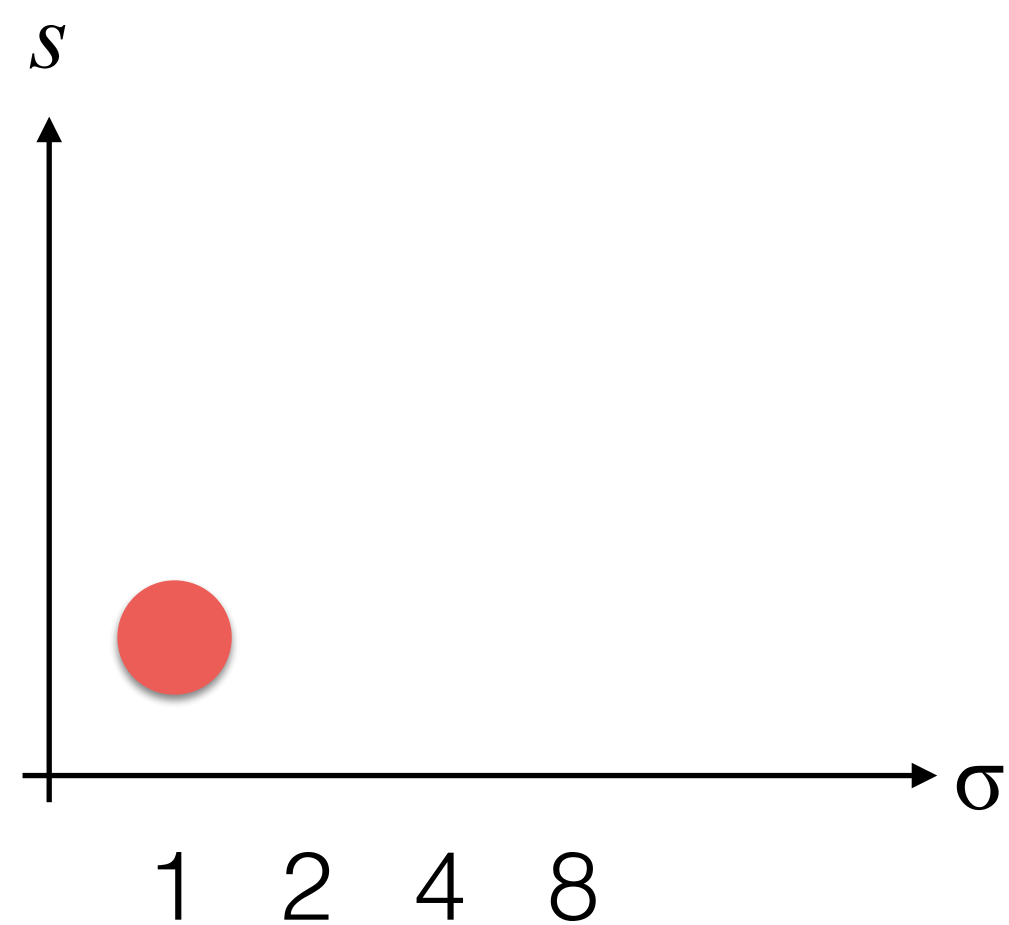
Scale Invariant: The Approach

This is our start!



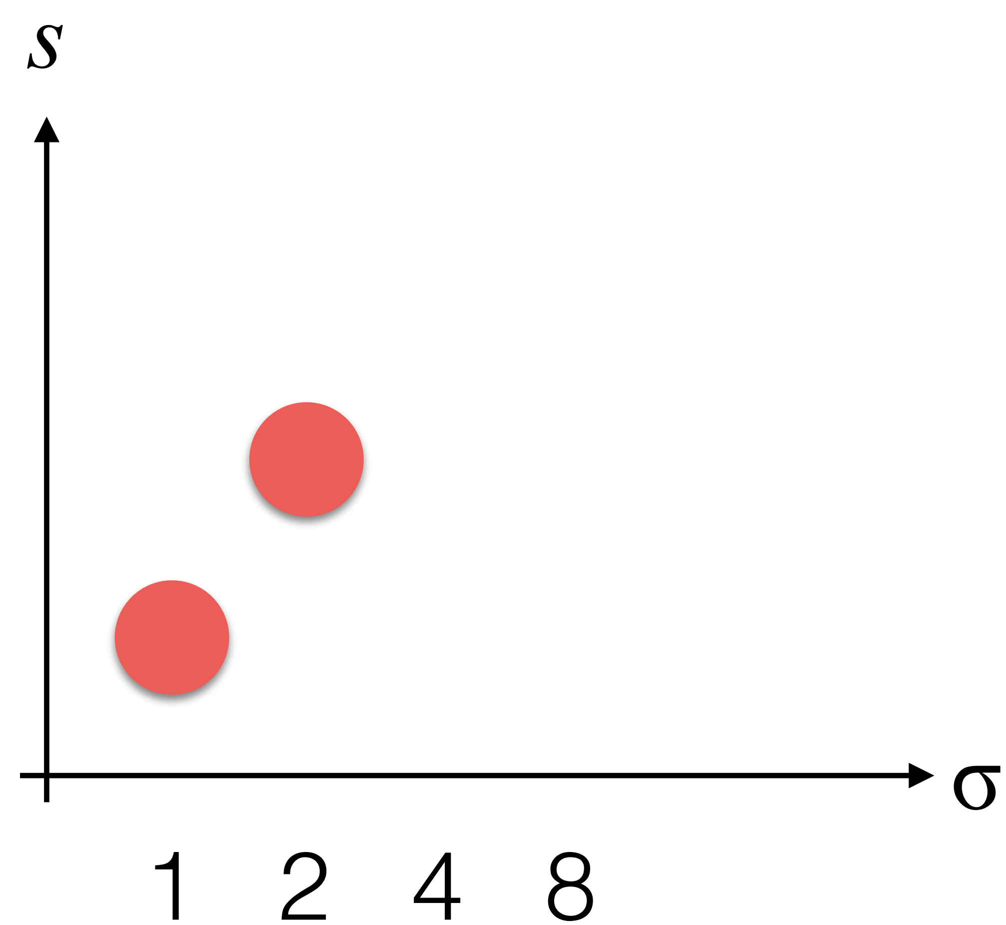
Scale Invariant: The Approach

$\sigma = 1$

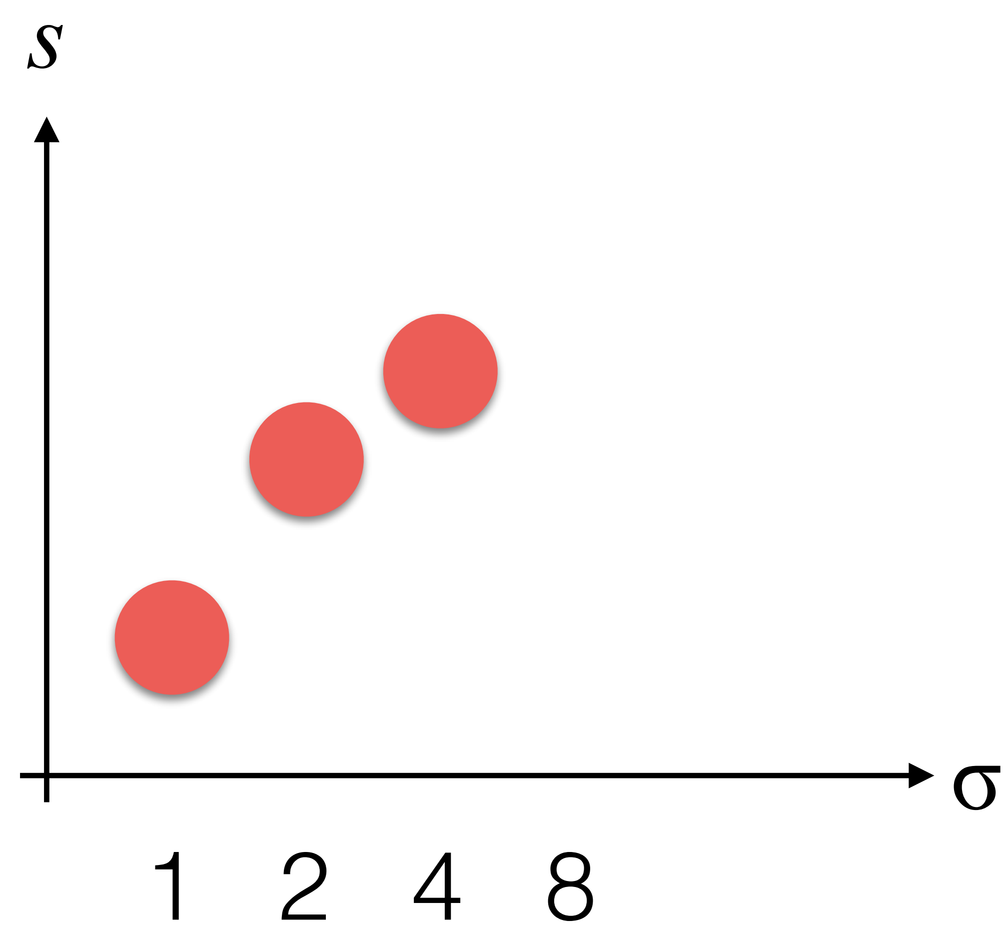


Scale Invariant: The Approach

$\sigma = 2$



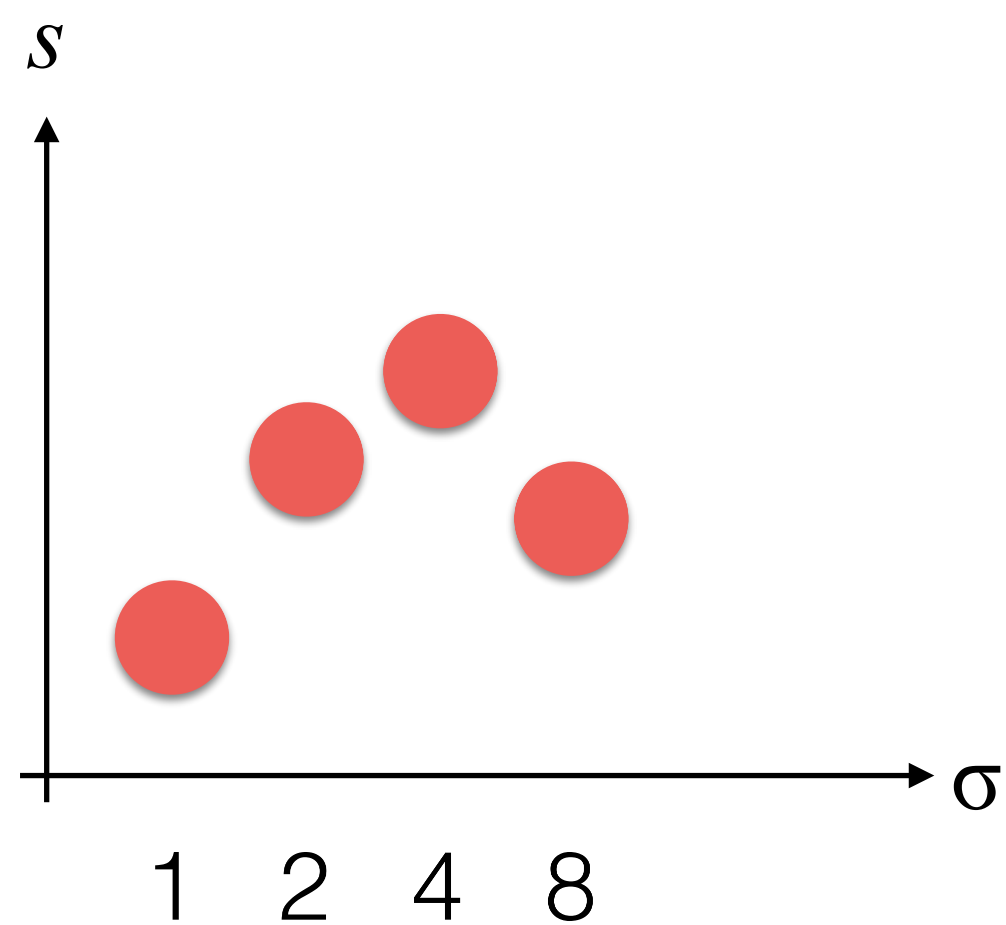
Scale Invariant: The Approach



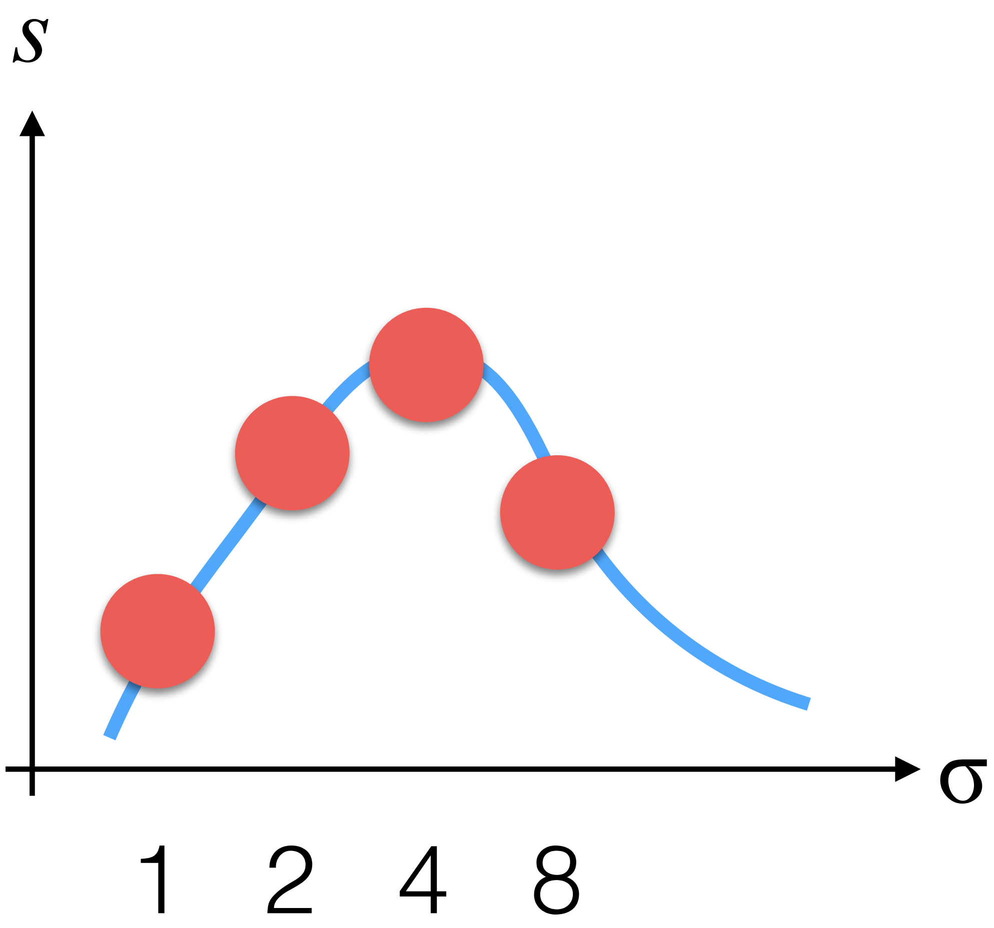
$\sigma = 4$

Scale Invariant: The Approach

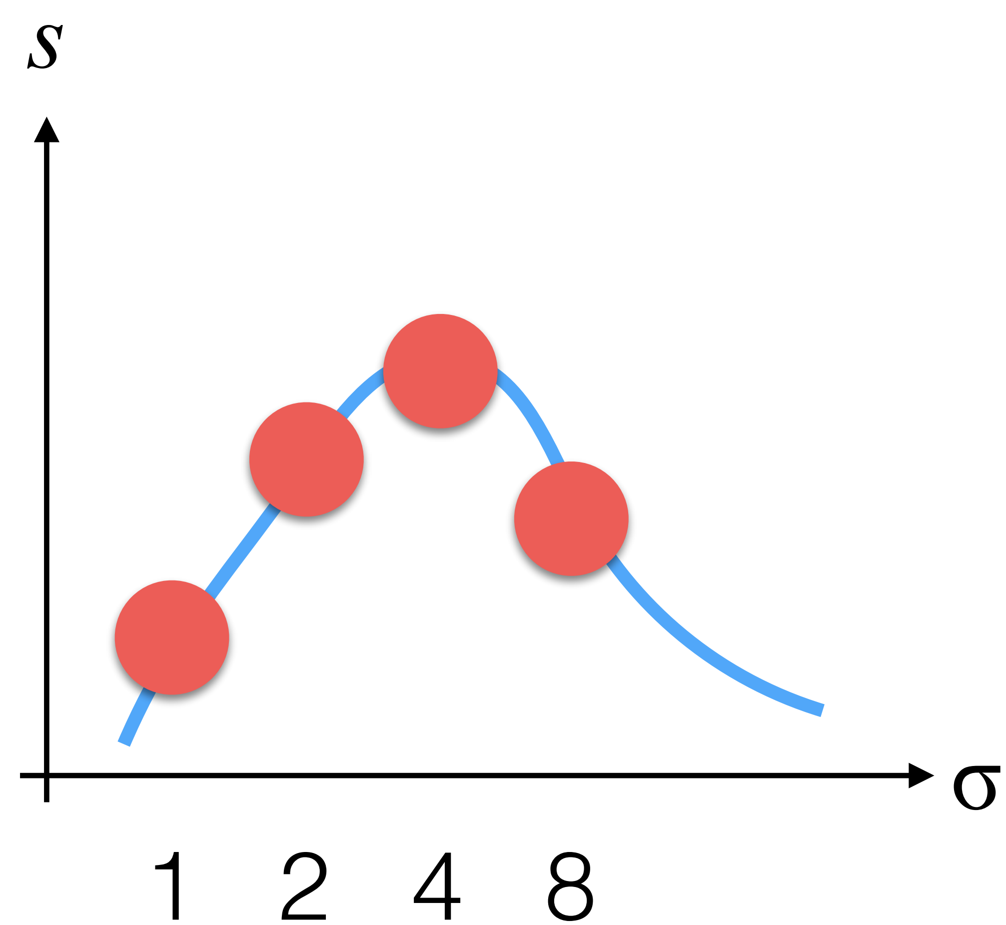
$\sigma = 8$



Scale Invariant: The Approach

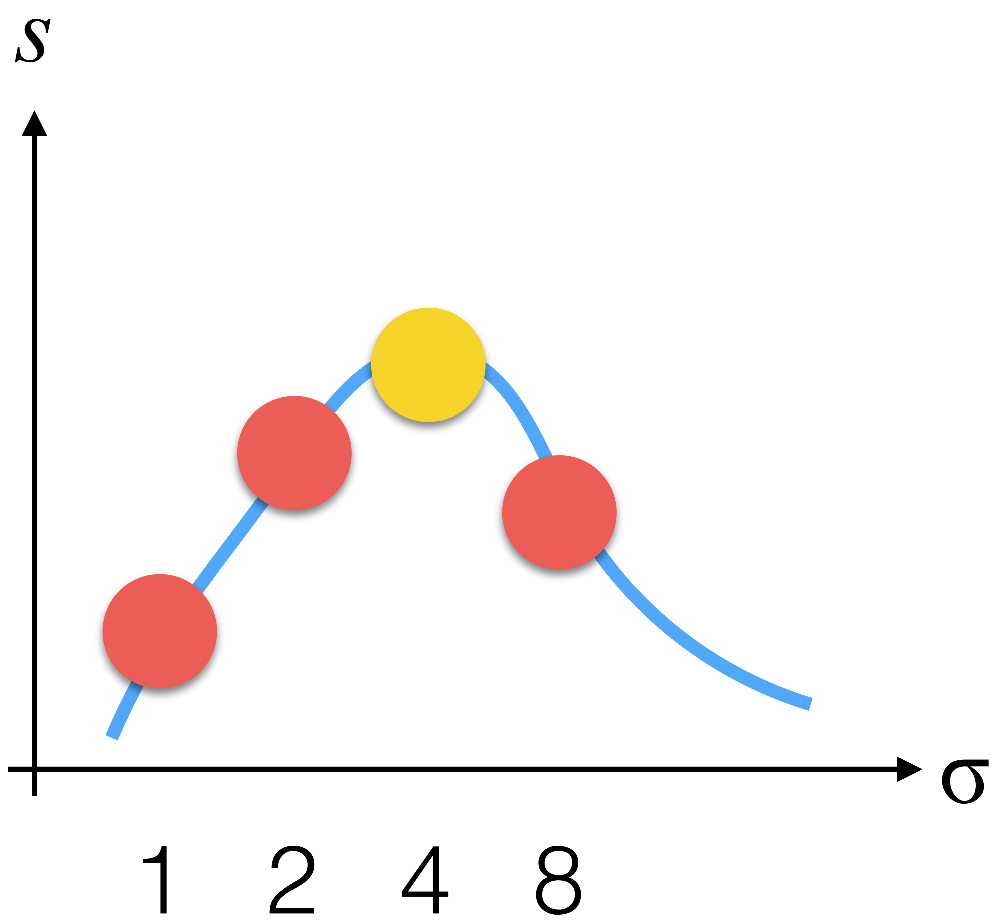


Scale Invariant: The Approach



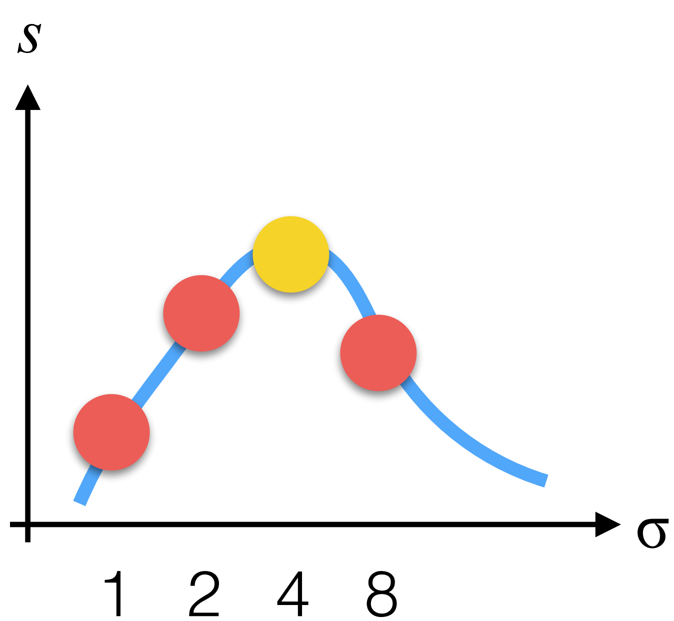
Which is σ for which *s* is the maximum?

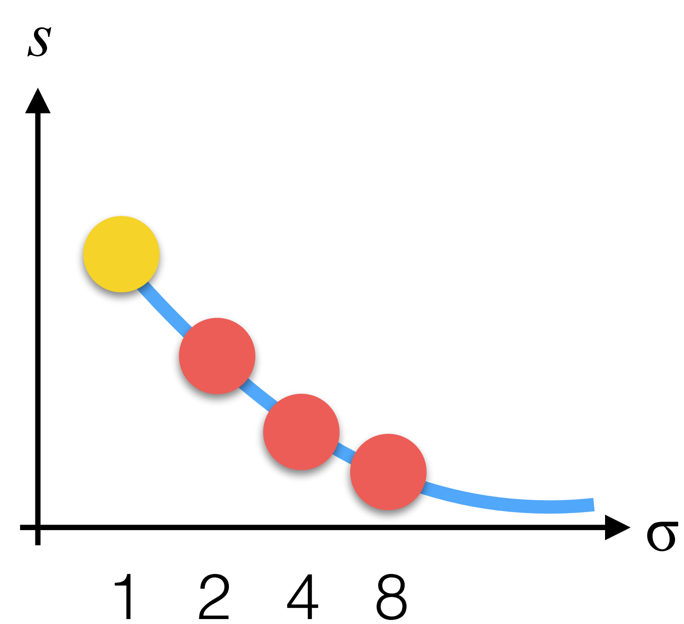
Scale Invariant: The Approach



It is $\sigma = 4$

Scale Invariant: The Approach





Extraction of Features

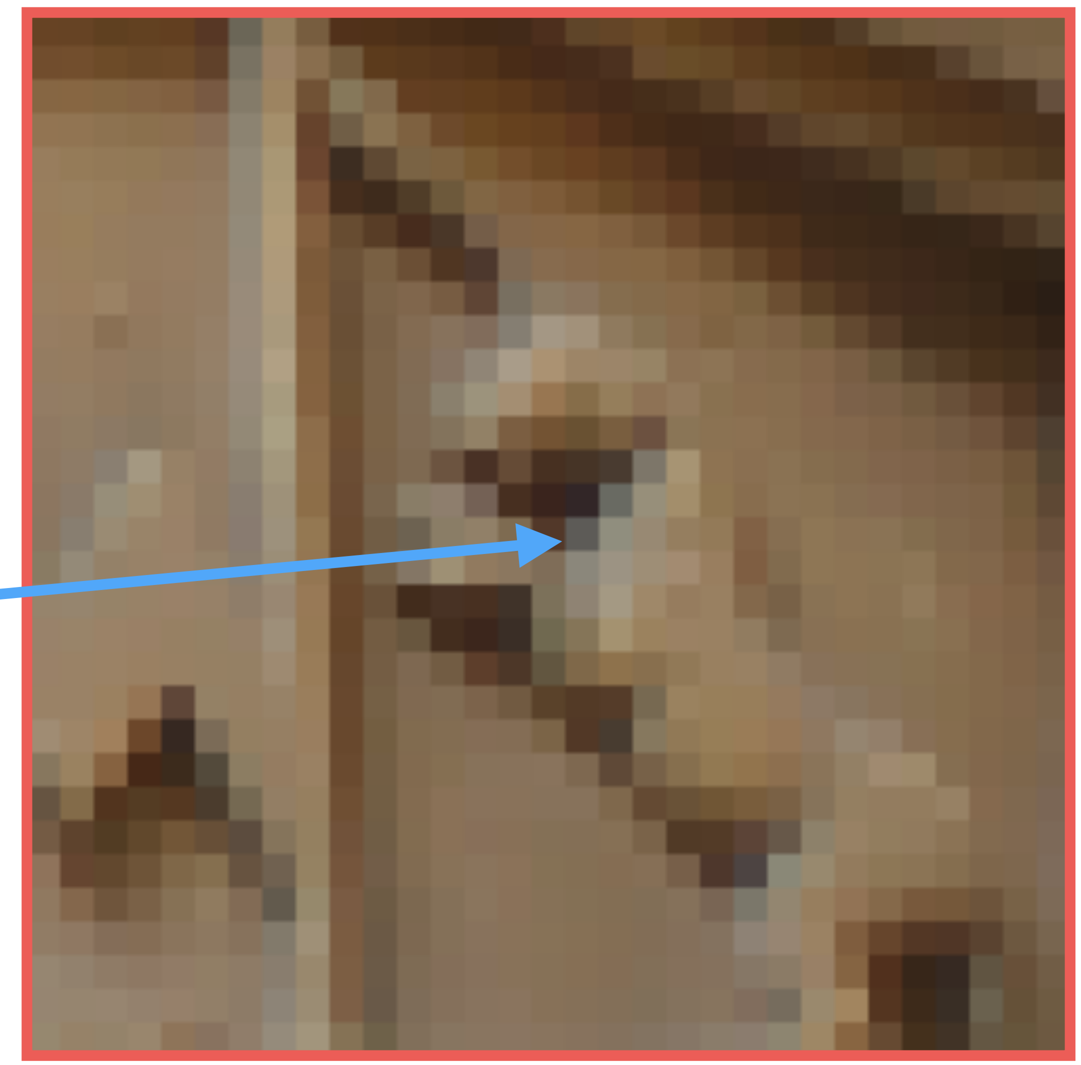
- General overview:
  - We compute the scale for each pixel using the sigma value at which we have the maximum value of the signature function.
  - We compute the Harris Corner using the scale to increase the size of the local window; i.e., the scale of the window will be multiplied by the sigma value.

Feature Descriptors

Feature Descriptors

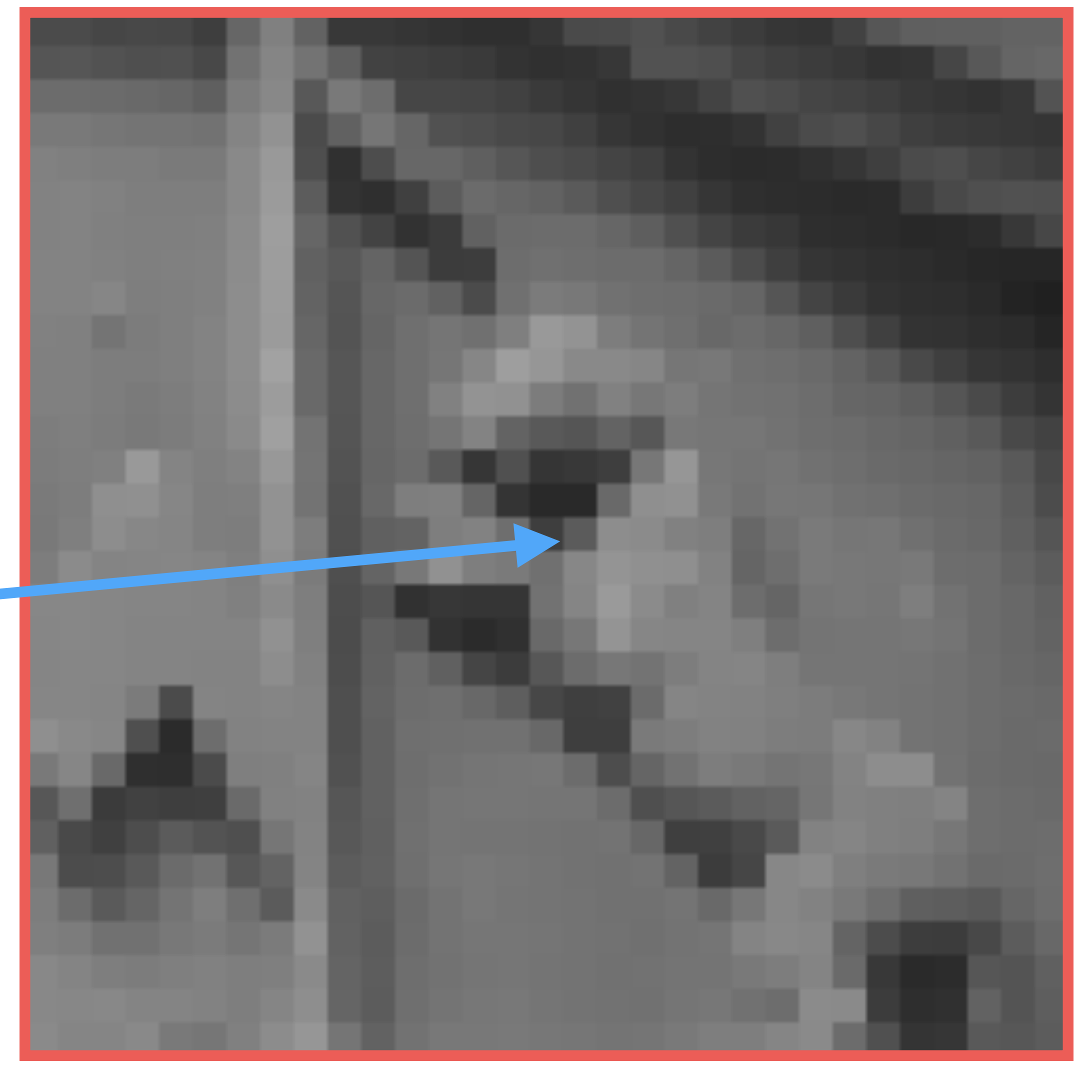
- Once we found our features (i.e., corners), we need to describe in a meaningful and possibly unique way.
  - Why?
    - We want compare corners between images in order to find correspondences between images.

Feature Descriptors



A patch, P, is a sub-image centered in a given point (u, v).

Feature Descriptors



A patch, P, is a sub-image centered in a given point (u, v).

Feature Descriptors

- There are many local features descriptors in literature:
    - BRIEF/ORB descriptor.
    - SIFT descriptor.
    - SURF descriptor.
    - etc.

Feature Descriptors

- Good properties that we want are invariance to:
  - Illumination changes.
  - Rotation.

BRIEF Descriptor

- The descriptor creates a vector of n binary values:

$$\mathrm{BRIEF}(P) = \mathbf{b} = [0, 1, 0, 0, \ldots, 1]^\top$$

- For efficiency, it is encoded as a number:

$$n_{\mathbf{b}} = \sum_{I=1}^{n} 2^{i-1} b_i$$

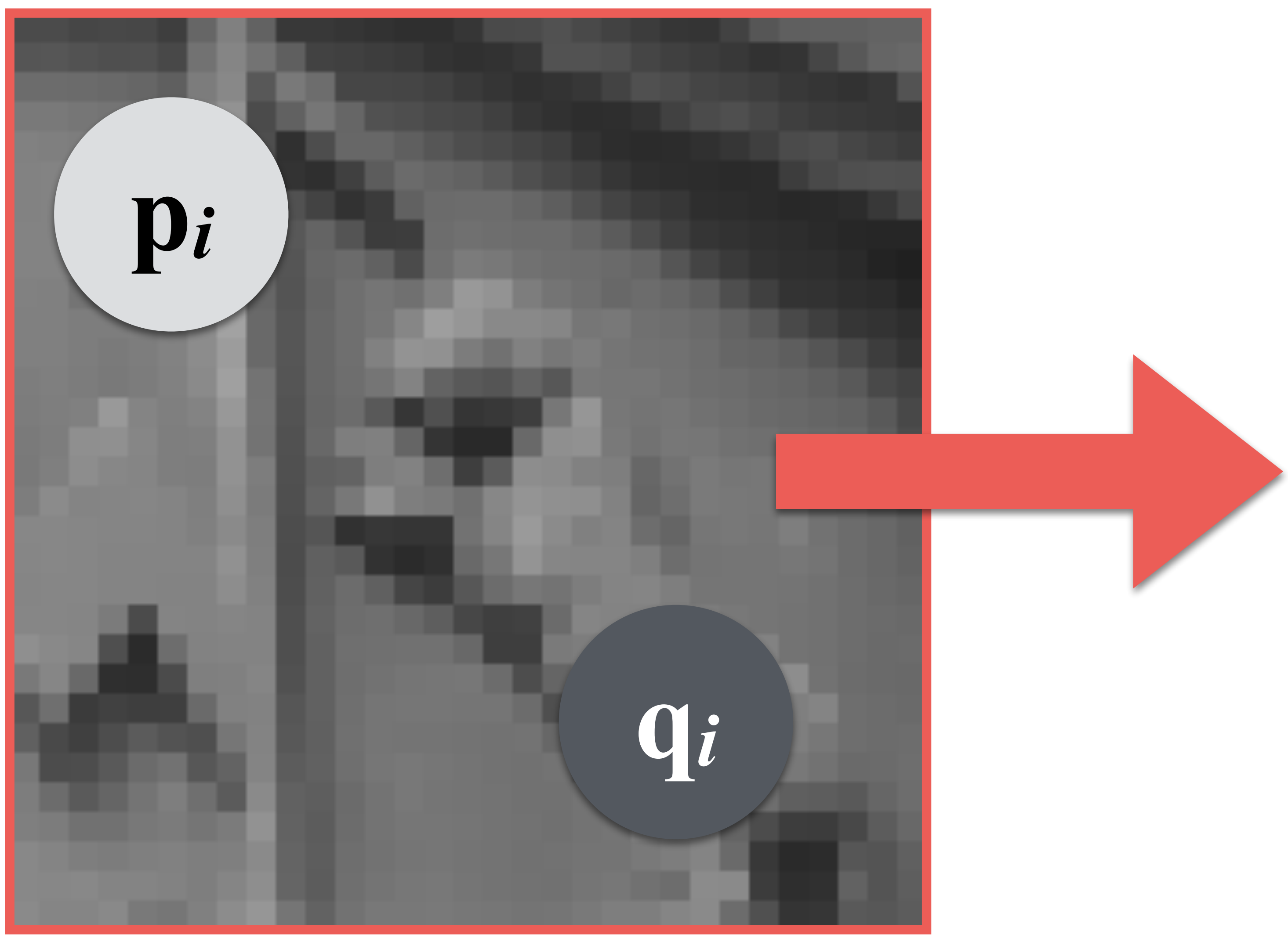
BRIEF Descriptor

- Given a patch, P, of size $S \times S$ an element of $\mathbf{b} = \{b_0, \ldots, b_n\}$ is defined as

$$b_i(\mathbf{q}_i, \mathbf{p}_i) = \begin{cases} 1 & \text{if } P(\mathbf{p}_i) < P(\mathbf{q}_i), \\ 0 & \text{otherwise} \end{cases}$$

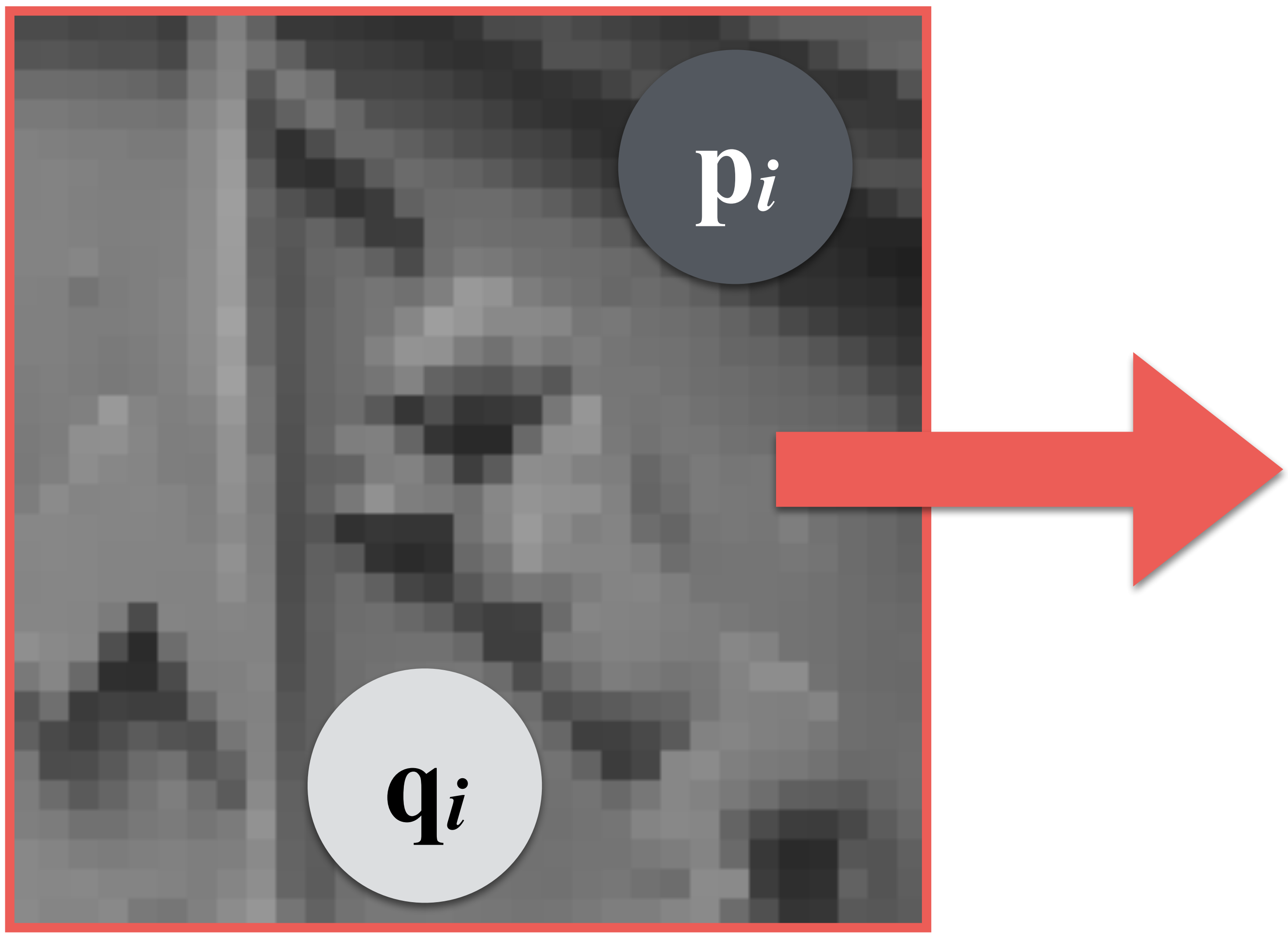
where $\mathbf{p}_i$ and $\mathbf{q}_i$ are two random points in P.

BRIEF Descriptor: Example



$b_i = 0$

BRIEF Descriptor: Example



$b_i = 1$

BRIEF Descriptor: Test

- Let's say we have two descriptor $\mathbf{b}^1$ and $\mathbf{b}^2$. How do we check if they are describing the same corner?
- We count the number of different bits in the two vectors (Hamming distance):

$$D_H(\mathbf{b}^1, \mathbf{b}^2) = \sum_{i=1}^{n} \neg\text{xor}(b_i^1, b_i^2)$$

- **Higher the closer**:
  - This is a very computationally efficient distance function.

BRIEF Descriptor: Test

A	B	A XOR B = [(NOT A) AND B] OR [(NOT B) AND A]	NOT (A XOR B)
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

BRIEF Descriptor: Point-Set

- The optimal number of points' couple (i.e., the size of the descriptor; n) is **256**:
  - This value was computed from experiments testing different lengths: 16, 32, 64, 128, 256, and 512.
- Points can be generated in different ways:
  - Uniform distribution in the patch
  - $\mathbf{p}_i \sim \text{i.i.d. } G\left(0, \frac{S^2}{25}\right)$ and $\mathbf{q}_i \sim \text{i.i.d. } G\left(0, \frac{S^2}{25}\right)$

BRIEF Descriptor: Point-Set

- Points are pre-computed, ***only once***, generating a set:

$$A = \begin{bmatrix} \mathbf{p}_0, & \mathbf{p}_1, & \cdots & \mathbf{p}_n \\ \mathbf{q}_0, & \mathbf{q}_1, & \cdots & \mathbf{q}_n \end{bmatrix}$$

  - This set is ***always*** used for the extraction of all descriptors in all photos!
    - If this is not done, we cannot do comparisons because we are comparing different tests (e.g., comparing apples and oranges):
      - We need to keep ***consistency***

BRIEF Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Patent free.
- Disadvantage:
  - Rotation is an issue:
    - The method can handle rotations up to 10-15 degrees only.

BRIEF Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Patent free.
- Disadvantage:
  - Rotation is an issue:
    - The method can handle rotations up to 10-15 degrees only.

BRIEF Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Patent free.
- Disadvantage:
  - Rotation is an issue:
    - The method can handle rotations up to 10-15 degrees only.

BRIEF Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Patent free.
- Disadvantage:
  - Rotation is an issue:
    - The method can handle rotations up to 10-15 degrees only.

ORB Descriptor

- The descriptor is a modified version of BRIEF and it can handle rotations!
- The first step of the algorithm is to compute the orientation of the current patch P.
- **Idea**: we determine the image's "*center of mass*", and we compute the angle between this "*center of mass*" and the center of the patch. This is a hint for the orientation of the patch.

ORB Descriptor: Patch Orientation

- We compute the patch orientation using Rosin moments of a patch:

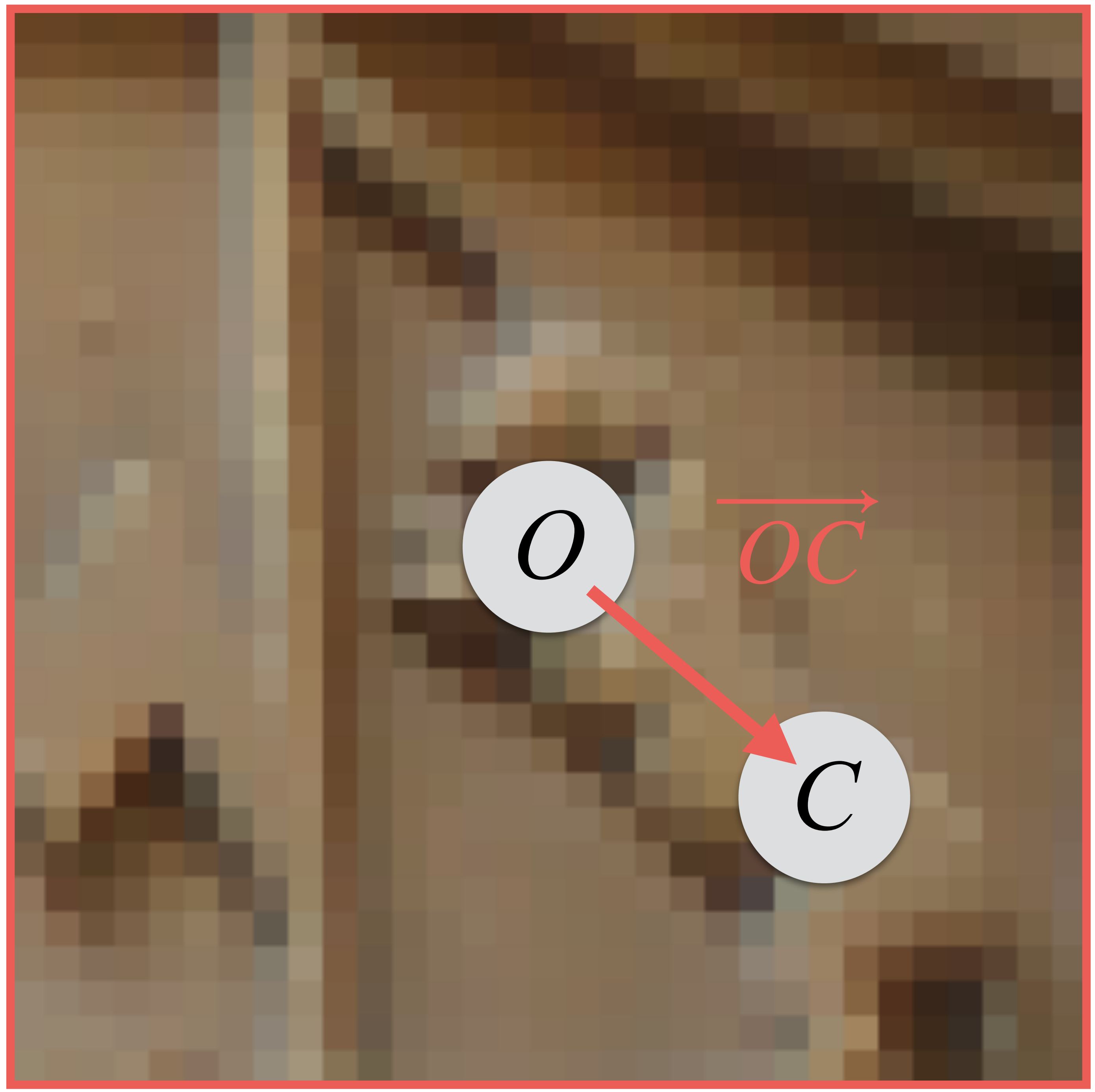
$$m_{a,b} = \sum_{x,y \in P} x^a y^b P(x, y)$$

- From this, we define the centroid, C, as

$$C = \left(\frac{m_{1,0}}{m_{0,0}} \frac{m_{0,1}}{m_{0,0}} \right)$$

- Now, we can create a vector from corner's center, O, to the centroid, C. This allows us to calculate the angle of rotation.

ORB Descriptor: Patch Orientation



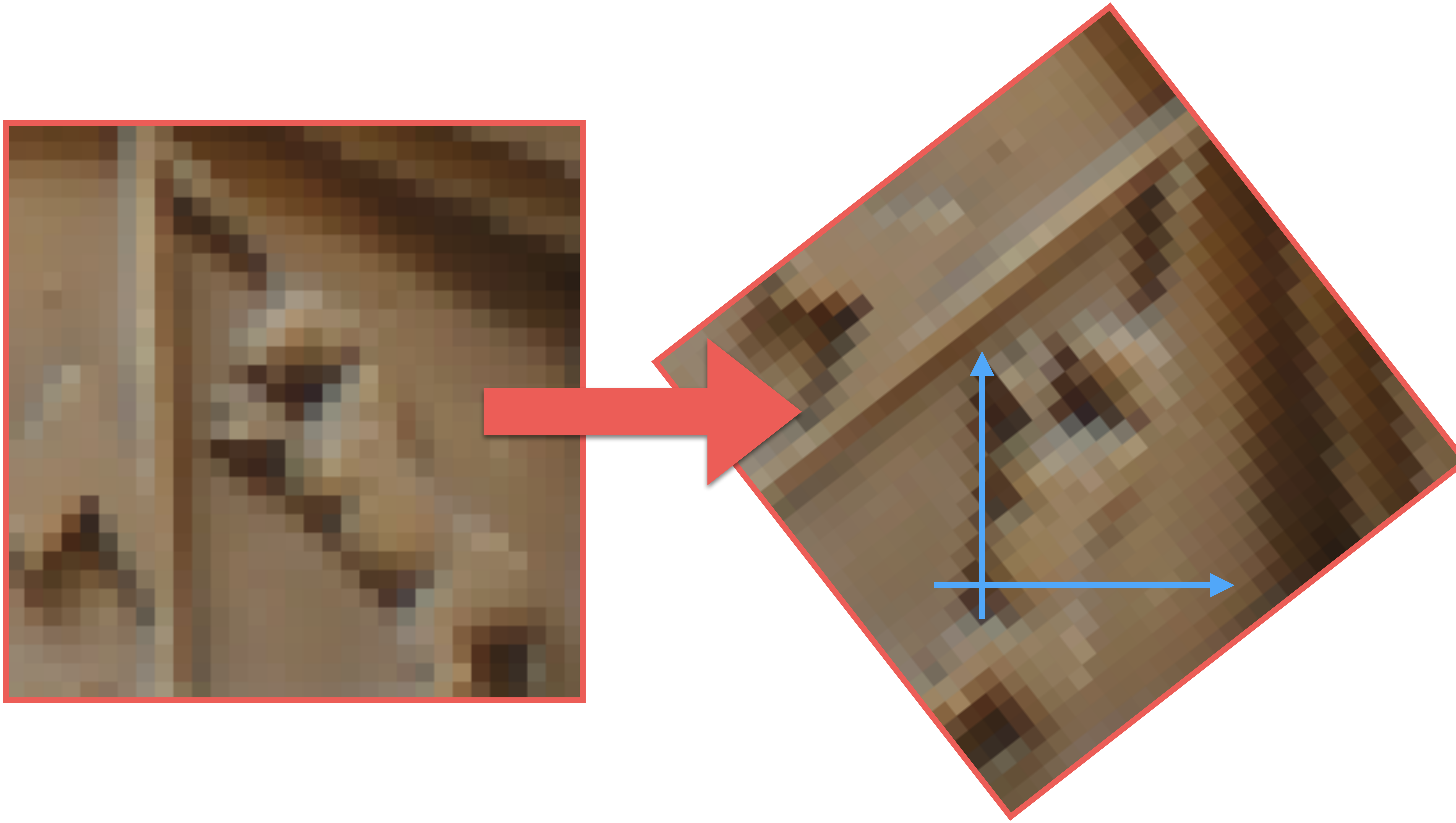
ORB Descriptor: Patch Orientation

- From this vector, the orientation of the patch can be computed simply as

$$\theta = \text{atan2}(m_{0,1}, m_{1,0})$$

- From this, we can rotate the patch P, but this operation is very computationally expensive:
    - We need to rotate each single point in the patch!

ORB Descriptor



ORB Descriptor: Patch Orientation

- Instead of rotating the whole patch, we can rotate only the points stored in A as:

$$A_\theta = \mathbf{R}_\theta \cdot A$$

where $\mathbf{R}_\theta$ is a 2D rotation matrix:

$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

- **NOTE**: we need to rotate less points!

ORB Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Invariant to rotation.
  - Patent free.
- Disadvantage:
  - Not robust as SIFT.

SIFT Descriptor

- It is the state-of-the-art descriptor.
- It was introduced in 1999, but it is still the king.

SIFT Descriptor: Patch Orientation

- The first step is to compute the orientation of P.
- We compute the horizontal (P_x) and vertical (P_y) gradients of the P.
- For each pixel at coordinates (i, j) in the patch we compute its orientation and magnitude:

$$m(i,j) = \sqrt{P_x(i,j)^2 + P_y(i,j)^2}$$

$$\theta(i,j) = \operatorname{atan2}\Big(P_y(i,j), P_x(i,j)\Big)$$

SIFT Descriptor: Patch Orientation

- A histogram, H, of directions is created for each orientation taking into account its magnitude.
- We repeat this process for all gradients in the patch!
  - Note that H is initialized as a vector of zeros.

SIFT Descriptor: Patch Orientation

- Let's say, we have a histogram H with 18 bins (b = 18).
  - This means each bin has a size (k) in degree of 20°:
    - $k = 360/b = 360/18$
- Now, we have to insert a gradient, $m = 10$ and $\theta = 45°$, from our patch in H we need to process a gradient in the patch.
  - First, we compute the index of the bin to update:

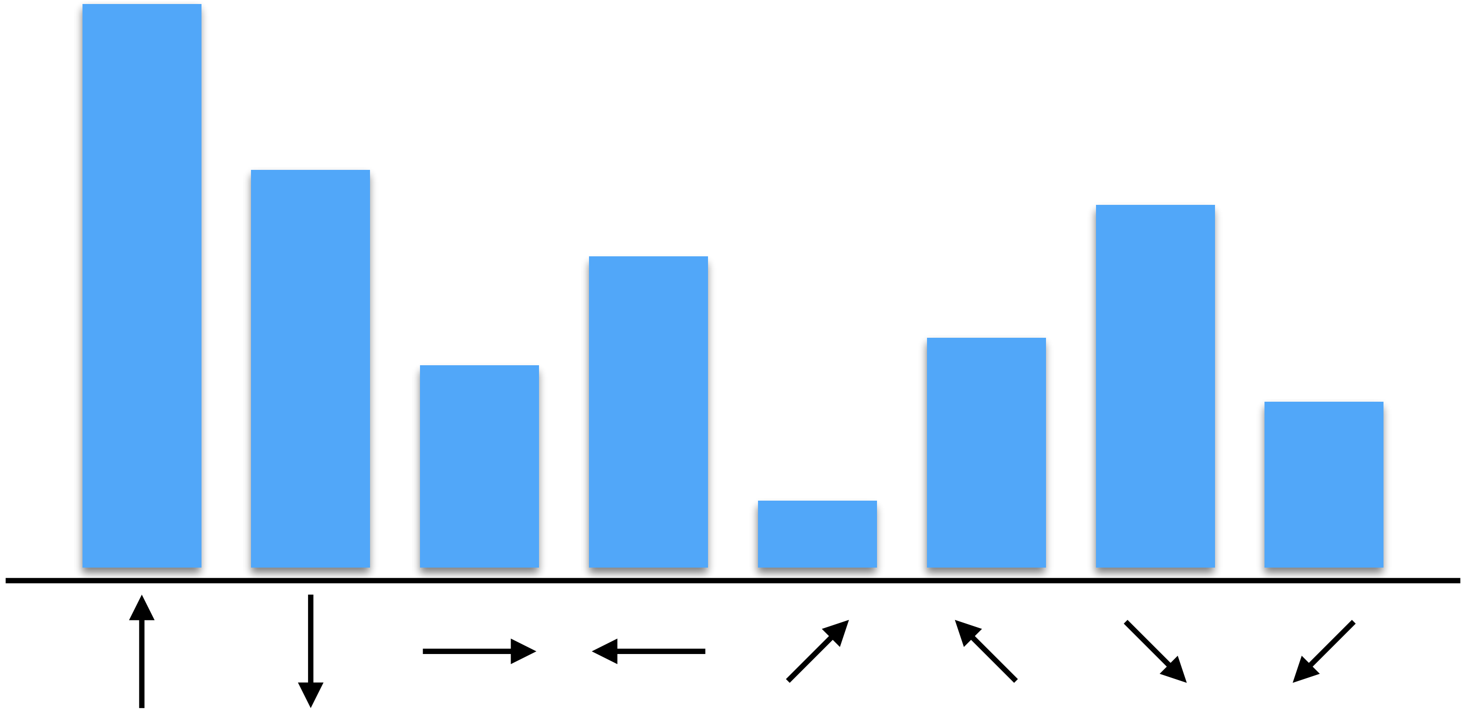
$$i = \left\lfloor \frac{\theta}{k} \right\rfloor = \left\lfloor \frac{45}{20} \right\rfloor$$

  - Then, we update H as

$$H(i) = H(i) + m = H(i) + 10$$

SIFT Descriptor: Patch Orientation

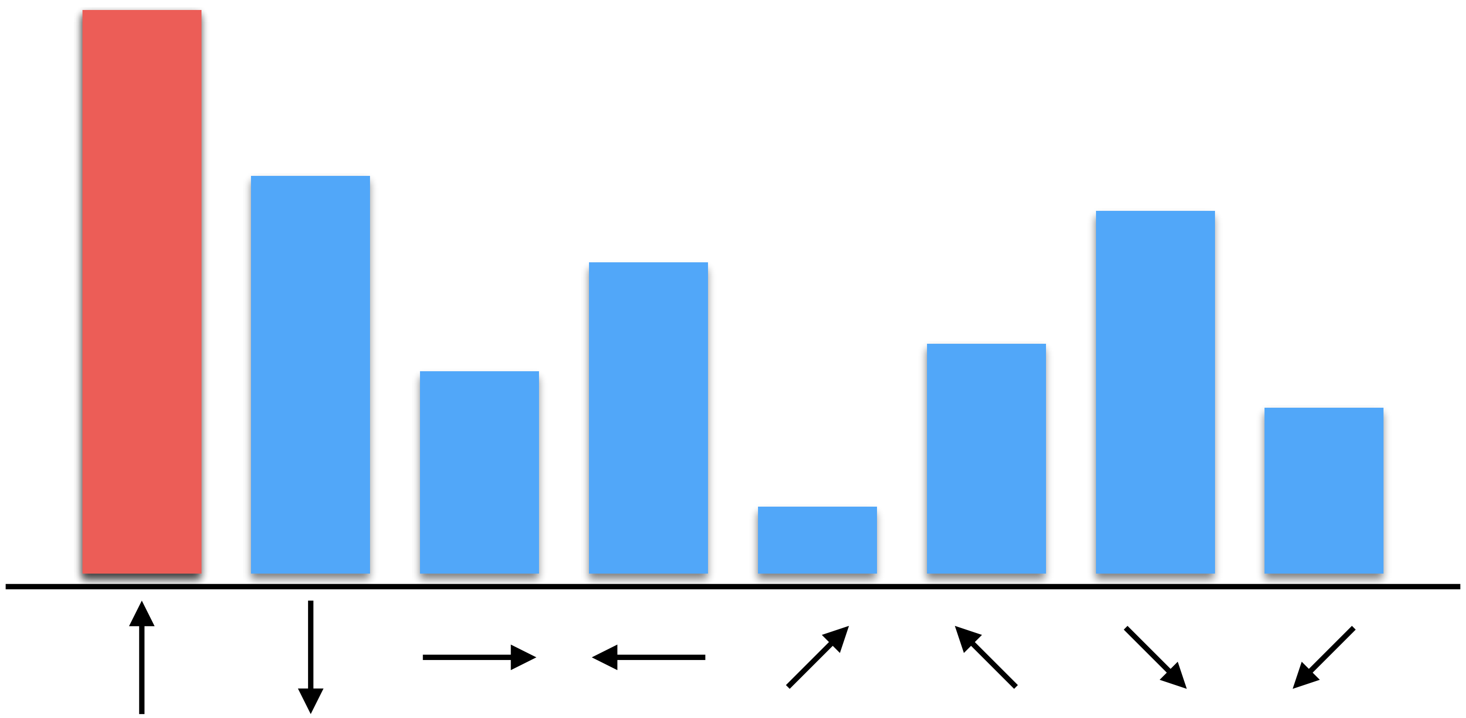
- Finally, we get this (an example with 8 bins; i.e., 8 directions):



- The patch orientation, α, is given by the highest peak:
  - If we have two equal peaks, we take the as winner the first one in histogram.

SIFT Descriptor: Patch Orientation

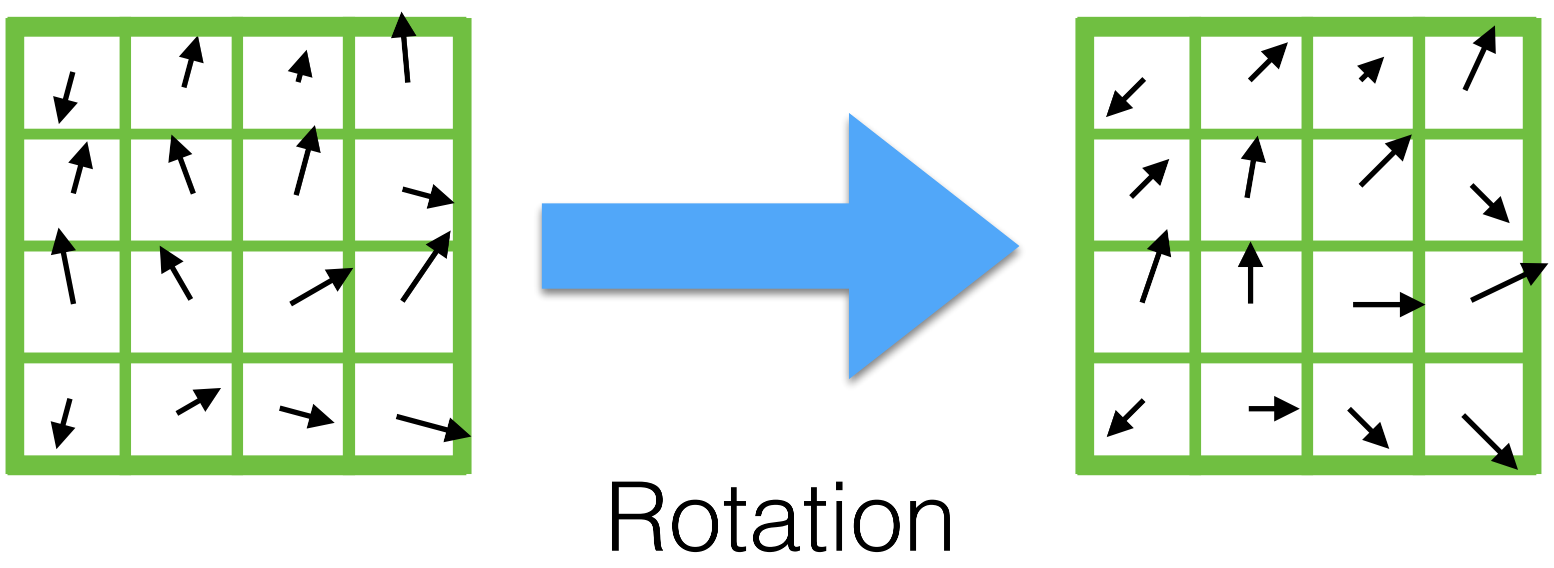
- Finally, we get this (an example with 8 bins; i.e., 8 directions):



- The patch orientation, α, is given by the highest peak:
    - If we have two equal peaks, we take the as winner the first one in histogram.

SIFT Descriptor

- Once we have α, we can rotate all gradients in the patch using it.
  - This ensures to be invariant to rotations!



SIFT Descriptor

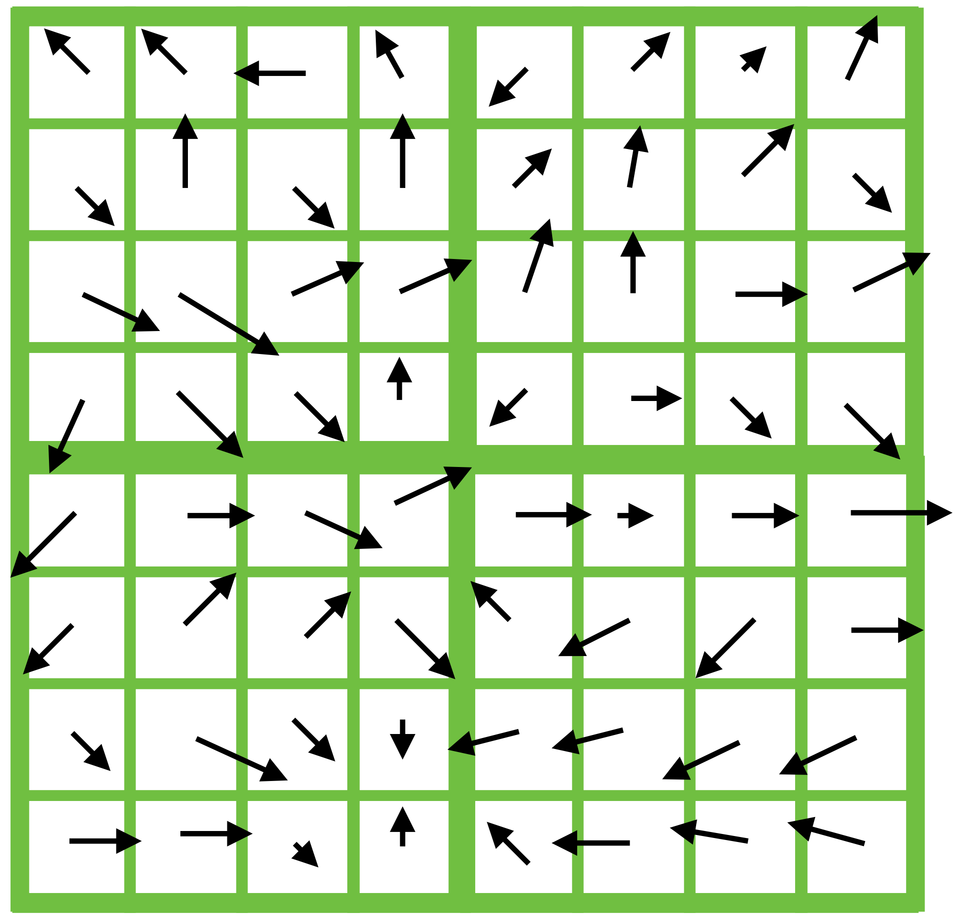
- Why do we rotate the gradients? It is computationally faster:
  - In theory, we should rotate the patch and then recompute the gradients.
  - This is computationally expensive!

SIFT Descriptor

- At this point, we divide the patch into 4x4 blocks. For each block, we compute a new histogram of directions.

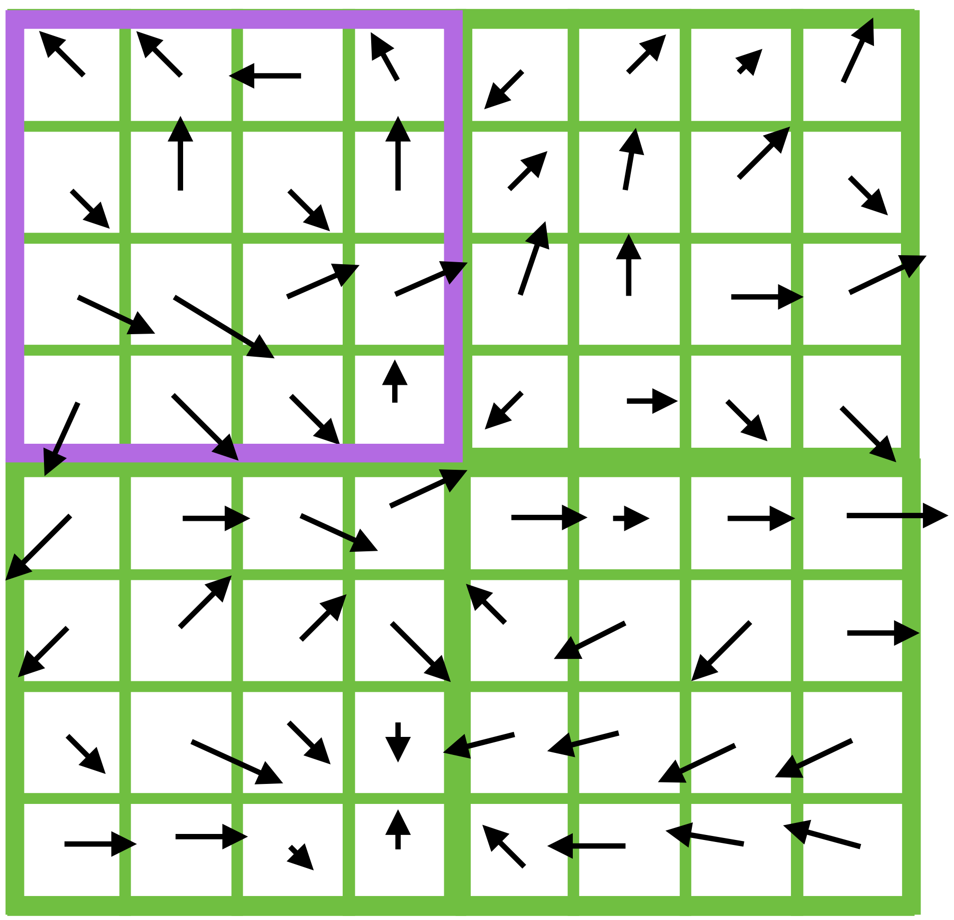
- The final SIFT descriptor is the concatenation (flattening) of all these histograms.

SIFT Descriptor: Example with 2x2 Blocks

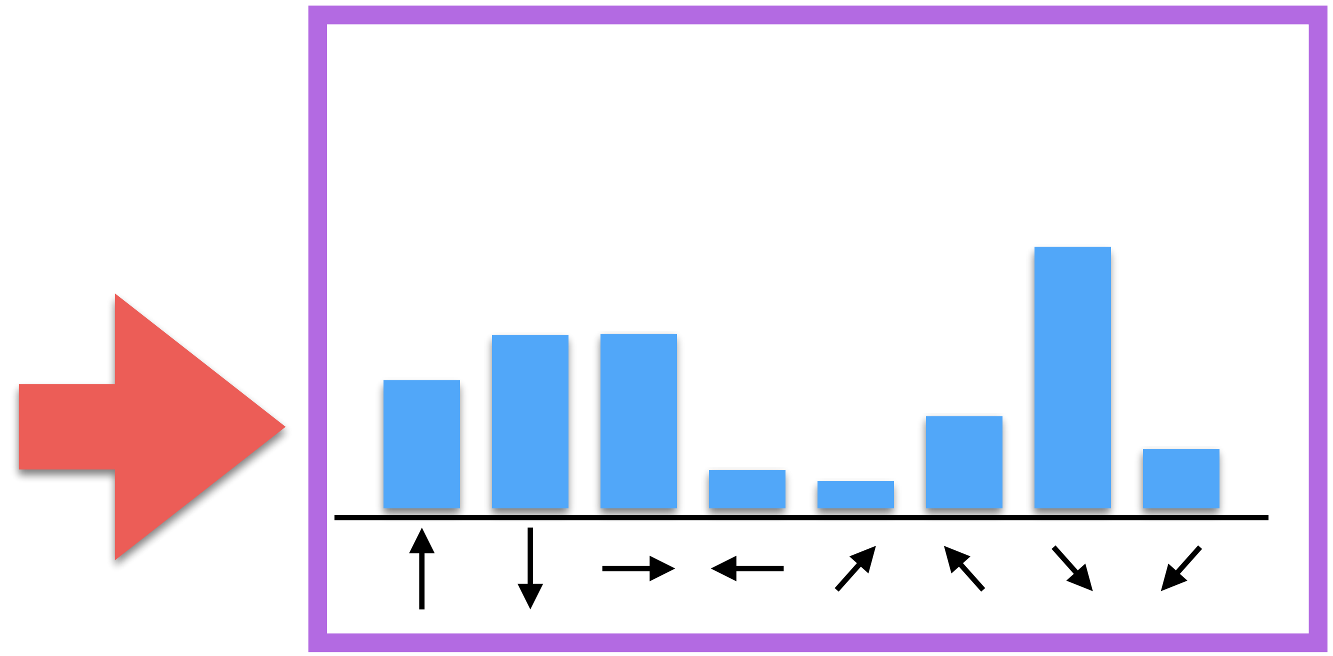


Patch and its gradients

SIFT Descriptor: Example with 2x2 Blocks

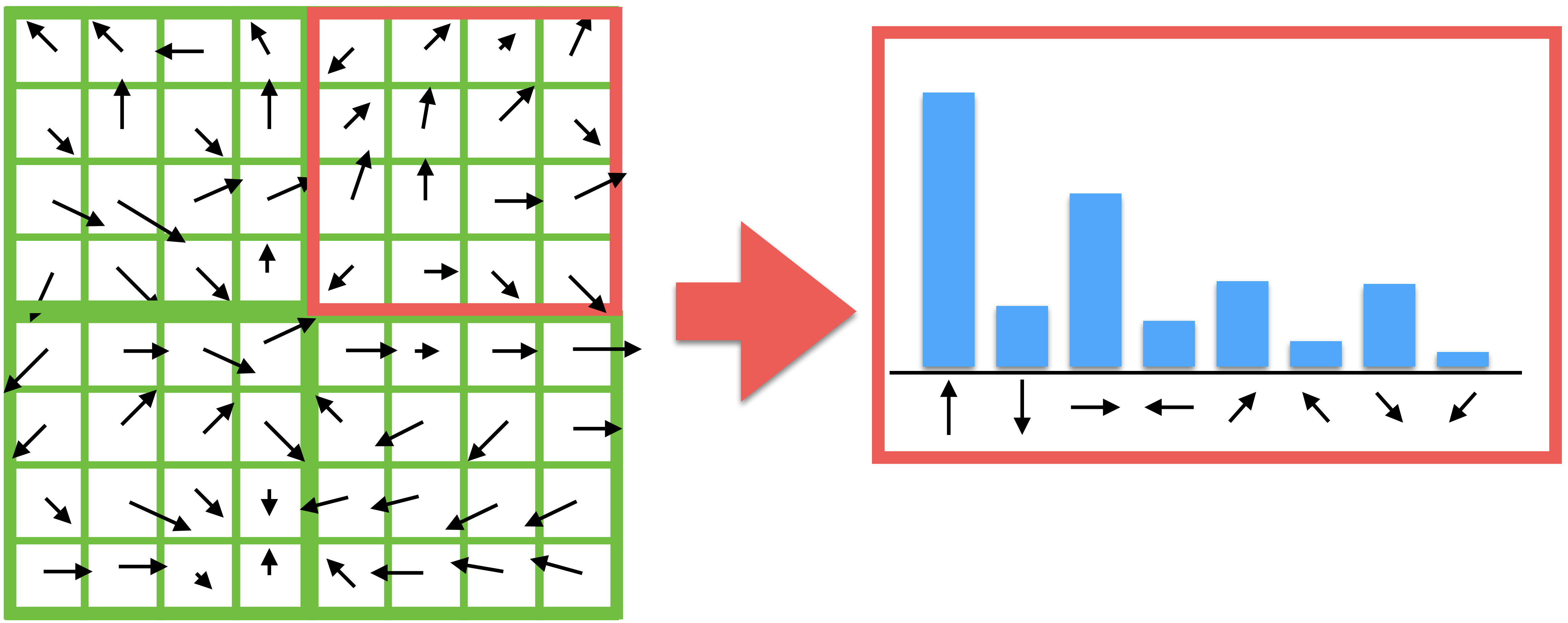


Patch and its gradients



We compute the histogram for the first block in violet

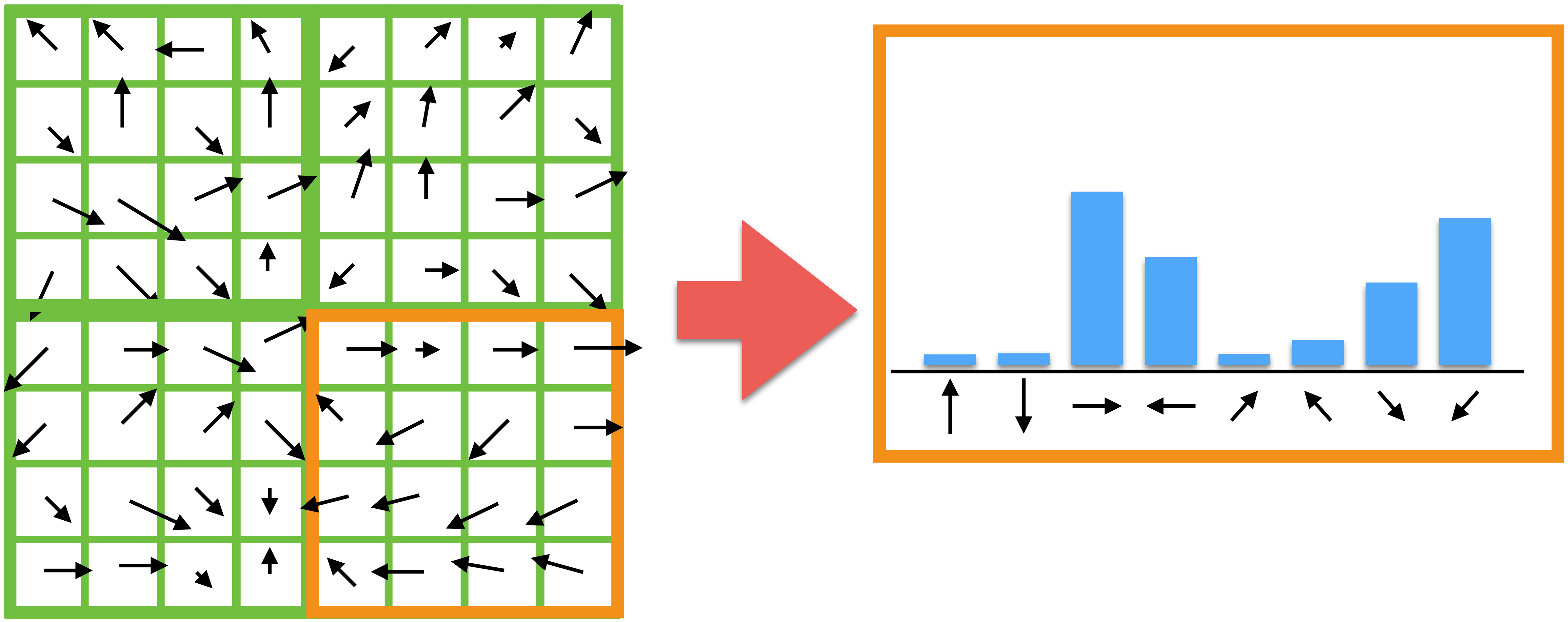
SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the second block in red

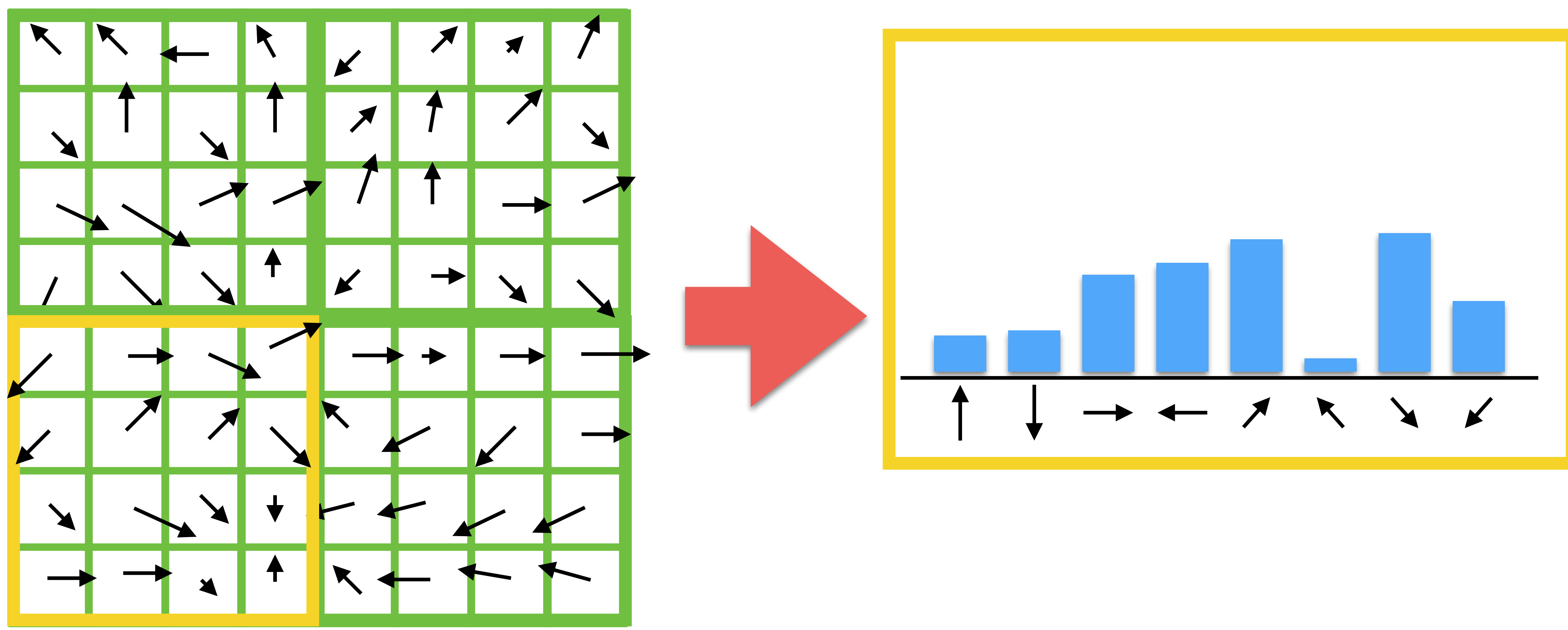
SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the third block in orange

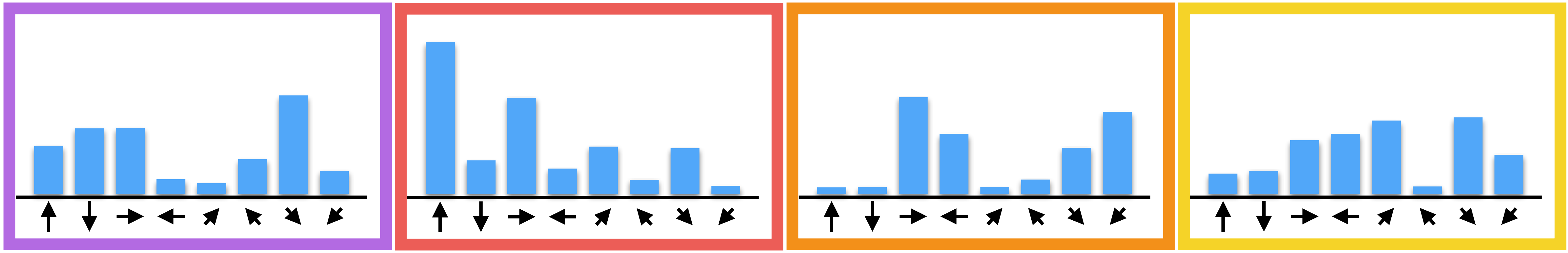
SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the fourth block in yellow

SIFT Descriptor: Example with 2x2 Blocks



The ***final descriptor*** is the concatenation of the histogram of all blocks.
Note that this can be encoded as a vector; in this example the vector has size equal to:

$$4x8 = \mathbf{32}$$

4 because we have 2x2 Blocks
8 because we have 8 direction for each histogram.

SIFT Descriptor: Test

- We test the differences as distance between histograms:

$$D_2(\mathbf{h}^1, \mathbf{h}^2) = \sqrt{\sum_{i=1}^{n} (h_i^1 - h_i^2)^2}$$

- **The lower the closer**:
  - This is the opposite compared to BRIEF/ORB.

SIFT Descriptor

- Advantages:
  - Invariant to illumination changes.
  - Invariant to rotation.
- Disadvantages:
  - Slower than BRIEF/ORB.
  - More memory than binary methods.
  - ~~Patented!~~ It is patent-free from 12th of Aprile 2020!

Matching Images

Matching: An Image Against Another One

- **Input**: two descriptor lists (*they can be of equal or different size*), $\mathbf{desc}_1$ and $\mathbf{desc}_2$, respectively of image I_1 and I_2.

- **Output**: a vector with indices of matches for each list:

    - The output is called $\mathbf{M}_{12}$ if we match I_1 against I_2

    - The output is called $\mathbf{M}_{21}$ if we match I_2 against I_1

Matching:
How the Output is Encoded Example 1

- Let's say we have 4 descriptors in $\mathbf{desc}_1$
- Let's say we have 3 descriptors in $\mathbf{desc}_2$
- Let's say that we want to match I_1 against I_2, this means that we want to compute $\mathbf{M}_{12}$.

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

$$\mathbf{M}_{12} = [\ ]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d^2_1 \\ d^2_2 \\ d^2_3 \end{bmatrix}$$

We find out that the first descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [\ ]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d^2_1 \\ d^2_2 \\ d^2_3 \end{bmatrix}$$

We find out that the first descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2,]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d^2_1 \\ d^2_2 \\ d^2_3 \end{bmatrix}$$

We find out that the second descriptor of $\mathbf{desc}_1$ matches with the third descriptor of $\mathbf{desc}_2$.

$\mathbf{M}_{12}$ =[2,]

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the second descriptor of $\mathbf{desc}_1$ matches with the third descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3,]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the third descriptor of $\mathbf{desc}_1$ matches with the first descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3,]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d^2_1 \\ d^2_2 \\ d^2_3 \end{bmatrix}$$

We find out that the third descriptor of $\mathbf{desc}_1$ matches with the first descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2,\ 3,\ 1,]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the fourth descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3, 1, 2]$$

Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d^2_1 \\ d^2_2 \\ d^2_3 \end{bmatrix}$$

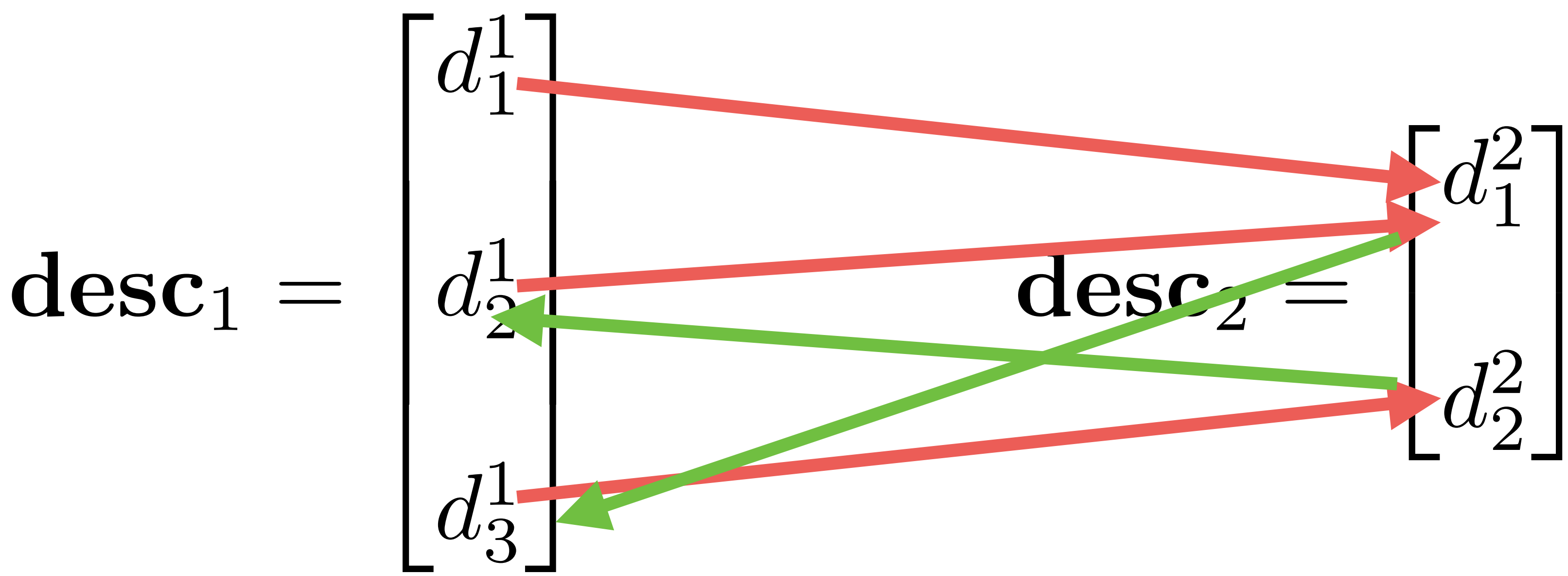
We find out that the fourth descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3, 1, 2]$$

Matching: How the Output is Encoded Example 2

- Let's say we have 3 descriptors in $\mathbf{desc}_1$
- Let's say we have 2 descriptors in $\mathbf{desc}_2$
- Let's say that we match I_1 against I_2, obtaining $\mathbf{M}_{12}$. Then, we match I_2 against I_1 obtaining $\mathbf{M}_{21}$.

Matching: Example 2



$\mathbf{M}_{12} = [1,1,2]$ $\mathbf{M}_{21} = [3,2]$

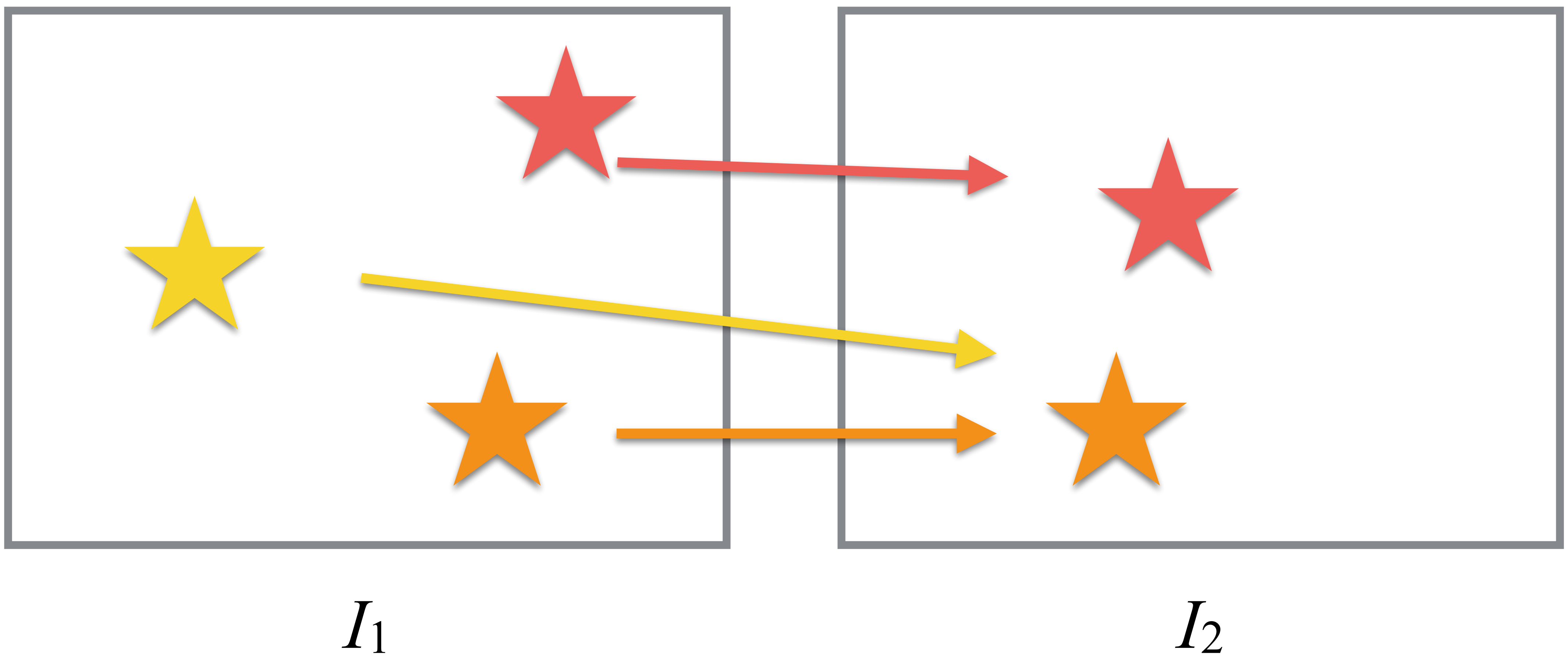
Matching: Example 2

- From this example, we can notice that:
  - The matching operator is ***NOT*** an invertible function:
    - Therefore, $\mathbf{M}_{12}$ and $\mathbf{M}_{21}$ can be very different!
  - Why? Let's see it!

Matching: Example 2

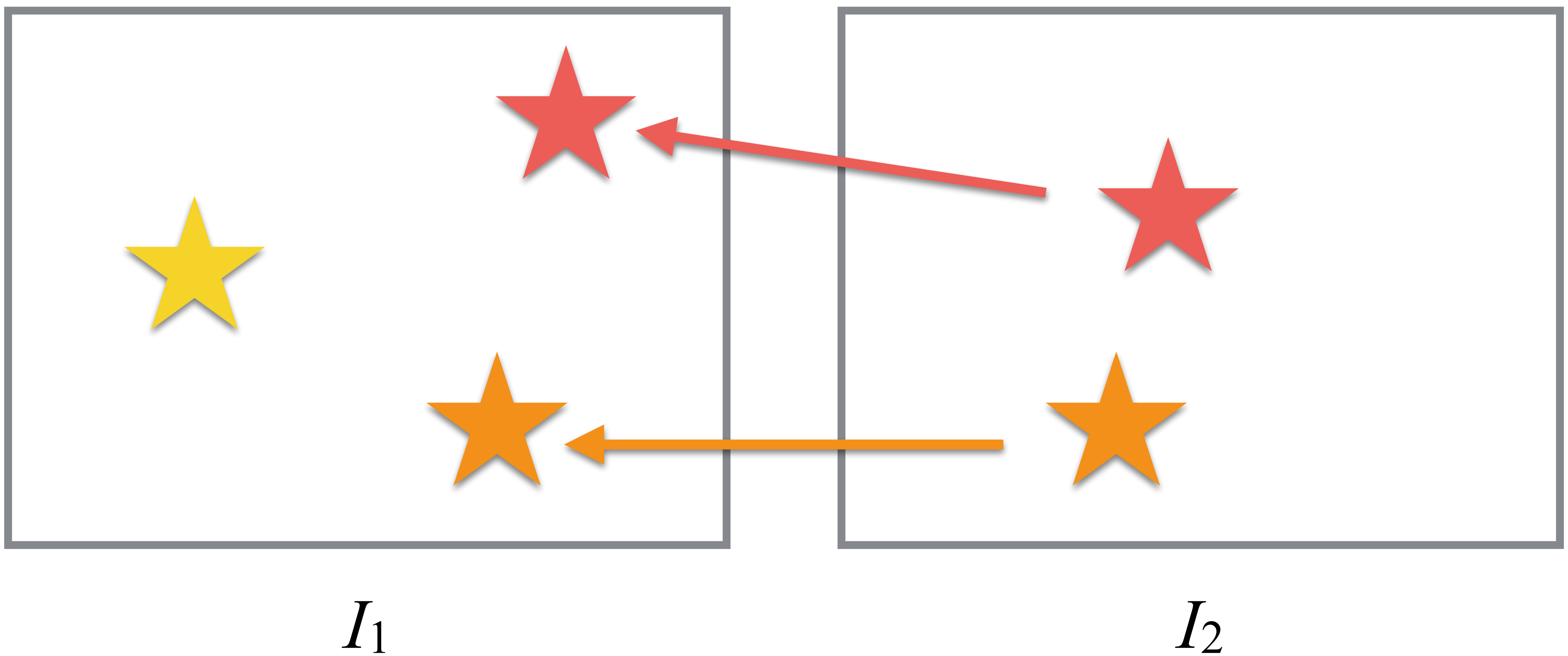
Let's say that stars in this example are feature-point.

Matching: Example 2



When we match I_1 against I_2, we have three matches. This is because we need to match the **yellow** star with something in the other image no matter what.

Matching: Example 2



When we match I_2 against I_1, we do not have a match between the **yellow** star in I_1 and the **red**/**orange** one in I_2 because the other **red**/**orange** star in I_1 is closer than the **yellow** star!

Matching: Brute Force Algorithm

- A simple method to find a ***matched descriptor*** in $\mathbf{desc}_2$ for each descriptor in $\mathbf{desc}_1$:
    - For each descriptor in $\mathbf{desc}_1$, we test it against all descriptors in $\mathbf{desc}_2$, and we keep as matched one **the closest** (in terms of distance; either Hamming or Euclidean).

Matching: Brute Force Algorithm

For each descriptor $\mathbf{d}^1{}_i$ in $\mathbf{desc}_1$:

matched(i) = -1;

matched_dist = **BOTTOM**;

For each descriptor $\mathbf{d}^2{}_j$ in $\mathbf{desc}_2$:

if Closer($D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$, matched_dist)

matched(i) = j;

matched_dist = $D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$;

endif

$D(,)$ is a distance function; it can be Hamming, Euclidean, etc.

Matching: Brute Force Algorithm

For each descriptor $\mathbf{d}^1{}_i$ in $\mathbf{desc}_1$:

matched(i) = -1;

matched_dist = **BOTTOM**;

For each descriptor $\mathbf{d}^2{}_j$ in $\mathbf{desc}_2$:

if Closer($D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$, matched_dist)

matched(i) = j;

matched_dist = $D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$;

endif

$D(,)$ is a distance function; it can be Hamming, Euclidean, etc.

Matching: Brute Force Algorithm

For each descriptor $\mathbf{d}^1{}_i$ in $\mathbf{desc}_1$:

matched(i) = -1;

matched_dist = **BOTTOM**;

For each descriptor $\mathbf{d}^2{}_j$ in $\mathbf{desc}_2$:

if Closer($D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$, matched_dist)

matched(i) = j;

matched_dist = $D(\mathbf{d}^1{}_i, \mathbf{d}^2{}_j)$;

endif

BOTTOM = +Inf for SIFT
BOTTOM = 0 for BRIEF/ORB

$D(,)$ is a distance function; it can be Hamming, Euclidean, etc.

Matching:
Brute Force Algorithm

- Advantage:
  - It is **exhaustive** (i.e., it takes a lot of time!) and finds the ***best solution***!
- Disadvantage:
  - This method is very slow:
    - Let's say we have n descriptors in $\mathbf{desc}_2$ and n in $\mathbf{desc}_2$. In the worst case, we need to compare roughly n^2 descriptors. This becomes an issue when we have more than 100 descriptors per image!

Matching: Improving Efficiency

- How can we improve (approximating results)?
- **Hashing**: the idea is to group similar descriptors in k groups or buckets that have a constant size.

Matching: Improving Efficiency

- We create k bucket.

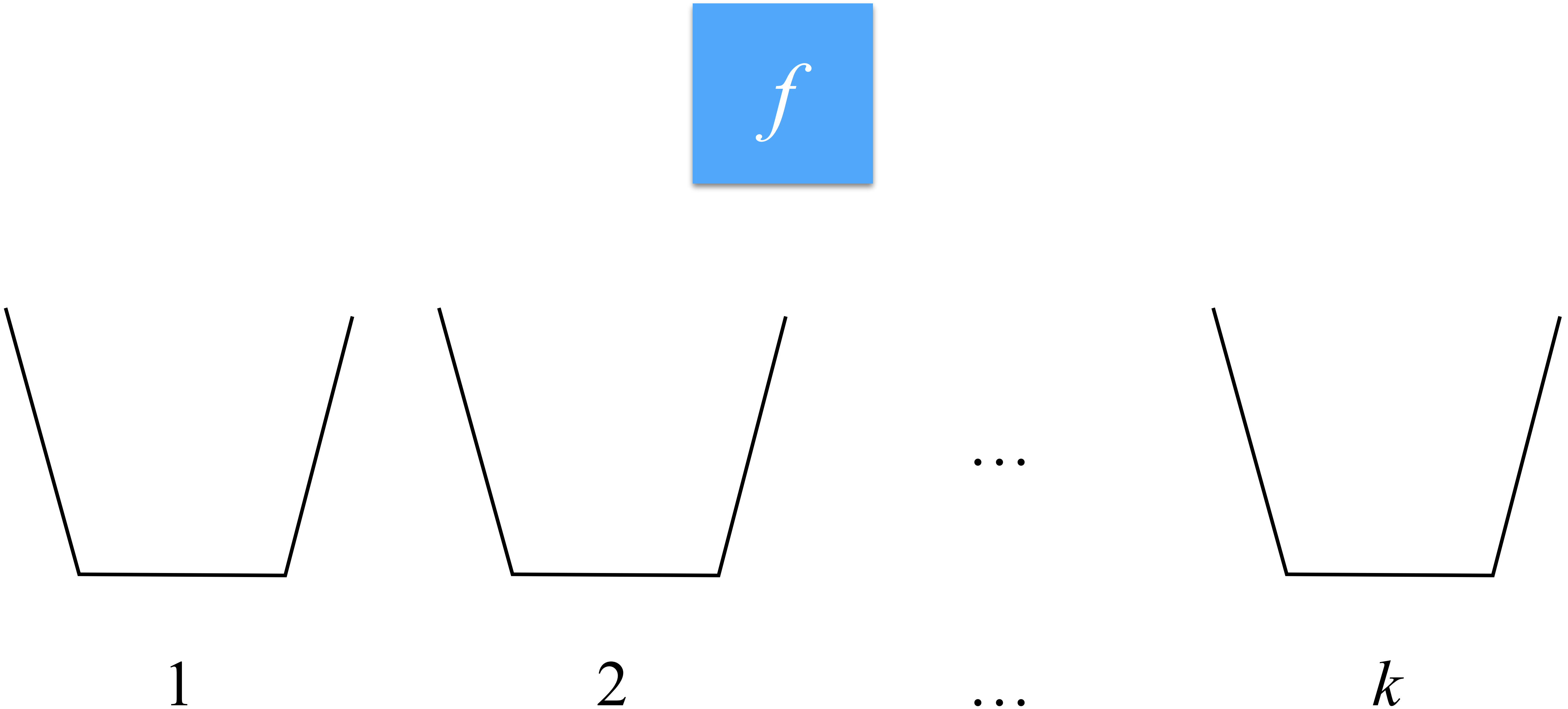
  - Each ***descriptor*** in $\mathbf{desc}_2$ of I_2 is assigned to a bucket using a function f, called hash function. This is defined as:

$$f\text{: } \textbf{\textit{descriptor}} \rightarrow [1,k] \text{ (positive integer numbers!)}$$

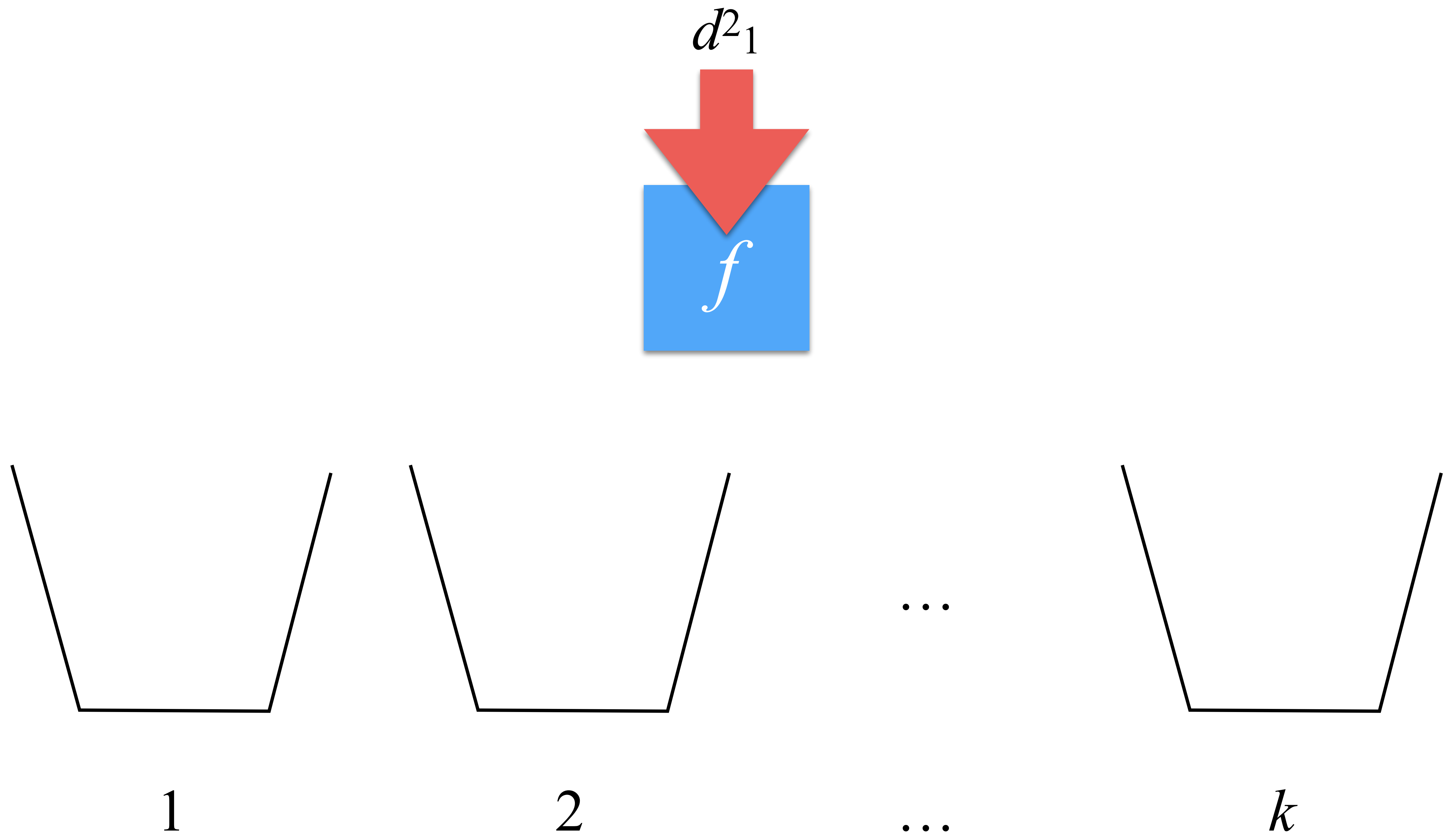
  - This means that f generates a number in $[1, k]$ given a descriptor.

    - For example, an f for BRIEF/ORB, where the descriptor is a 256-bit number, is the modulo operation.

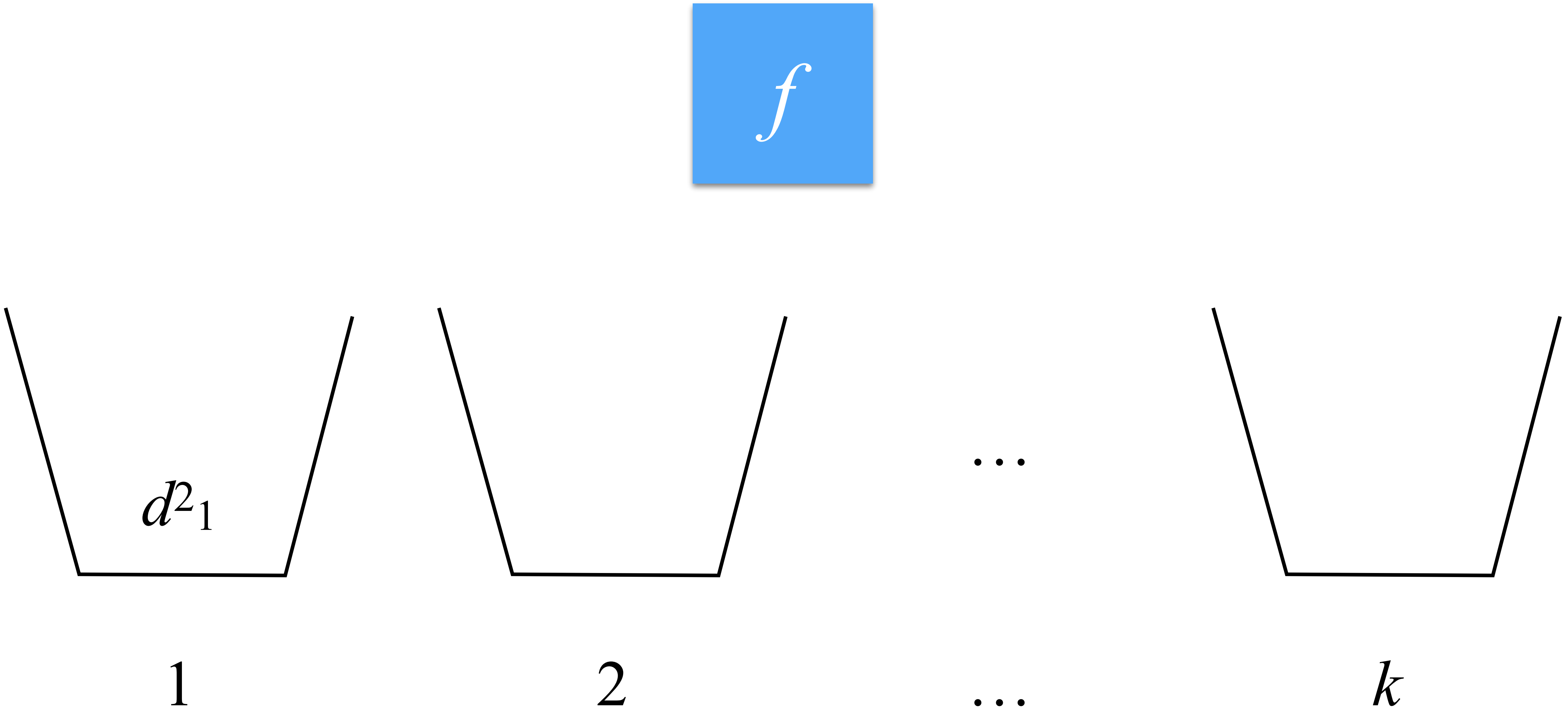
Matching: Improving Efficiency



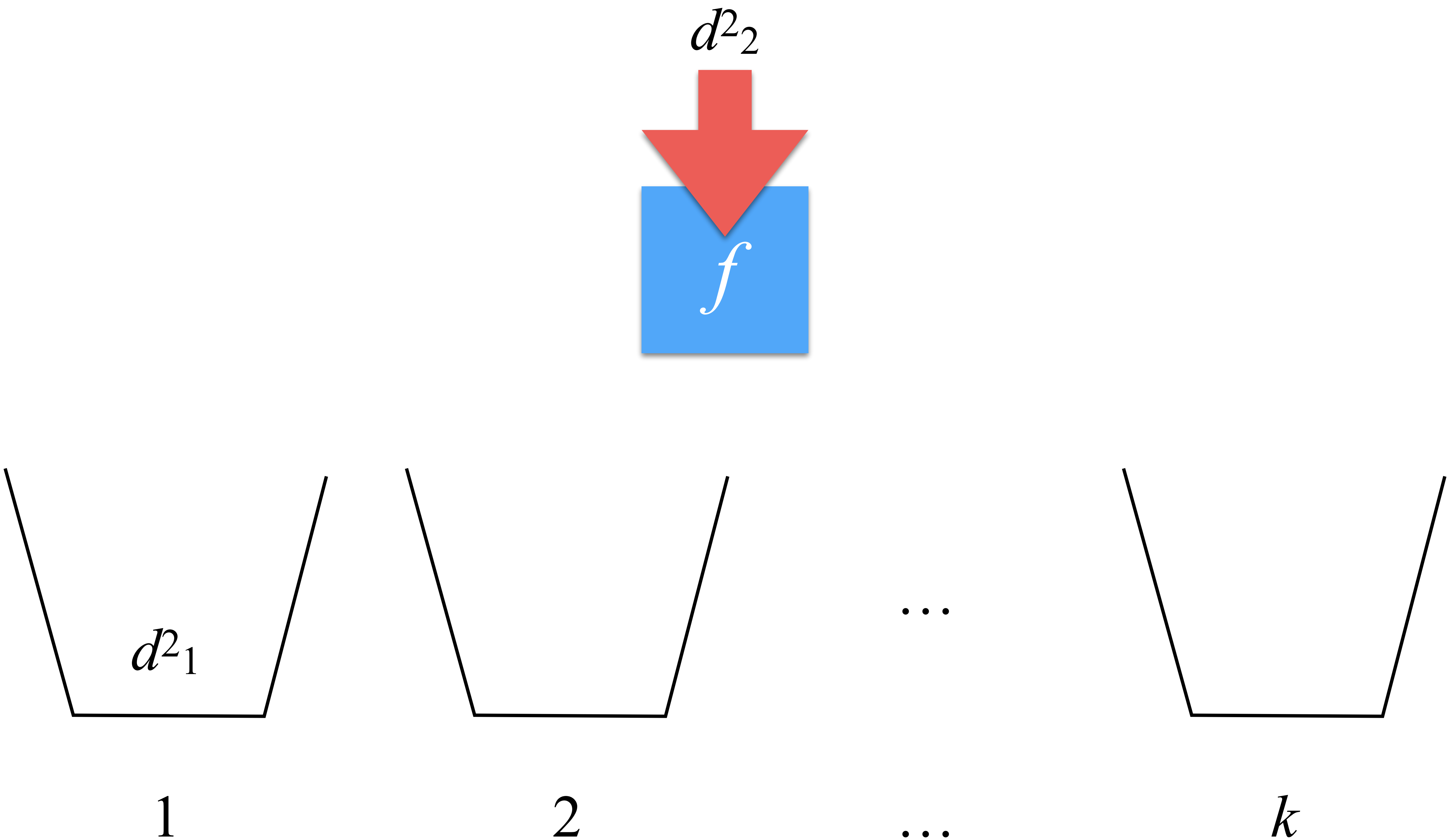
Matching: Improving Efficiency



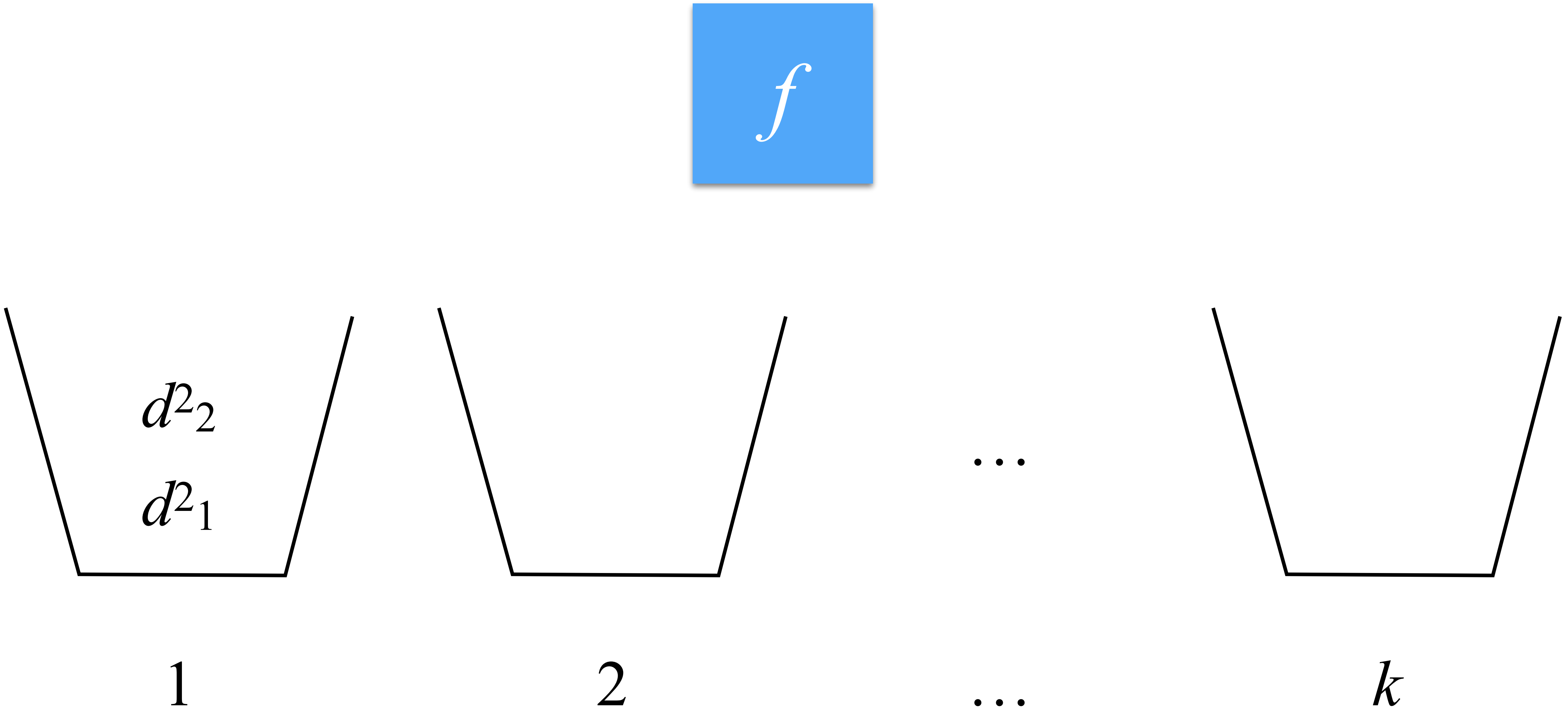
Matching: Improving Efficiency



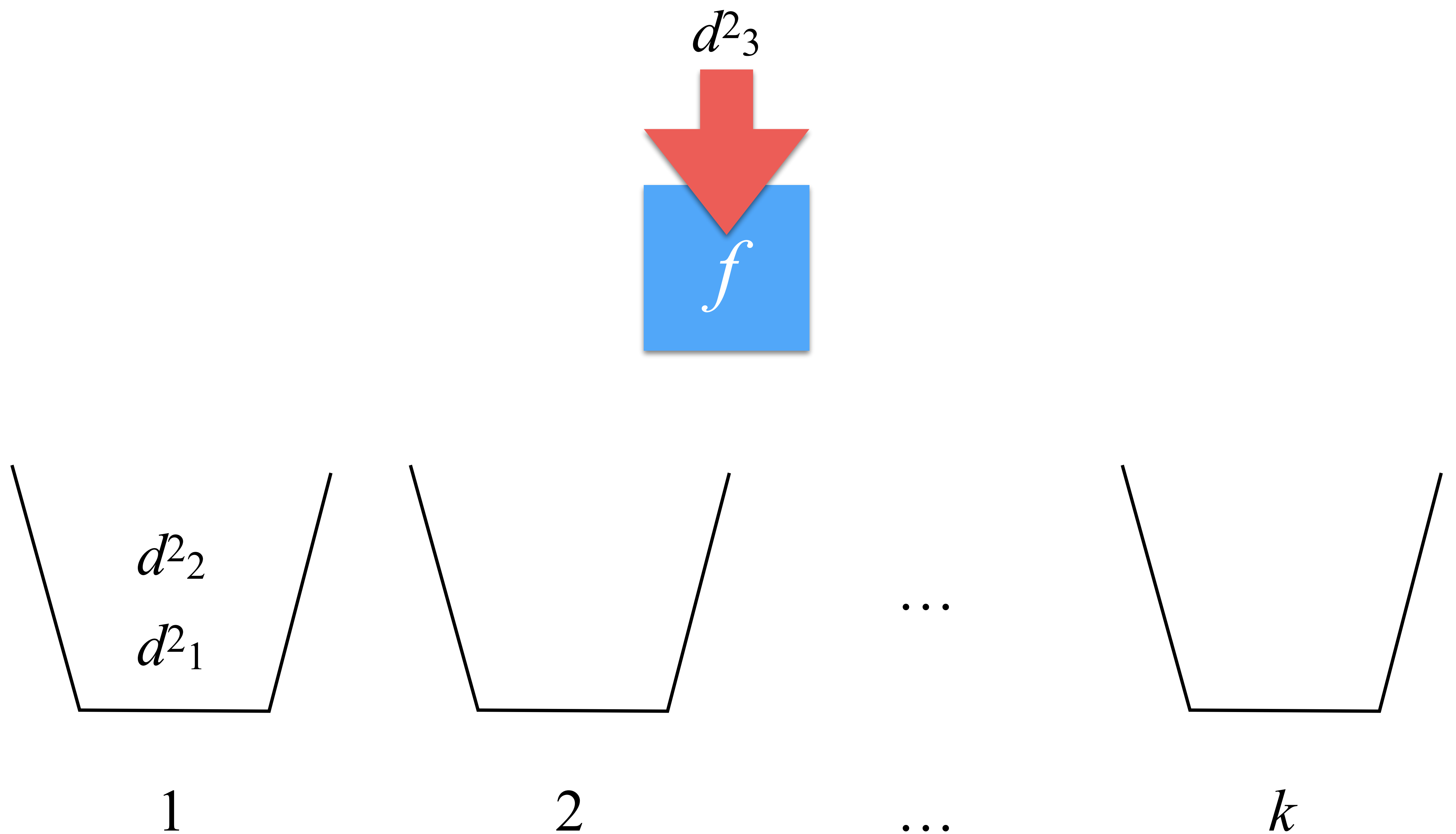
Matching:
Improving Efficiency



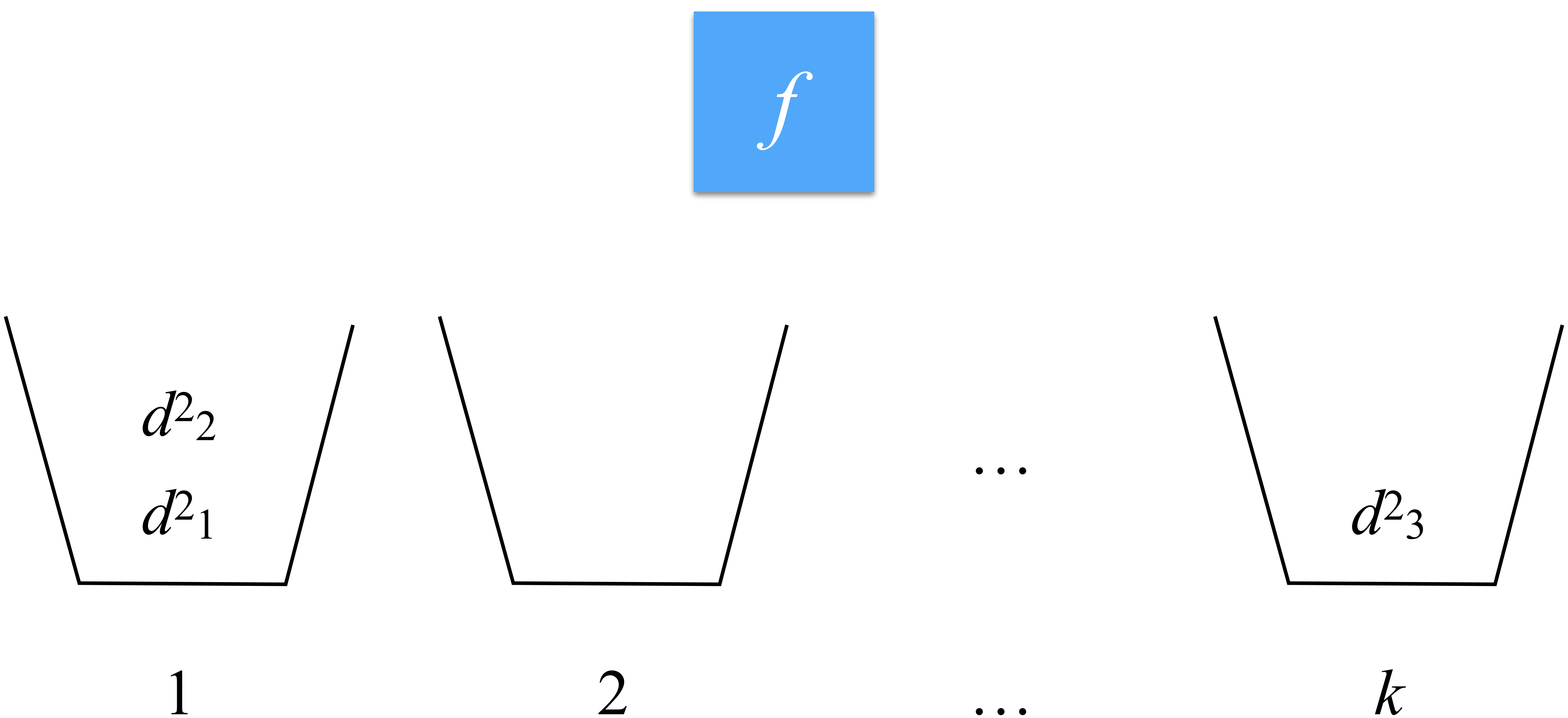
Matching: Improving Efficiency



Matching: Improving Efficiency



Matching: Improving Efficiency

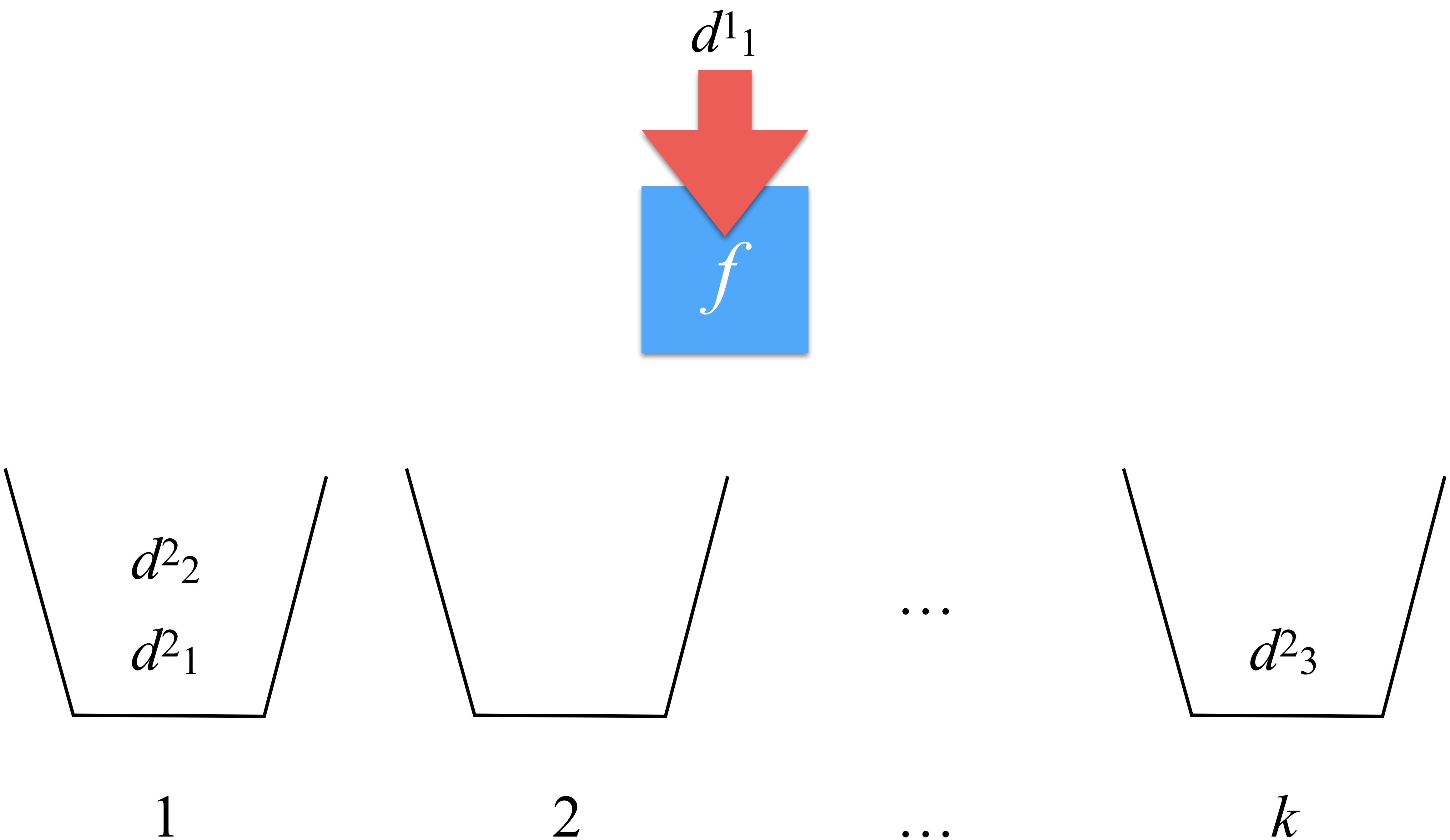


etc.

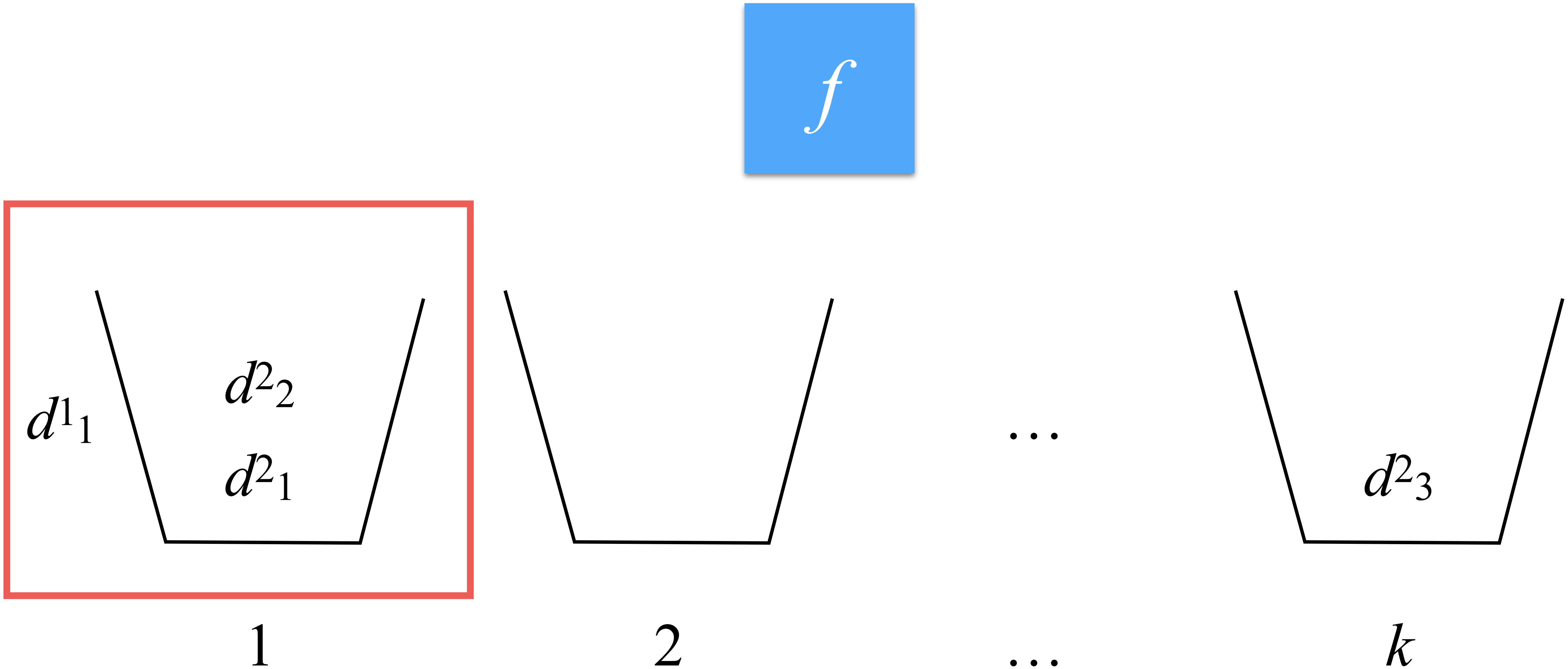
Matching: Improving Efficiency

- Now, we have all descriptors of I_2 into buckets.
- To find a match for a descriptor $d^1{}_i$ of I_1, we apply f to $d^1{}_i$. In this way, we obtain a bucket number, let's call it r.
- Finally, we run the brute force method between $d^1{}_i$ and all the descriptors that are in r.

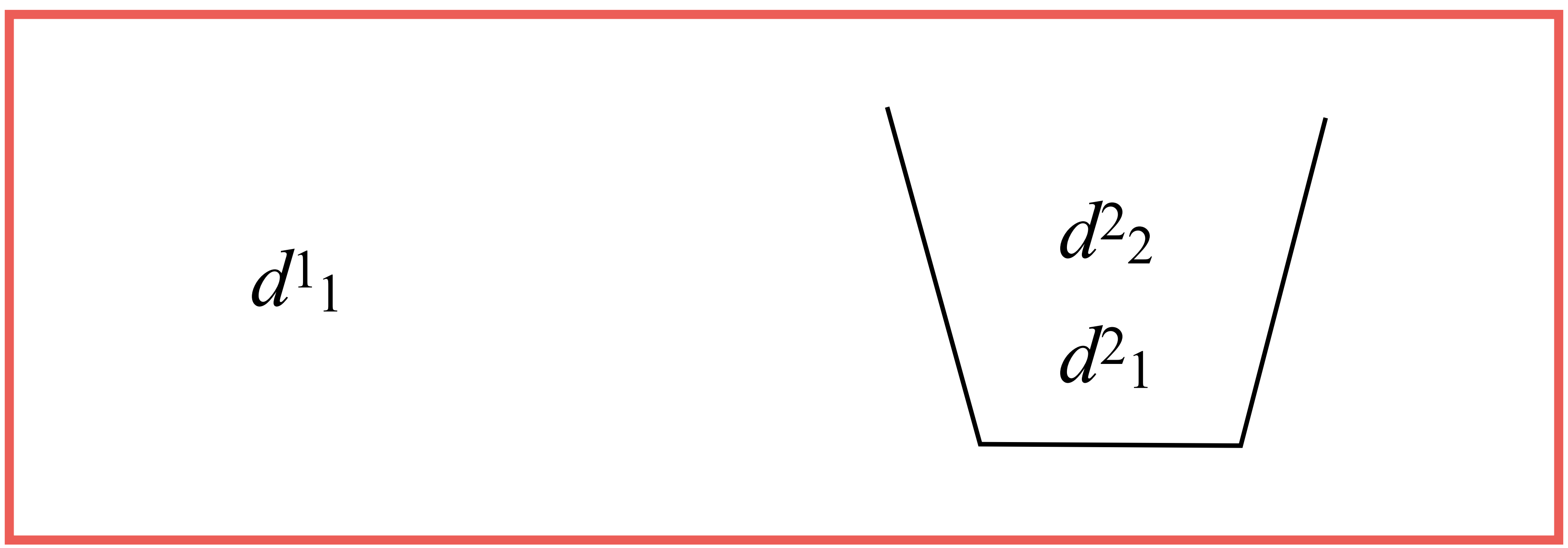
Matching:
Improving Efficiency



Matching: Improving Efficiency



Matching: Improving Efficiency

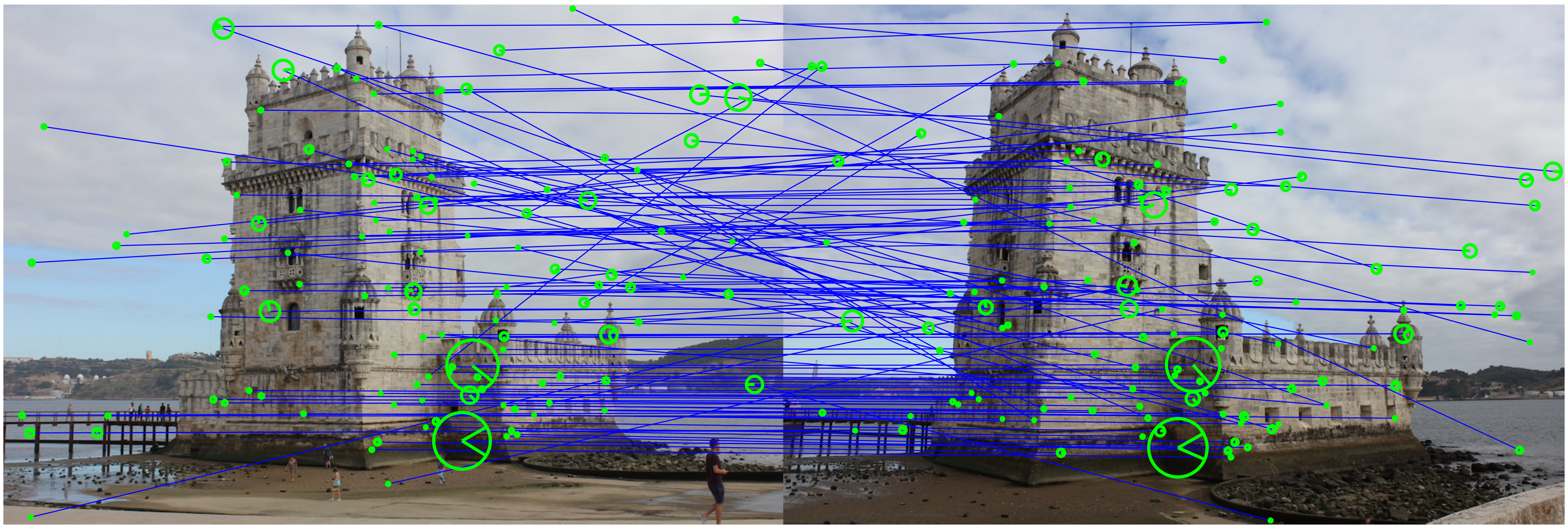


We run brute-force: we compare $d^1{}_1$ with descriptor in the bucket.

Matching: Improving Efficiency

- Advantages:
  - It is faster, we run the brute force method for a subset of descriptors.
- Disadvantages:
  - It is not exact, it is an ***approximation***:
    - We test only a sub-set of descriptors.
  - If f is not well crafted, we may have **distant** (i.e., not similar) descriptors in the same bucket.

Matching: Example



Matching

- Once we have know matches between images, we can understand which images are near each others:
    - This is important for the triangulation of points, and the camera calibration step.

that’s all folks!