

3D Models

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3D Models

- A 3D model is a computational representation of a real-world object. This is typically:
 - C0
 - Closed (not always!)
 - Discretized



3D Models

- Two main representations:
 - **Boundary representations** (b-rep): a 3D object is represented as a collection of connected surface elements; i.e., the boundary between solid and non-solid
 - **Volume representations**: a 3D object is represented by its interior volume. For example, 3D volumes or volume mesh (FEM)

Our focus is on

boundary representations

Polygonal Meshes

Surfaces

- A 2-dimensional region of 3-dimensional space
- A portion of space having length and breadth but no thickness

3D Representation: Polygonal Meshes

- Discretize the surface in a set of simple primitives:
 - Many points
 - **Triangles**
 - Quads
 - Polygons
 - Our focus is on:
 - simplicial complexes, e.g., triangles!

Why triangular meshes?

- Two main practical reasons:
 - Data-structures are straightforward
 - Graphics hardware (e.g., a GPU) uses triangles;

Why triangular meshes?

- Two main theoretical reasons:
 - Nice theory, i.e., simplicial complexes
- Less limiting cases:
 - a triangle is always planar!
 - if we remove a vertex, we get another simplicial!

Simplex

- A k -simplex, σ , is convex combination of $(k + 1)$ points ($\mathbf{p}_i$) that are **linearly independent** in the k -dimensional Euclidian space, $\mathbb{R}^k$:

$$\mathbf{x} = \sum_{\mathbf{p}_i \in \sigma} \alpha_i \cdot \mathbf{p}_i$$

$$\sum_i \alpha_i = 1 \quad \wedge \quad \alpha_i \geq 0 \quad \forall i$$

- A point $\mathbf{p}_i$ is called a vertex.
- k is the order of the simplex.

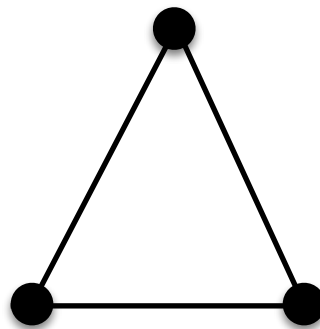
Simplices Example



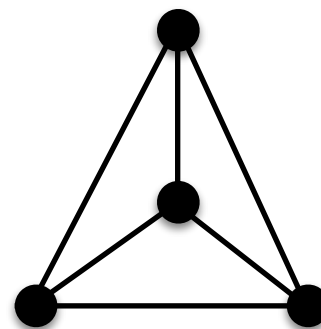
$k=0$



$k=1$



$k=2$



$k=3$

Sub-Simplex

- A sub-simplex σ' is called a **face** of a simplex σ if it is a sub-set of vertices of σ .

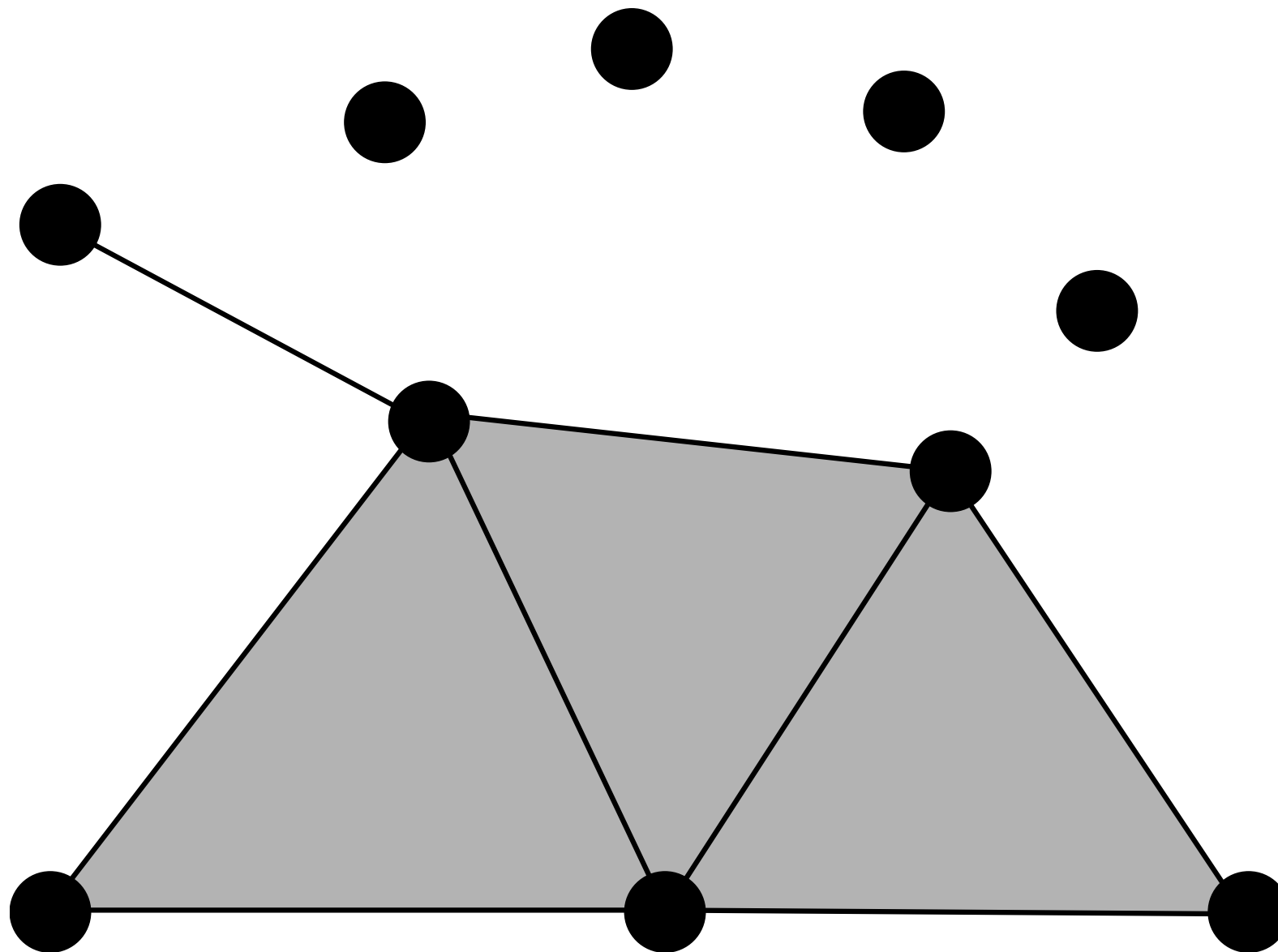
A Simplicial k -Complex

- A simplicial k -complex, Σ , is a finite collection of n simplices such that:
 - (i) The intersection of any two simplices of Σ is a face of each of them
 - (ii) Every face of a simplex, σ , of is in Σ

NOTE: k is the maximum order of all σ in Σ

Simplicial Complexes

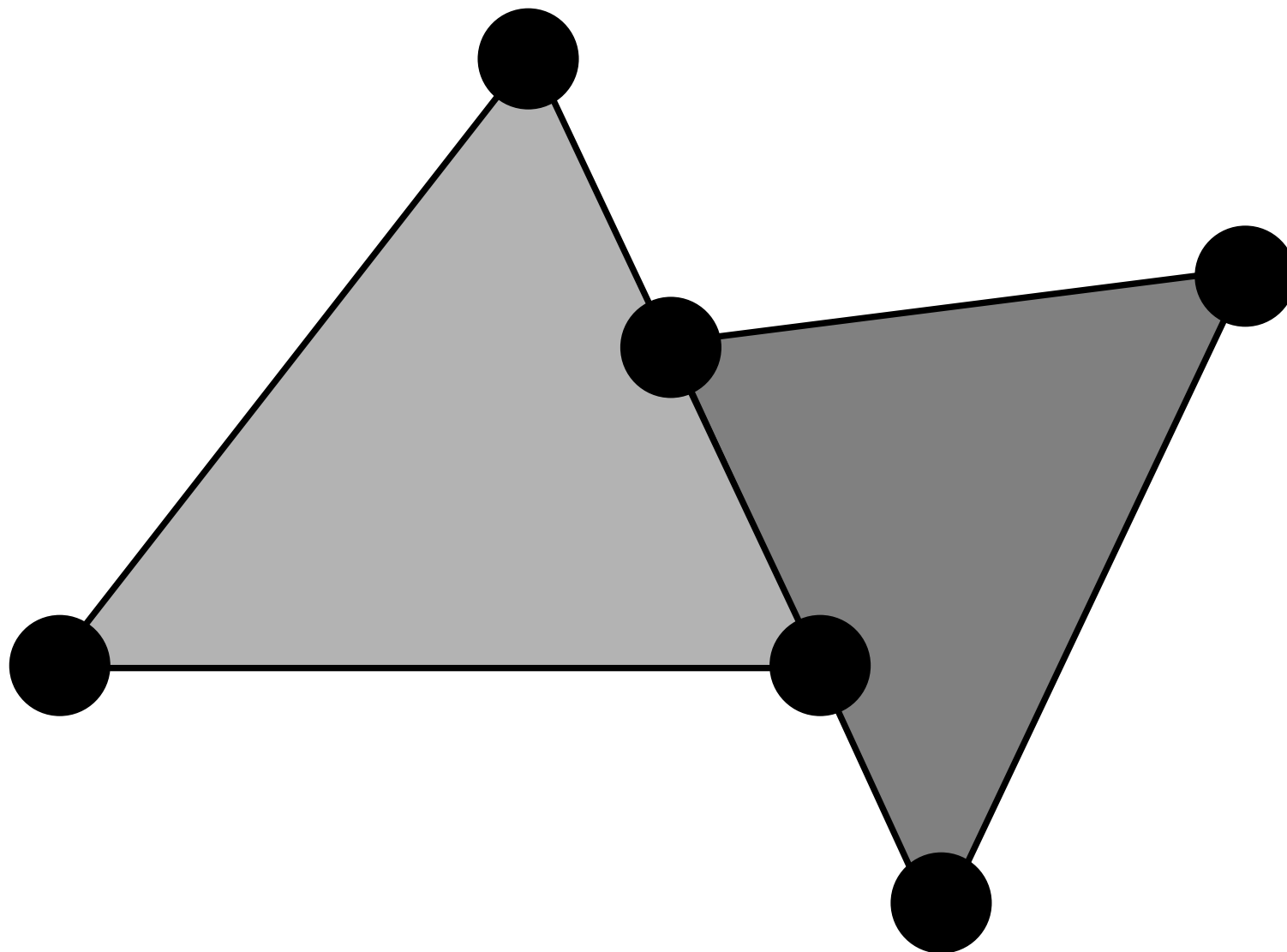
Example



GOOD!

Simplicial Complexes

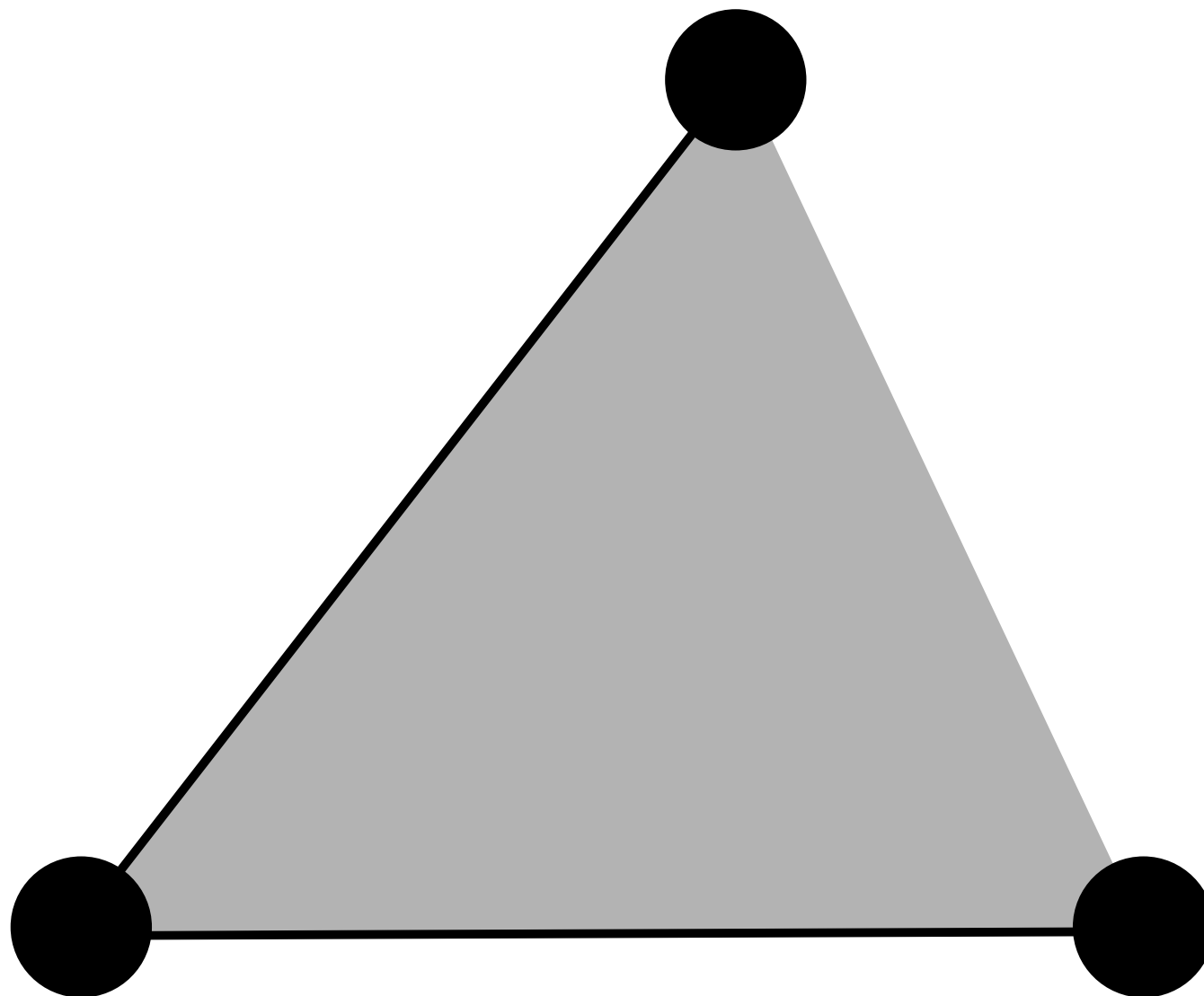
Example



BAD: is not valid! $\rightarrow$ Condition (i)

Simplicial Complexes

Example

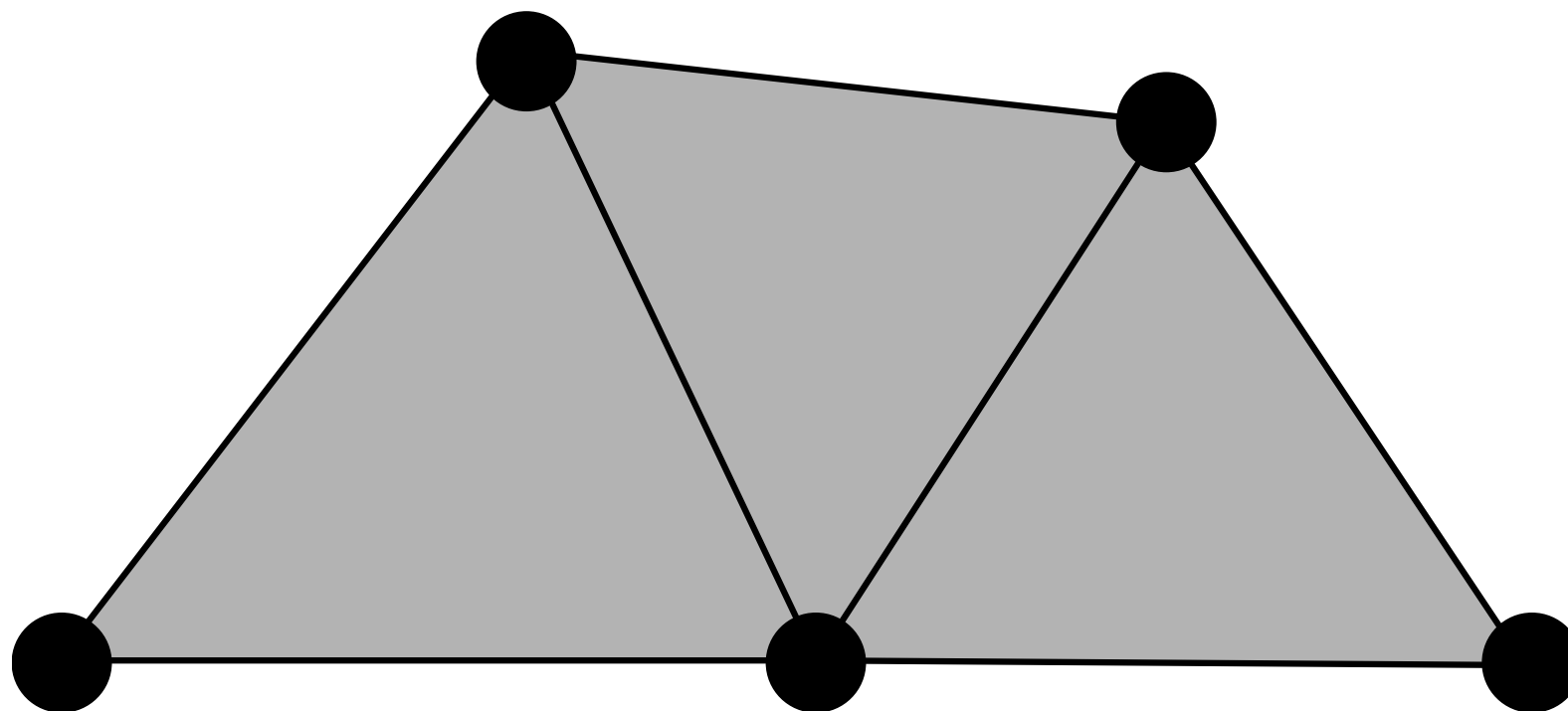


BAD: is not valid! —> Condition (ii)

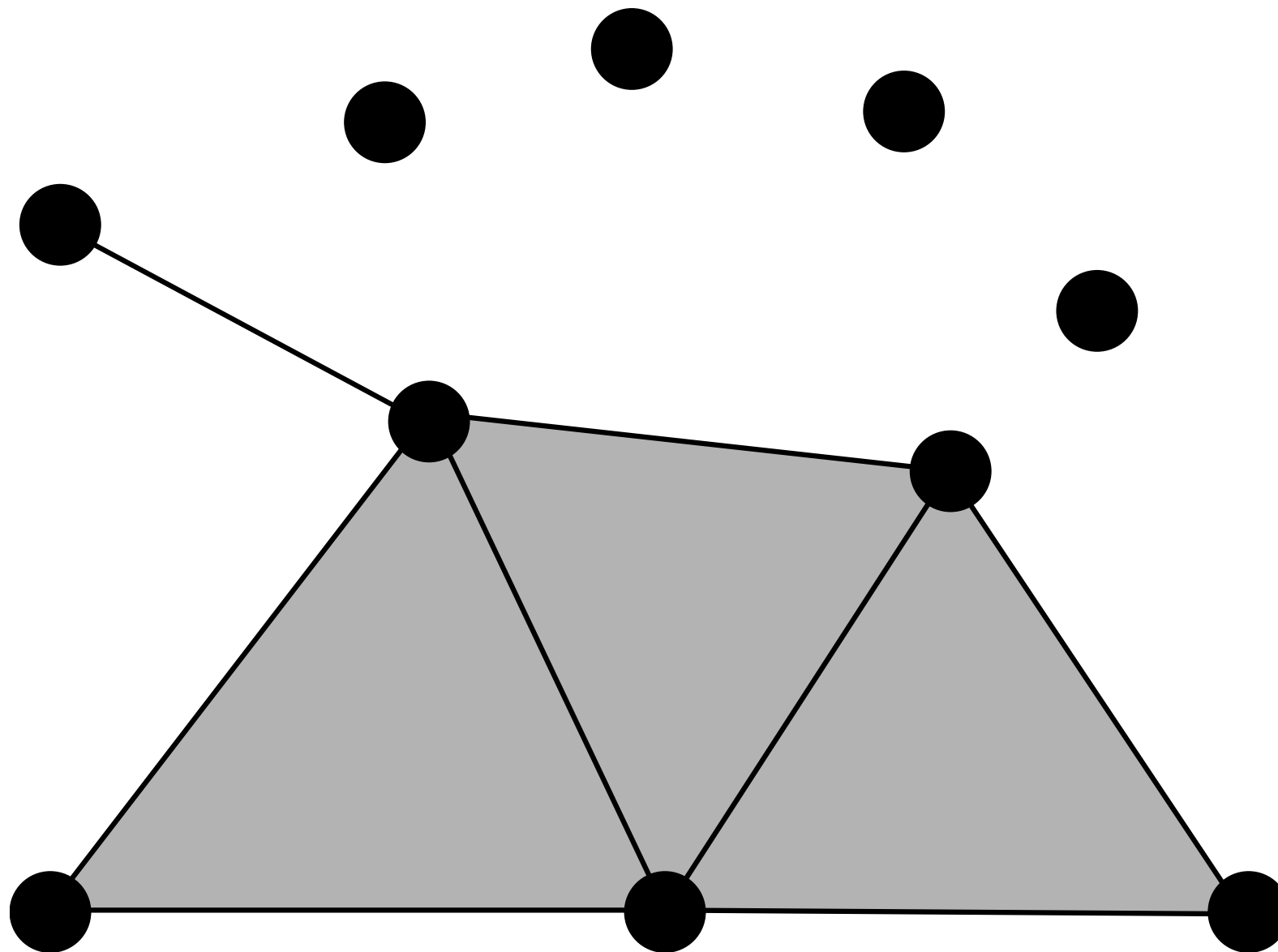
Simplicial Complexes

- A simplex, σ , is maximal in a simplicial complex, Σ , if it does not belong to any other simplex σ_2 of Σ .
- A k -simplicial complex, Σ , is maximal if all maximal simplices have order k .

A Maximal Simplicial Complex Example



A Non-Maximal Simplicial Complex Example



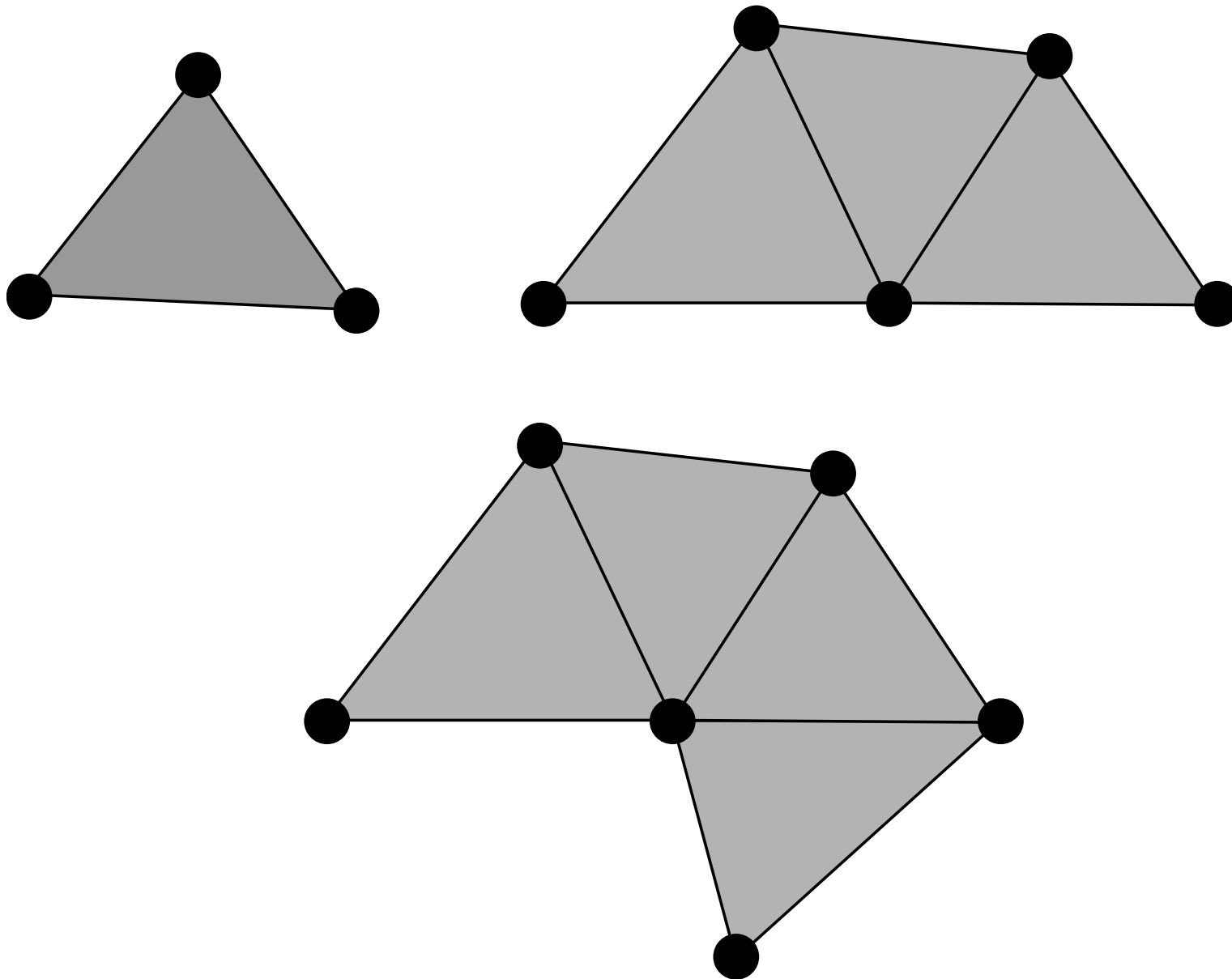
Manifoldness

- A surface, $S \in \mathbb{R}^3$ is manifold if and only if:
 - The neighborhood of each point is homeomorphic to an Euclidean space in two dimension or in other words:
 - The neighborhood of each point is homeomorphic to a disk or a semi-disk if the surface has boundaries!

Manifoldness

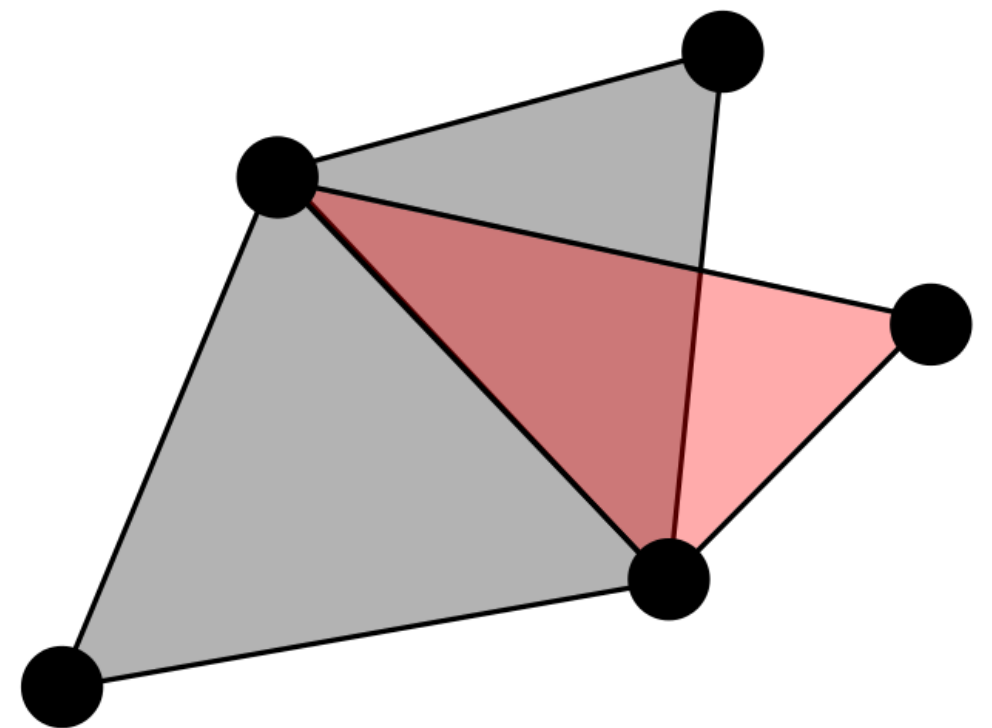
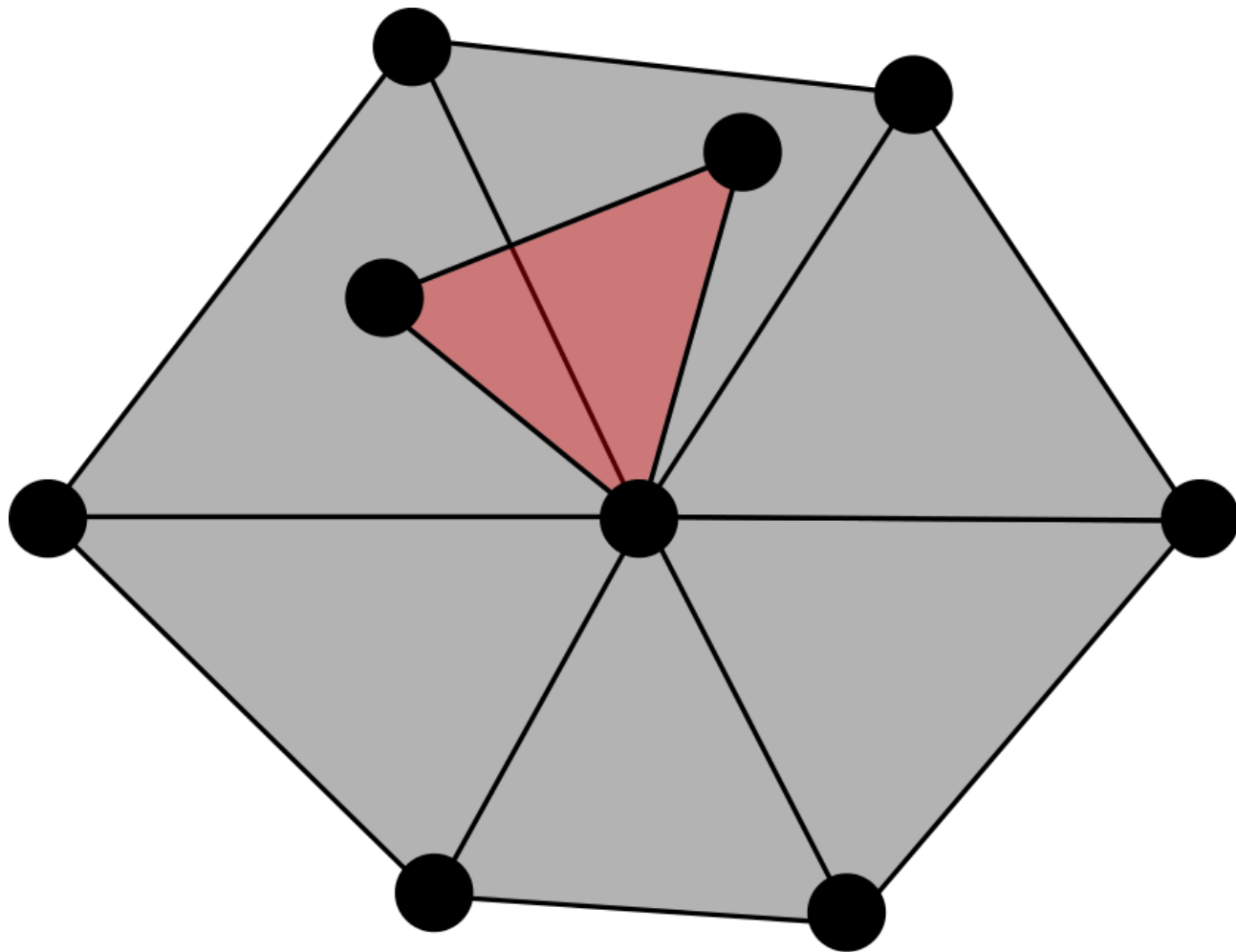
- In other words:
 - Each edge, E , is incident to only one or two faces!
 - The faces that are incident to a vertex form a closed or an open fan

Manifoldness Example



GOOD!

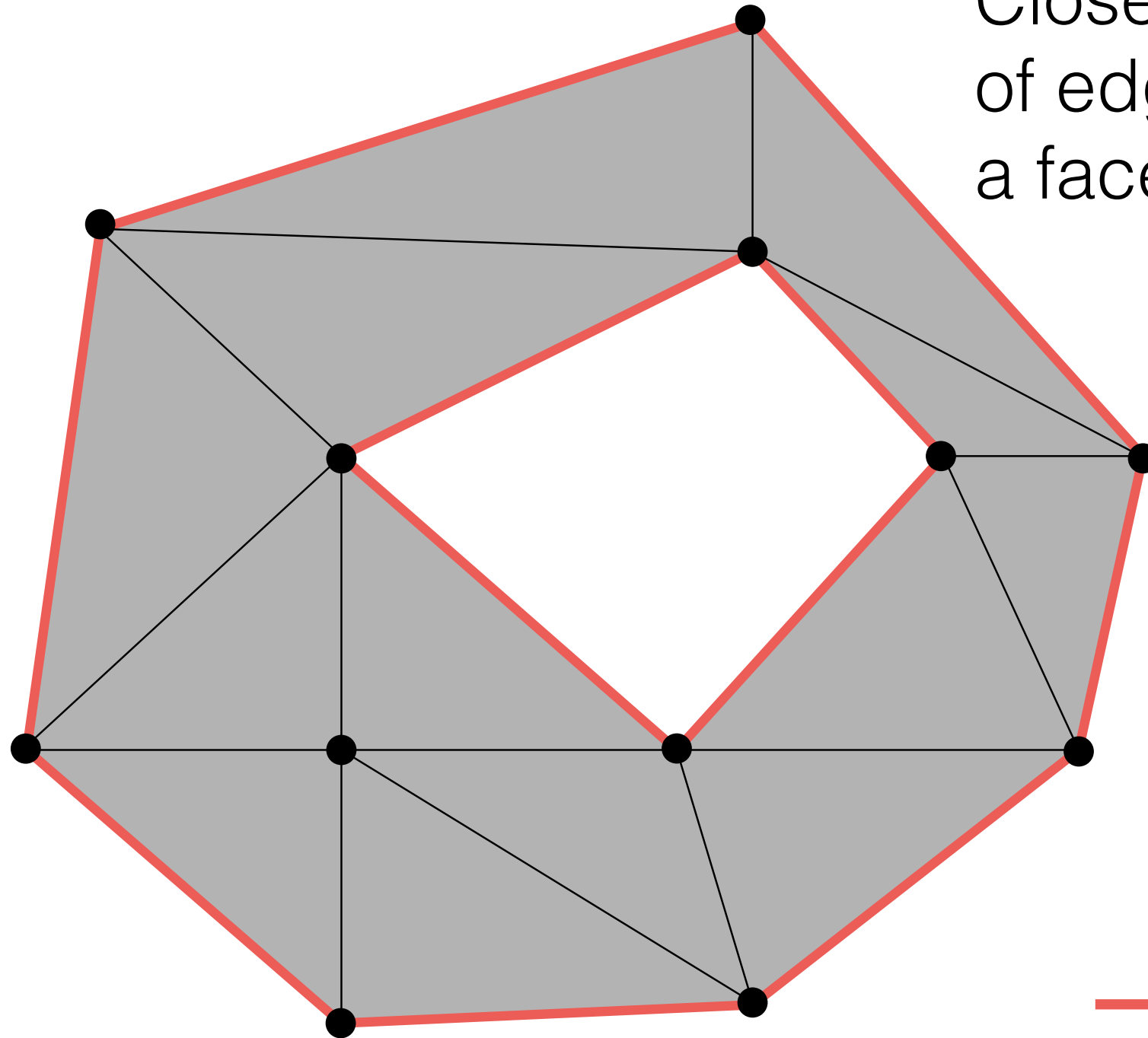
Manifoldness Example



BAD!

Borders

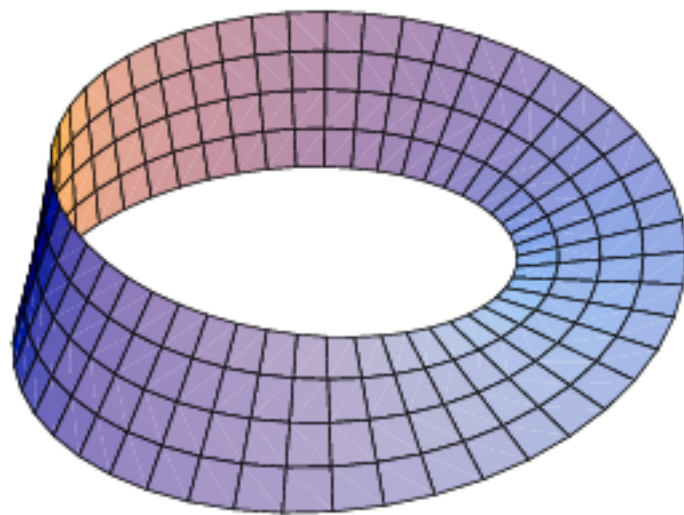
Closed sequence
of edges with only
a face



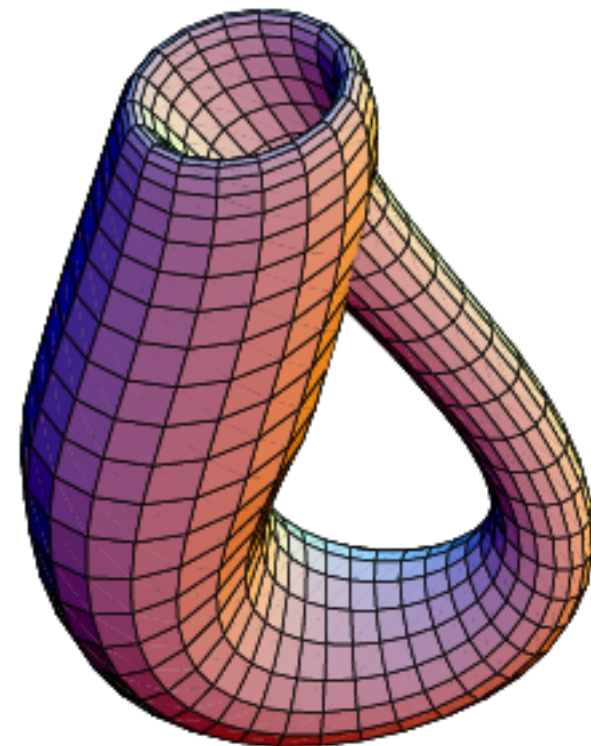
— Border

Orientability

- A surface, S , is orientable if it is possible to set a coherent normal to each point of the surface
- **NOTE:** Möbius strip and Klein bottle and non-manifold surfaces are not orientable:

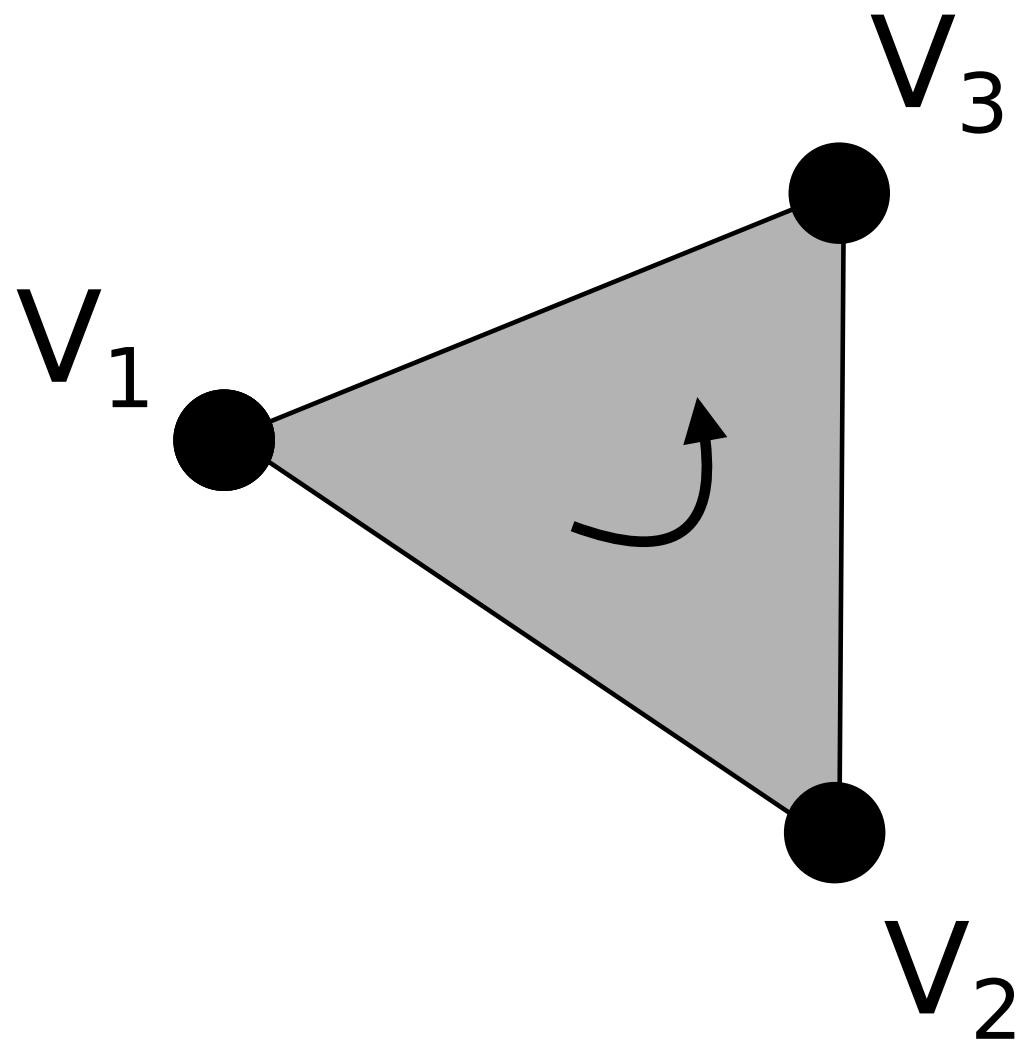


Möbius strip

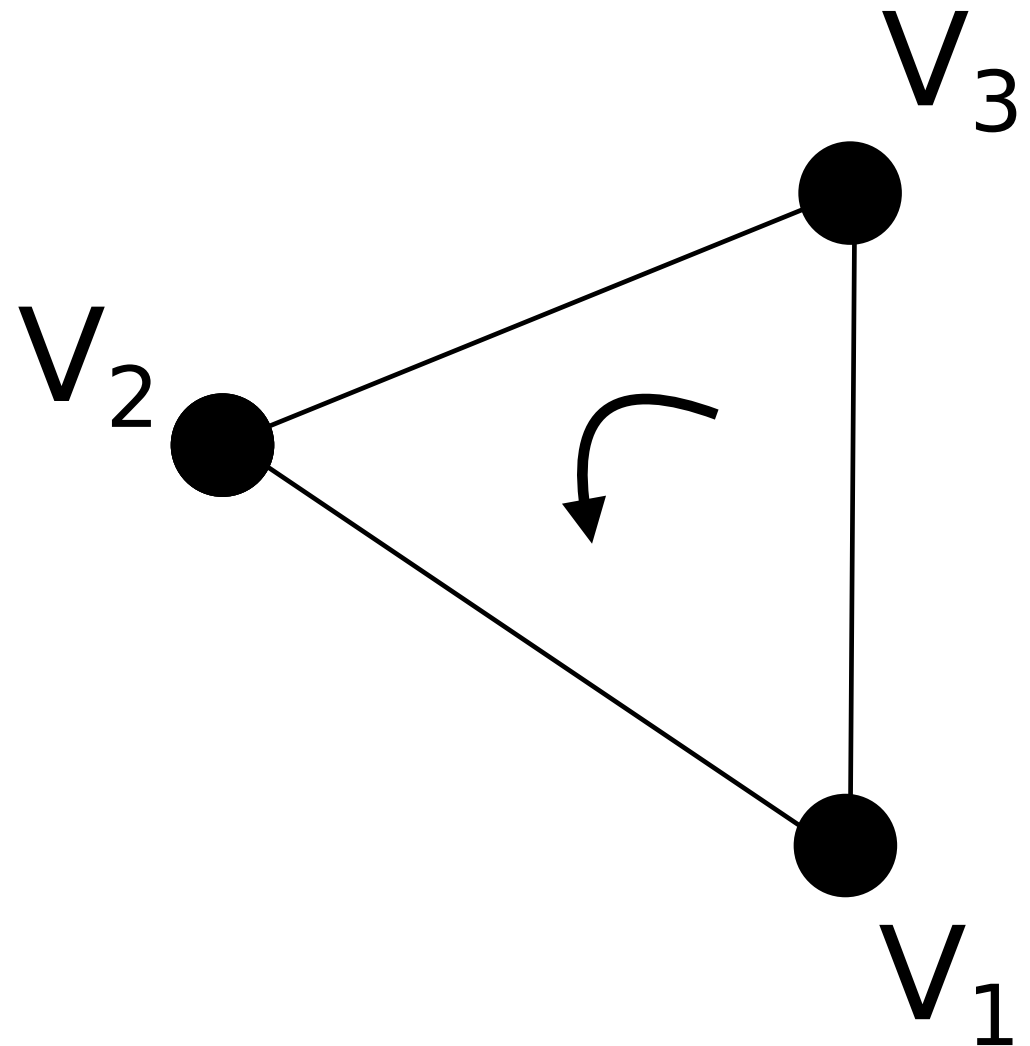


Klein bottle

Orientability



Front
(counter-clockwise)



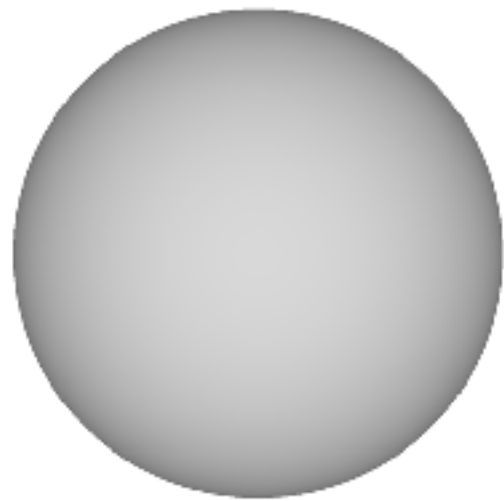
Back
(clockwise)

Mesh

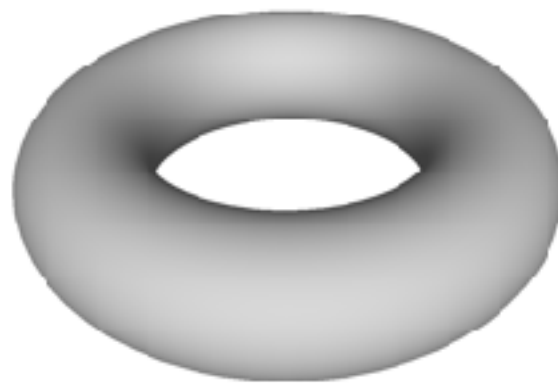
- A mesh is maximal 2-simplicial complexes that is a 2-manifold orientable surface.
- We can have non 2-manifold meshes
- We assume that they are maximal

Genus

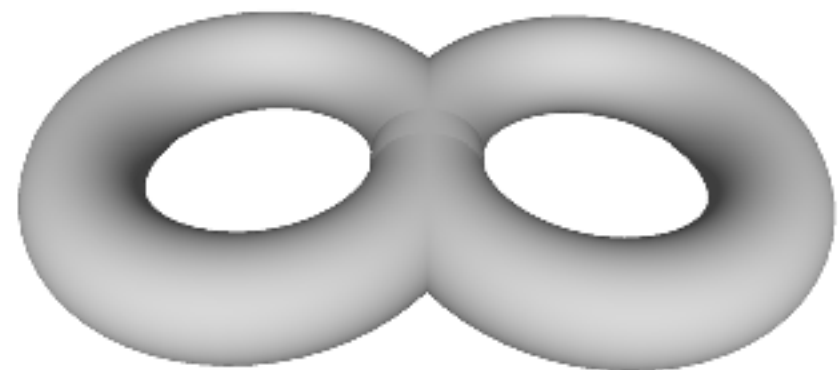
- The genus is the maximum number of cuttings along non-intersecting closed simple curves without rendering the resultant manifold disconnected



0



1



2

- Genus $\longrightarrow$ “the number of handles”

Euler Characteristic

- Given V vertices, E edges, and F faces of a polygonal closed and orientable surface with genus G , we have that:

$$2 - 2G = V - E + F$$

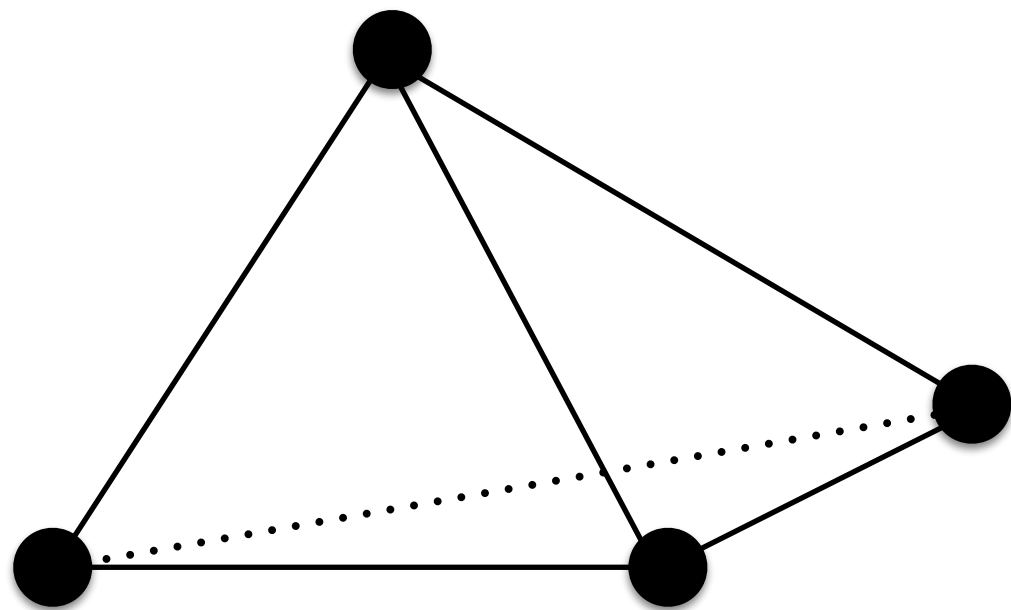
$$\chi = V - E + F$$

- More in general for a 2-manifold orientable polygonal mesh (with S connected components and B borders):

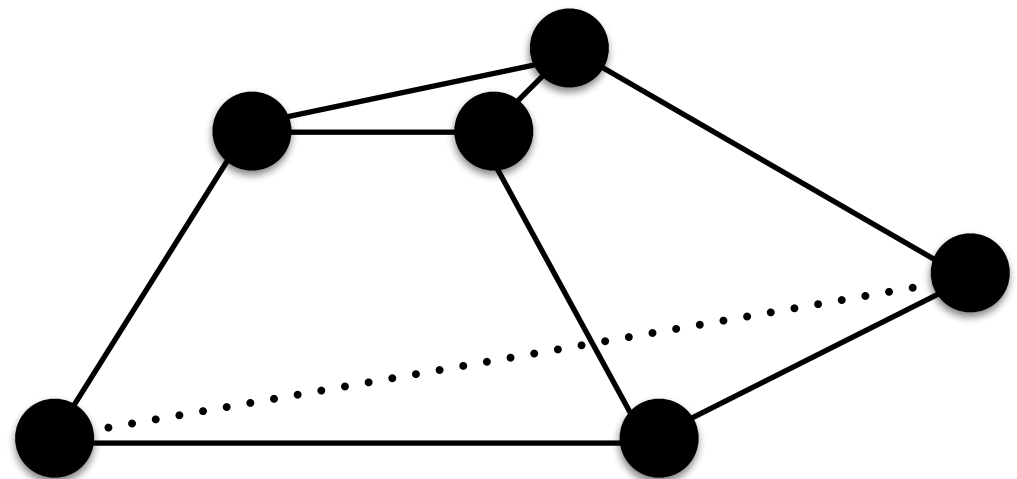
$$V - L + F = 2(S - G) - B$$

Euler Characteristic Example

The Euler characteristic is 2 for any simply connected polyhedron

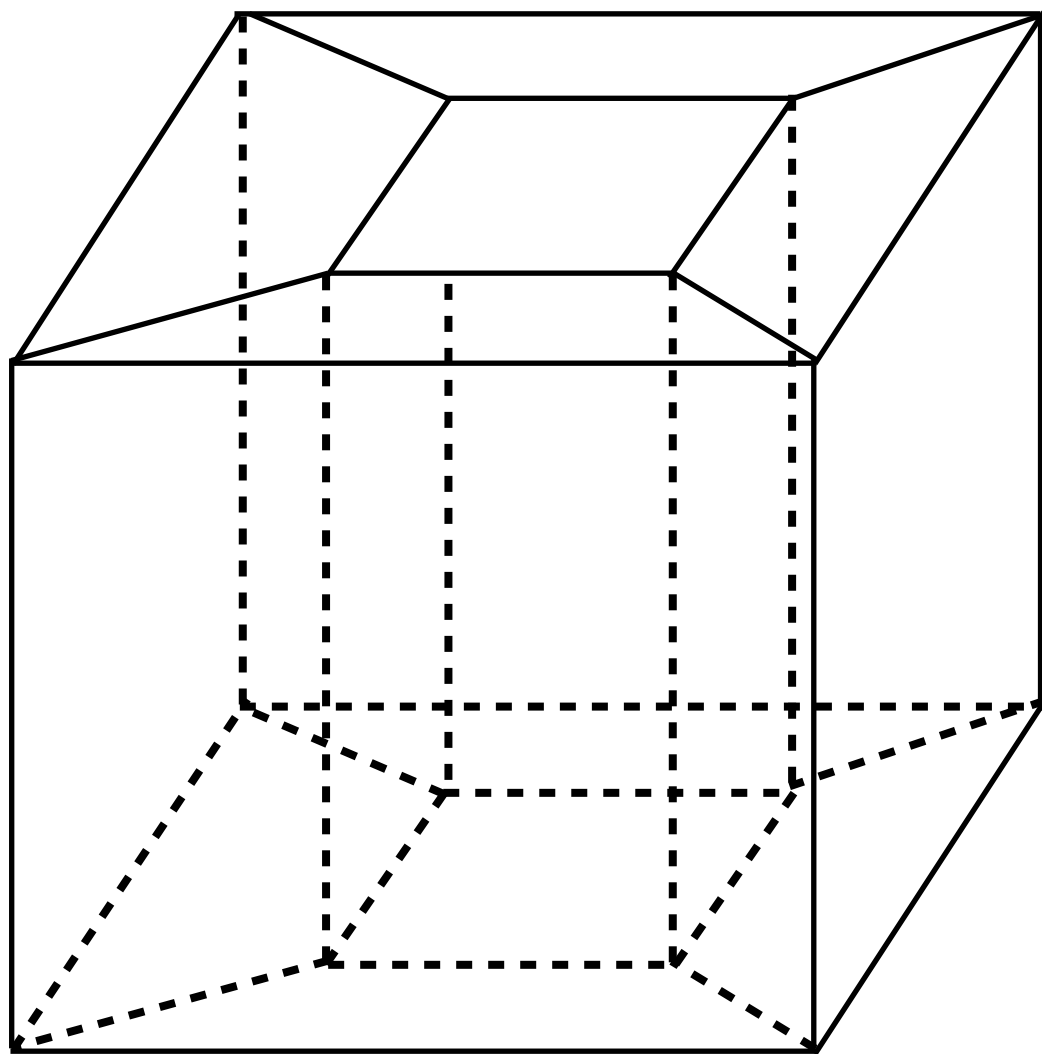


$$\chi = V - E + F$$
$$\chi = 4 - 6 + 4 = 2$$



$$\chi = V - E + F$$
$$\chi = 6 - 9 + 5 = 2$$

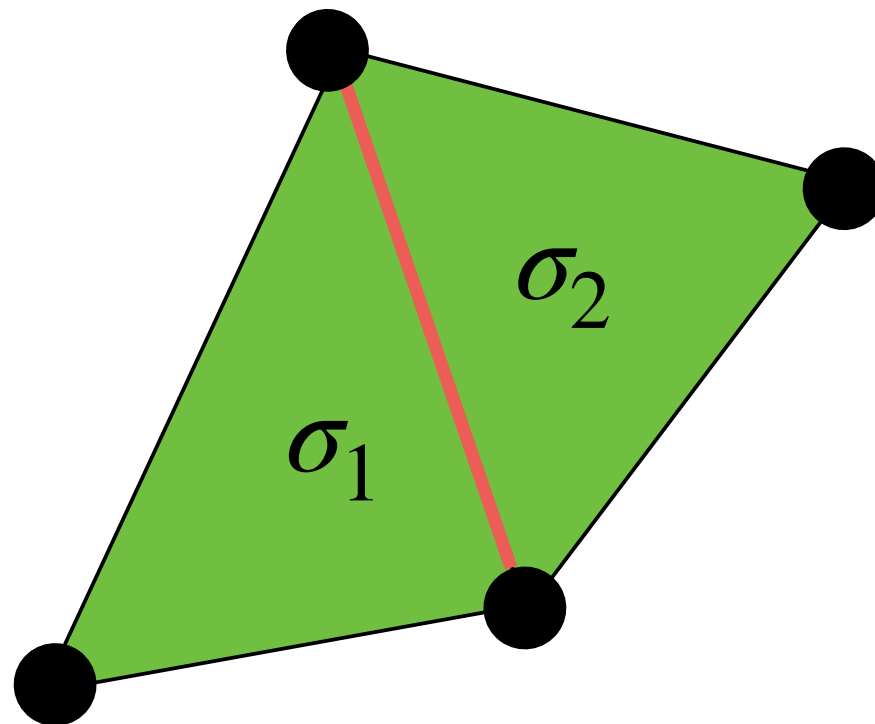
Euler Characteristic Example



$$\chi = V - E + F$$
$$\chi = 16 - 32 + 16 = 0 = 2 - 2g$$

Adjacency Relations

- Given two simplices, σ_1 and σ_2 , they are incident if σ_1 is a face of σ_2 or vice-versa:



Adjacency Relations

- Two k -simplices are m -adjacent ($k > m$) if a m -simplex exists such that it is a face of both.
- For example:
 - Two triangles sharing an edge are 1-adjacent
 - Two triangle sharing a vertex are 0-adjacent

Adjacency Relations

- An adjacency relations is an ordered couple of the following elements:
 - $E \longrightarrow$ edge
 - $F \longrightarrow$ Face
 - $V \longrightarrow$ Vertex
- For example: (E,E) , (V,V) , (F,F) , (E,F) , (F,E) , (E,V) , (V,E) , (F,V) , (V,F) , (E,V) , and (V,E) .

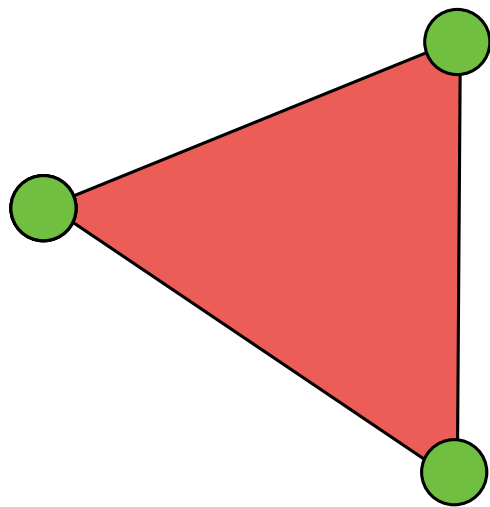
Adjacency Relations

Example

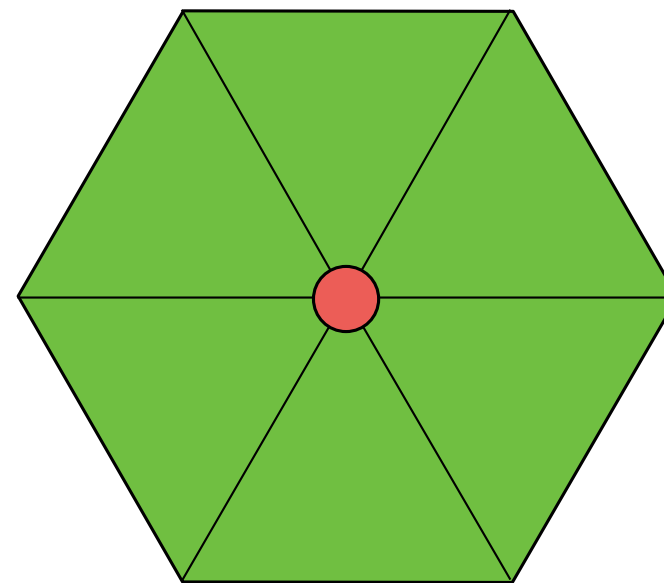
- Meaning of some relations:
 - FF $\longrightarrow$ adjacency between triangles
 - FV $\longrightarrow$ vertices of a triangle
 - VF $\longrightarrow$ triangles sharing a vertex

Adjacency Relations

Example



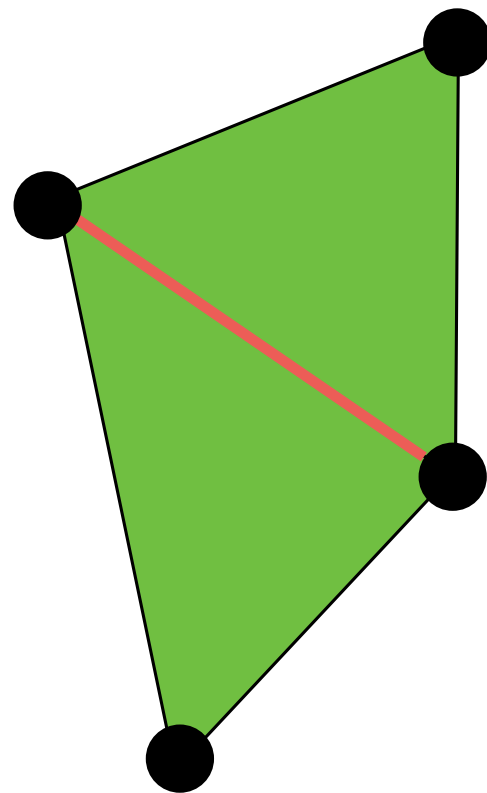
FV



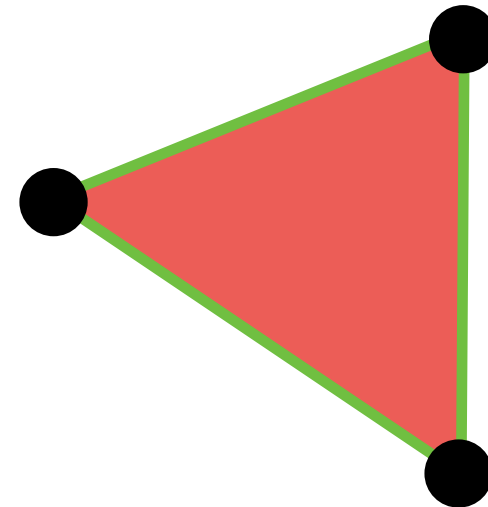
VF

Adjacency Relations

Example



EF

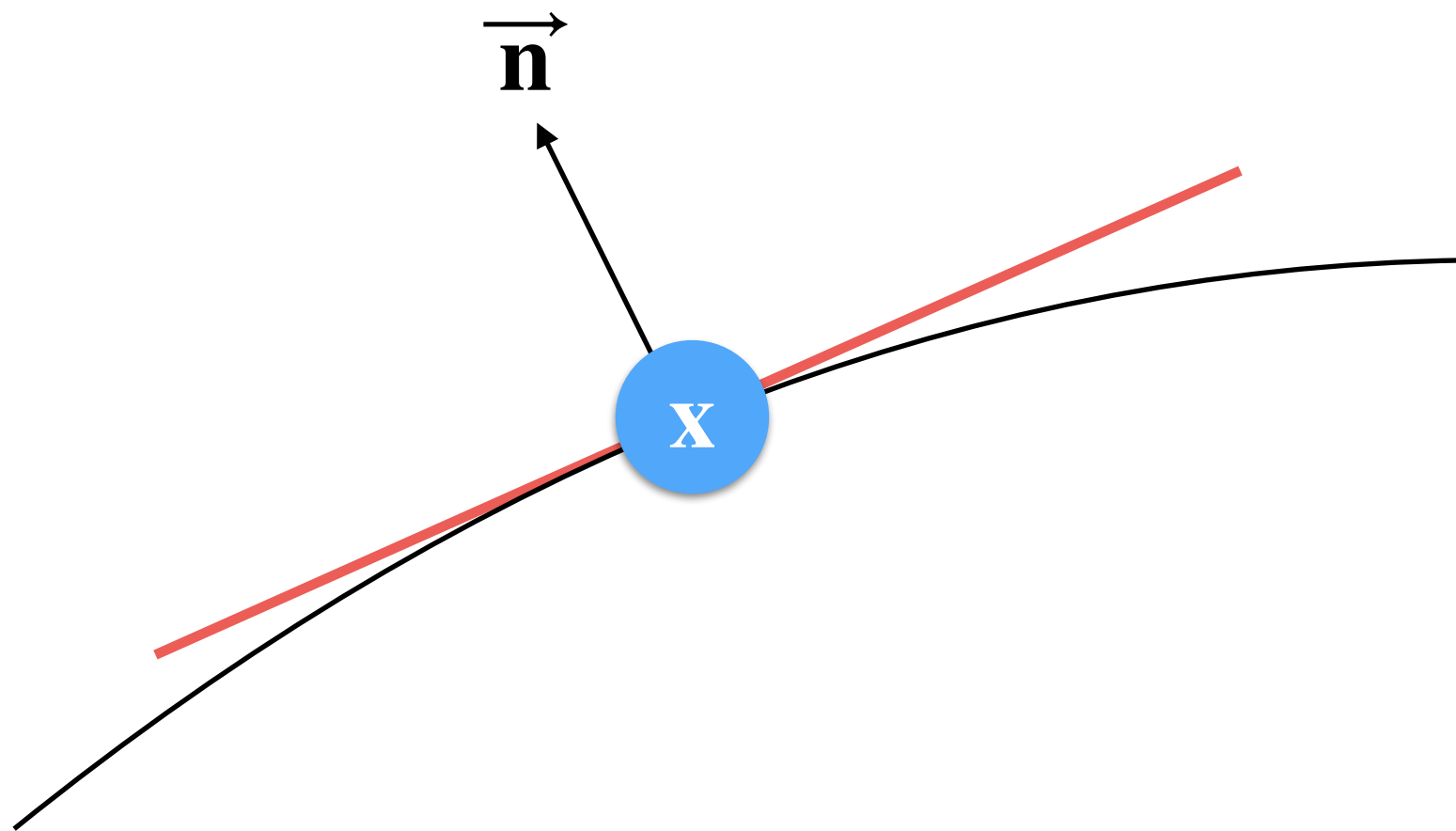


FE

Normals

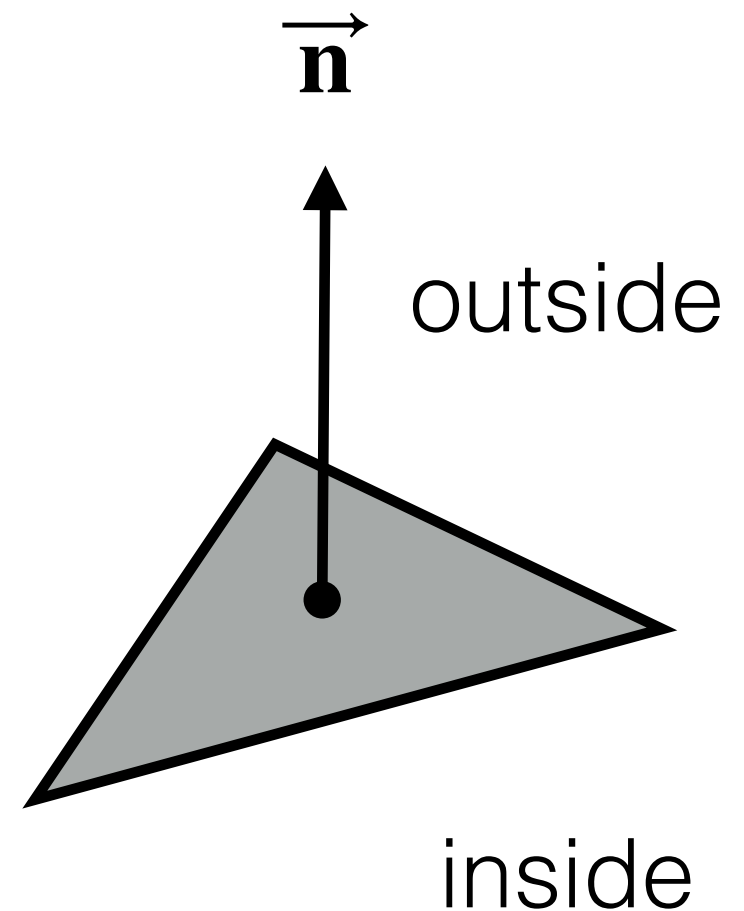
The Unit Normal

- The unit normal, $\vec{\mathbf{n}}$, to a point, $\mathbf{x}$, is the unit vector perpendicular to the tangent plane



The Unit Normal

- A normal is an important attribute for a vertex:
- It defines the direction of the object boundary



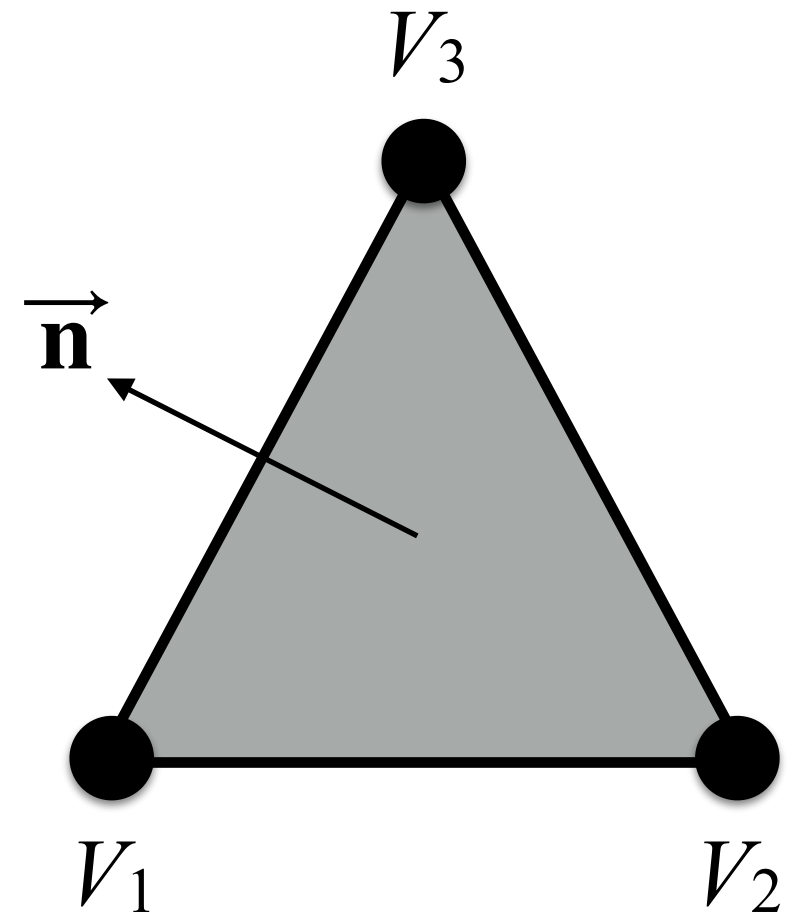
How to compute triangle normal?

- Given a triangle (V_1 , V_2 , and V_3), its normal (outer-pointing normal):

$$\vec{\mathbf{n}} = (V_3 - V_2) \times (V_1 - V_2)$$

$$\vec{\mathbf{n}} = \frac{\vec{\mathbf{n}}}{\|\vec{\mathbf{n}}\|}$$

- This means that vertices order is important! Typically is counter-clockwise

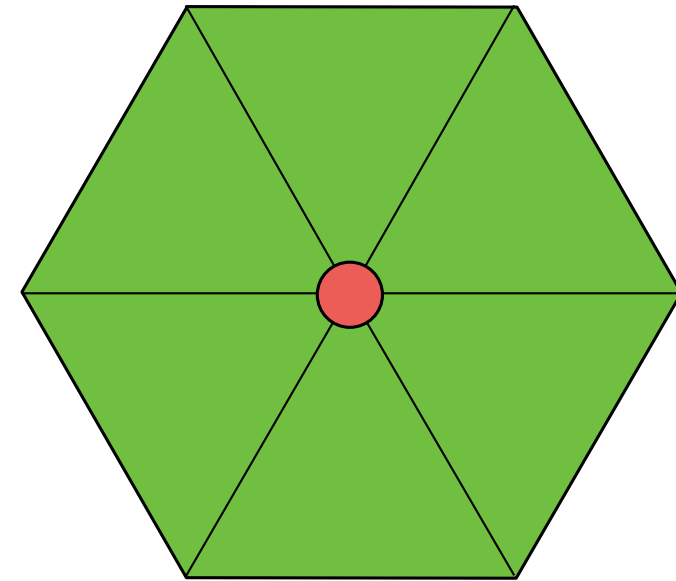


How to compute per vertex normal?

- We compute normals for each triangle
- For each vertex:
 - We compute the sum of normals of all triangles **VF** sharing that vertex:

$$\vec{n}_s(V) = \sum_{\{i|V \in T_i\}} \vec{n}_{T_i}$$

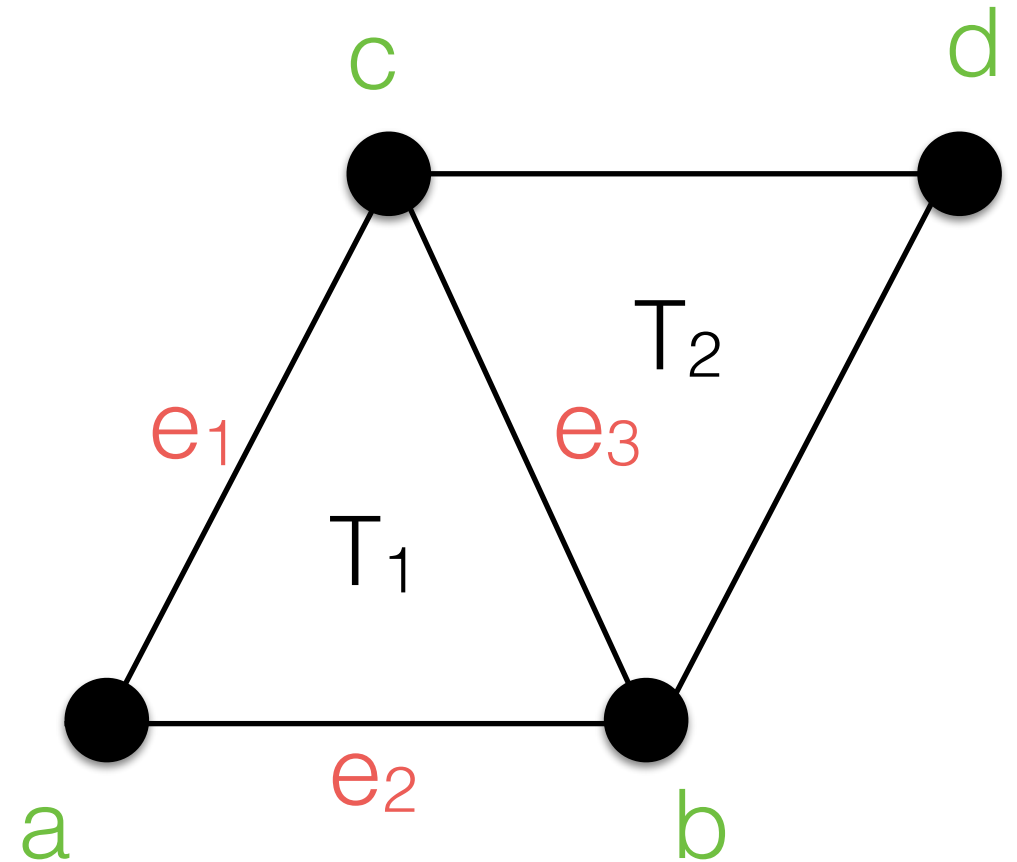
- We normalize this sum
- **Note:** per-vertex normals are useful but not correct!



Data Structures for 3D Meshes

List of Triangles

- For each triangle of the 3D model, we store its coordinates.
- For example:



Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)

Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)

Triangle 3: (1,2,-2); (3,-2,5); (-6,2,4)

....

Triangle n : (-8,2,7); (-2,3,9); (1,2,-7)

What's *very wrong*
with this??

Triangle 1: (3,-2,5); (2,2,4); (-6,2,4)

Triangle 2: (2,2,4) ; (0,-1,-2); (9,4,0)

Triangle 3: (1,2,-2); (3,-2,5); (-6,2,4)

....

Triangle n : (-8,2,7); (-2,3,9); (1,2,-7)

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Triangle 1: (**3,-2,5**); (**2,2,4**); (**-6,-2,4**)

Triangle 2: (**2,2,4**) ; (0,-1,-2); (9,4,0)

Triangle 3: (1,2,-2); (**3,-2,5**); (**-6,-2,4**)

....

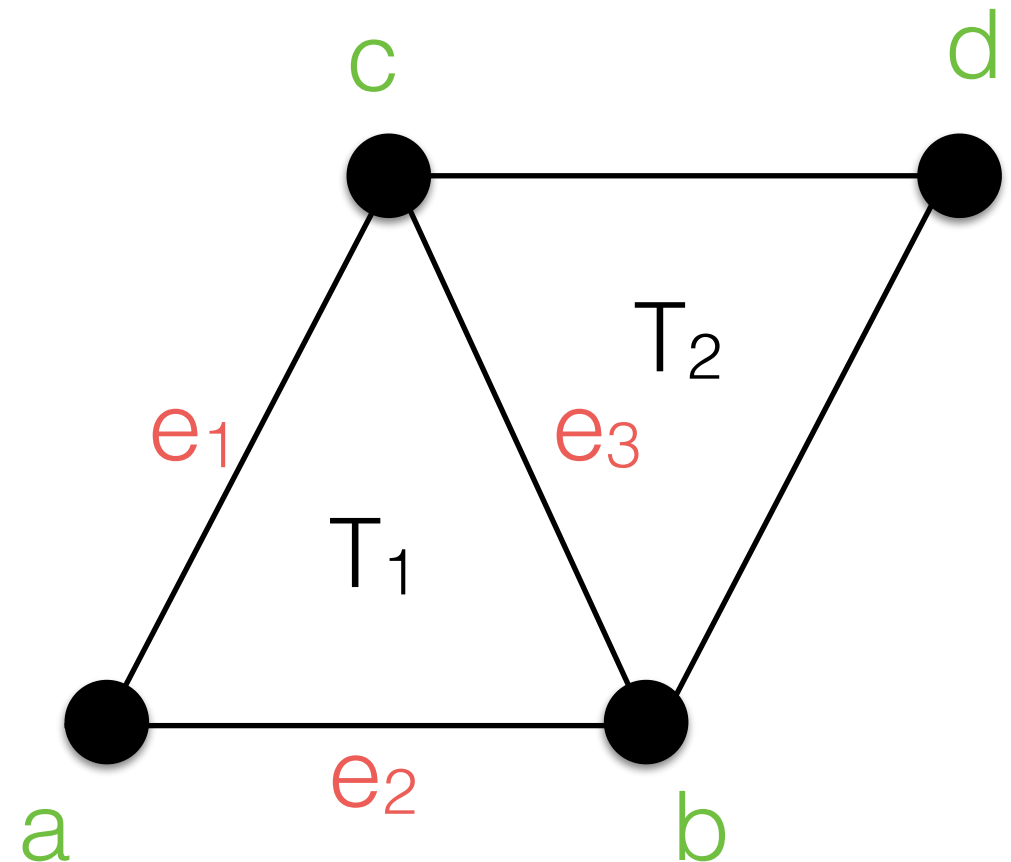
Triangle n : (-8,2,7); (-2,3,9); (1,2,-7)

List of Triangles

- Disadvantages:
 - Wasted disk and memory space:
 - Vertices are duplicated!
 - Memory: $|V| \times |T|$
 - Difficult to manage:
 - if we modify a vertex of a triangle, we will need to find and update its clones!
- How do we query neighbors?

List of Unique Vertices

- We store vertices in a list
- For each triangle of the 3D model, we store indices to the vertices' list



Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)
5. (1.0, -1.0, -1.0)
6. (1.0, -1.0, 1.0)
7. (1.0, 1.0, -1.0)
8. (1.0, 1.0, 1.0)

Faces:

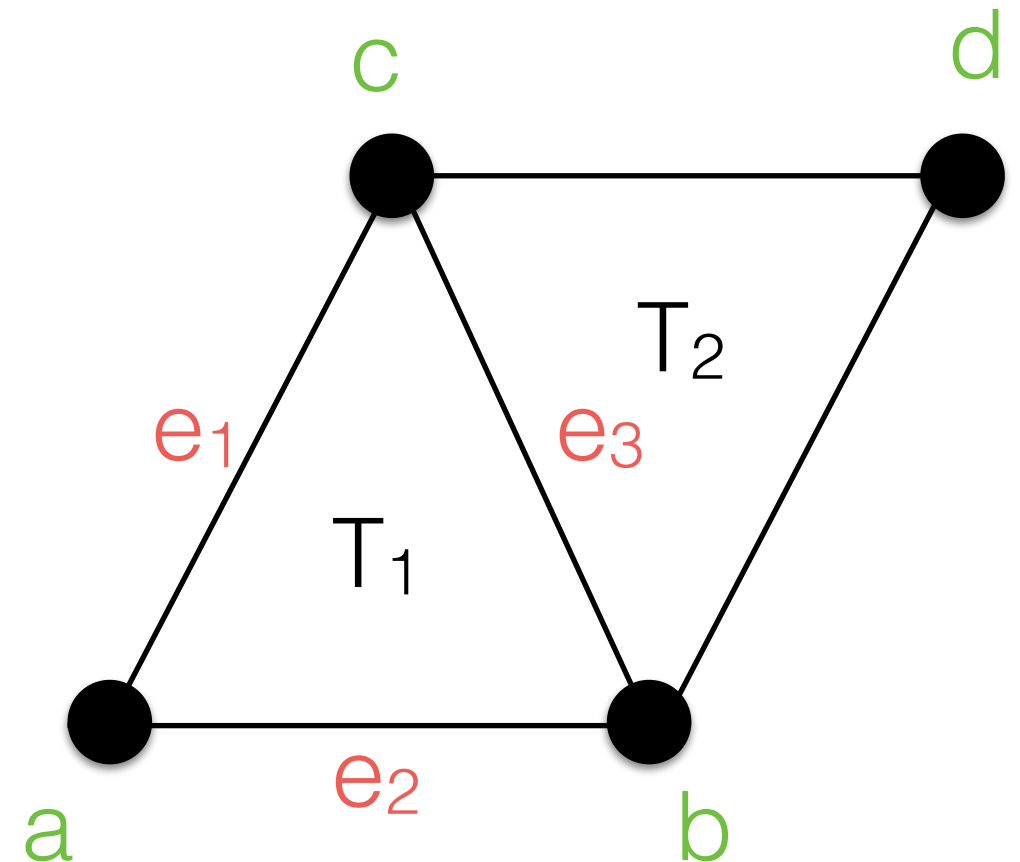
1. 1 2 4
2. 5 7 6
3. 1 5 2
4. 3 4 7
5. 1 7 5

List of Unique Vertices

- Wasted disk and memory space:
 - Common edges between two triangles are stored two times in the list of faces!
 - Memory: $|V| + |T|$
- Better management:
 - Easy to edit a vertex's attribute (e.g., its position)!
- How do we query neighbors?

List of Unique Edges

- We store vertices in a list
- For each edge, we store indices to the vertices' list
- For each triangle of the 3D model, we store indices to edges's list



Vertices:

1. (-1.0, -1.0, -1.0)
2. (-1.0, -1.0, 1.0)
3. (-1.0, 1.0, -1.0)
4. (-1, 1, 1.0)

Edges:

1. 1 2
2. 2 3
3. 4 2
4. 3 4
5. 1 3

Faces:

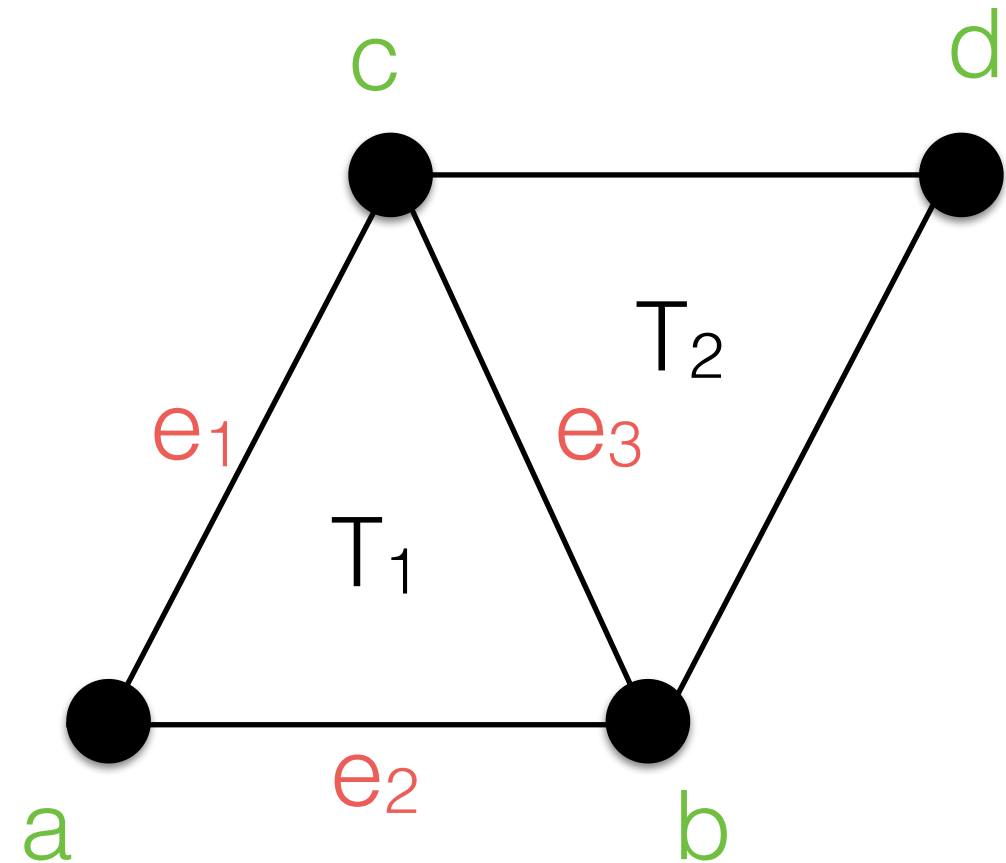
1. 1 2 5
2. 2 4 3

List of Unique Edges

- Better management:
 - Easy to edit an edge's attribute (e.g., its color)!
- We can do some queries, but not all of them!

Extended List of Unique Edges

- We add to an edge the indices of its left and right triangle
- This simplifies edge-face queries!



Vertices:

1. $(-1.0, -1.0, -1.0)$
2. $(-1.0, -1.0, 1.0)$
3. $(-1.0, 1.0, -1.0)$
4. $(-1, 1, 1.0)$

Edges:

1. 1 2
2. 2 3
3. 4 2
4. 3 4
5. 1 3

Faces:

1. -1 1
2. 1 2
3. -1 2
4. -1 2
5. 1 -1

Faces:

1. 1 2 5
2. 2 4 3

File Formats

File Formats

- There are many 3D file formats. The most used, and de-facto standard:
 - STL
 - PLY
 - OBJ
- Standards:
 - COLLADA: <https://www.khronos.org/collada/>
 - X3D: <http://www.web3d.org/x3d/>

STL File Format

- Standard Triangle Language (STL) created by 3D Systems
- This format represents only the 3D geometry:
 - No color/texture
 - No other attributes
- The format specifies both ASCII and binary representations

STL File Format

- Data structure: list of triangles
- Vertices are ordered using the right-hand rule
- 3D coordinates must be positive
- No scale metadata; i.e., units are arbitrary

STL File Format

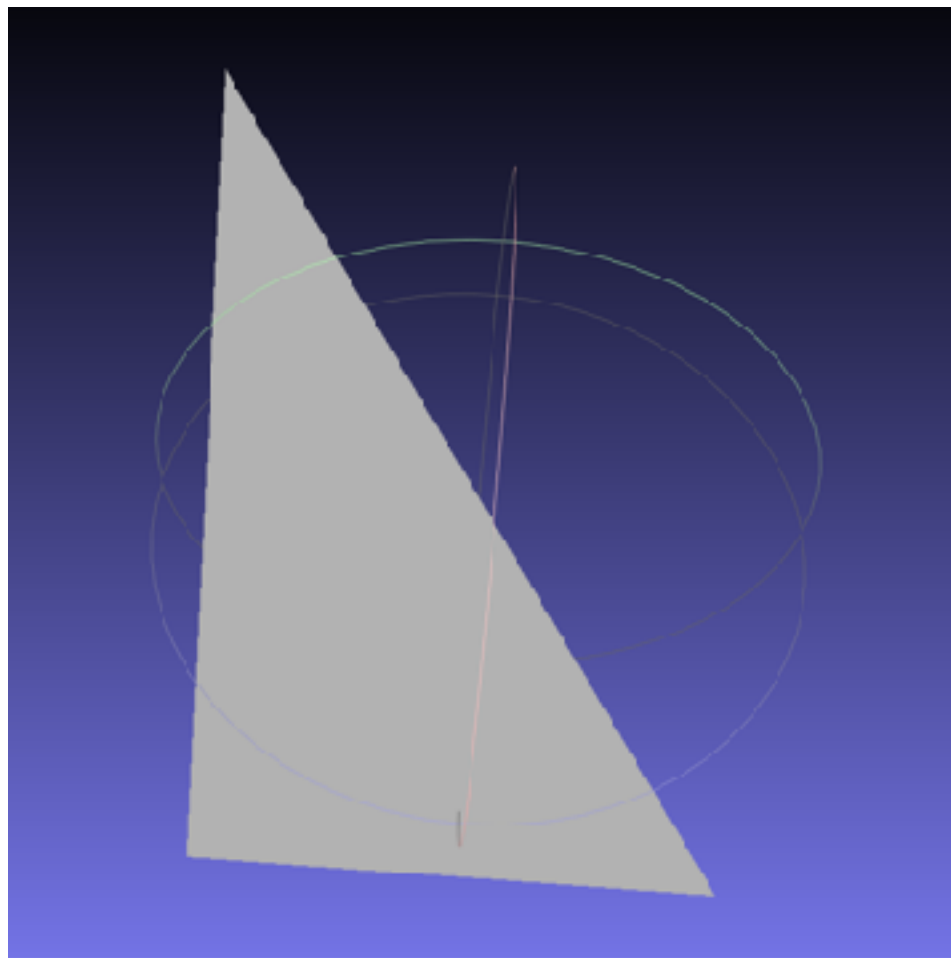
- The file begins as

```
solid name
```

- A face is defined as

```
facet normal nx ny nz
  outer loop
    vertex v1x v1y v1z
    vertex v2x v2y v2z
    vertex v3x v3y v3z
  endloop
endfacet
```

STL File Format: An Example



```
solid triangle
facet normal 0 1 0
  outer loop
    vertex 0.0 0.0 0.0
    vertex 1.0 0.0 0.0
    vertex 0.0 1.0 1.0
  endloop
endfacet
endsolid triangle
```

PLY File Format

- Polygon File Format (PLY) is a popular format created by Stanford University (Greg Turk)
- The format is very flexible:
 - we can add many attributes
 - we can define triangular and polygonal meshes
- The format specifies both ASCII and binary representations

PLY File Format

- Data structure: list of unique vertices
- No scale metadata; i.e., units are arbitrary
- The file is divided into two parts:
 - **Header** that specifies vertices and faces
 - **Body** that specifies the concrete data

PLY File Format: Header

- The file begins as

```
ply  
format ascii 1.0
```

- Vertex specification is defined as

```
element vertex num_vertices  
property float x  
property float y  
property float z
```

properties can be: char, uchar, short, ushort, int, uint float, double, etc.

PLY File Format: Header

- Faces are defined as

```
element face num_faces  
property list uchar int vertex_indices  
  
end_header
```

PLY File Format: Body

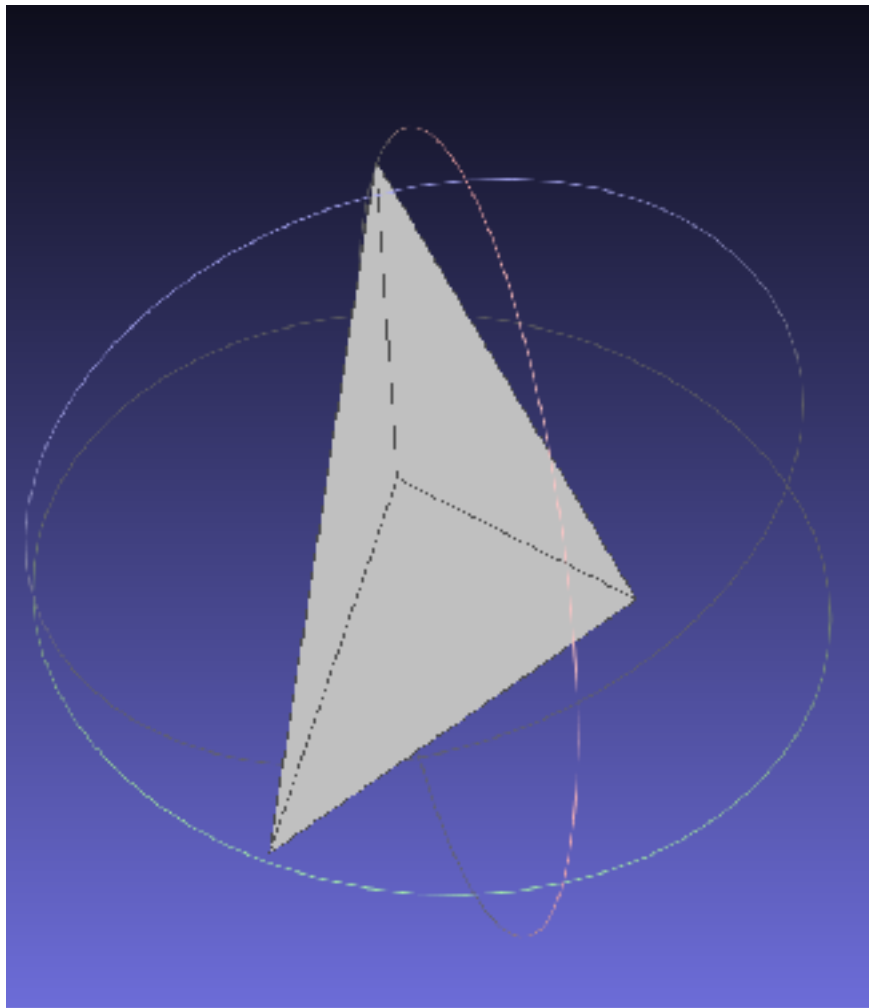
- Each i-th vertex is specified as

```
vix viy viz
```

- Each face is specified as

```
3 index_v1 index_v2 index_v2
```


PLY File Format: An Example



```
ply
format ascii 1.0
element vertex 4
property float x
property float y
property float z
element face 4
property list uchar int vertex_indices
end_header
-0.60 -0.97 0.37
-0.34 0.98 0.76
0.037 0.65 -1.06
0.88 -0.75 -0.25
3 1 3 2
3 0 1 2
3 0 3 1
3 3 0 2
```


Acknowledgements

- Some images and text are based on work by:
 - Dr. Paolo Cignoni:
 - <http://vcg.isti.cnr.it/~cignoni/>