# 3D from Photographs: Automatic Matching of Images

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# 3D from Photographs

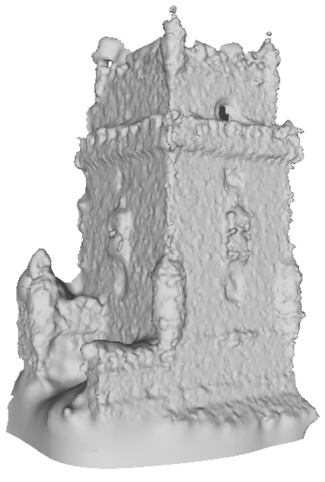
Photographs

Automatic Matching of Images

Camera Calibration

Dense Matching

Surface Reconstruction



3D model

# 3D from Photographs

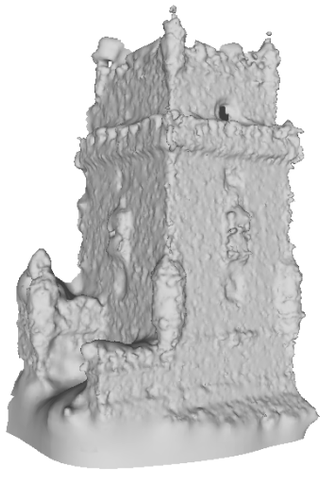
Photographs

Automatic Matching of Images

Camera Calibration

Dense Matching

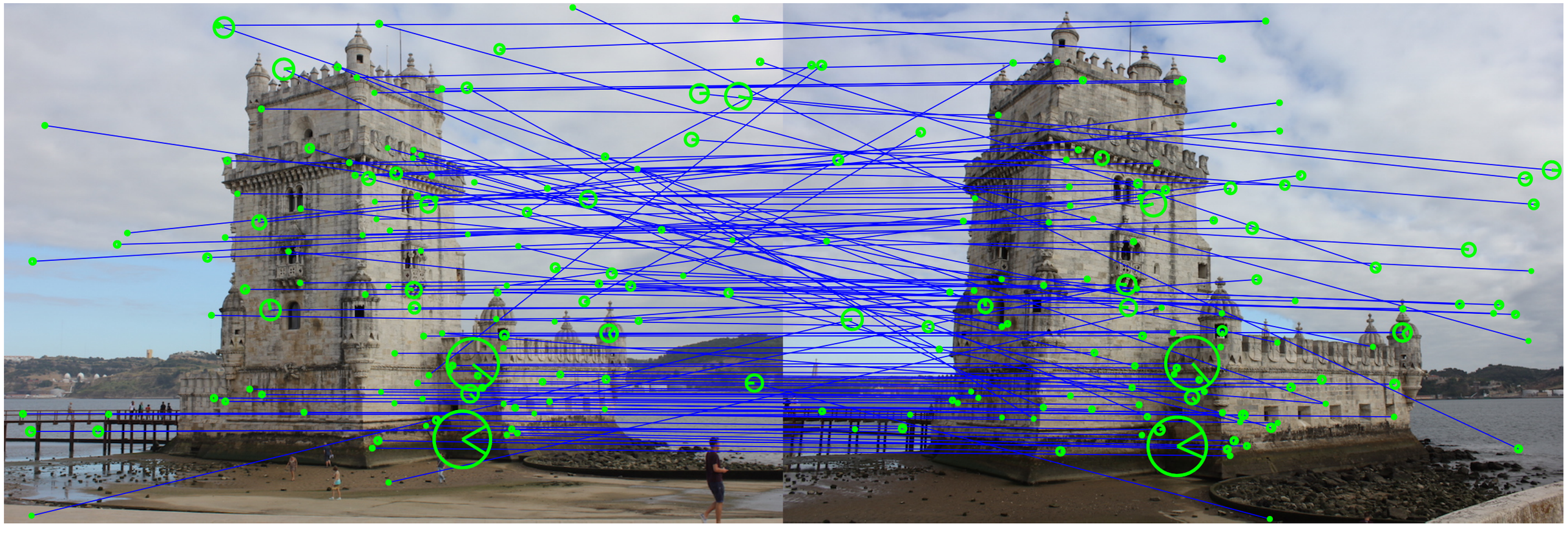
Surface Reconstruction



3D model

# The Matching Problem

- We need to find corresponding feature across two or more views:



# The Matching Problem

- Why?
  - 3D Reconstruction.
  - Image Registration.
  - Visual Tracking.
  - Object Recognition.
  - etc.

# The Matching Problem: Automatic Panorama Generation

Input Photographs

# The Matching Problem: Automatic Panorama Generation

Input Photographs

# The Matching Problem: Automatic Panorama Generation

Input Photographs

Panorama

# Extraction of Features

# Features

- A feature is a piece of the input image that is relevant for solving a given task.

- Features can be global or local.

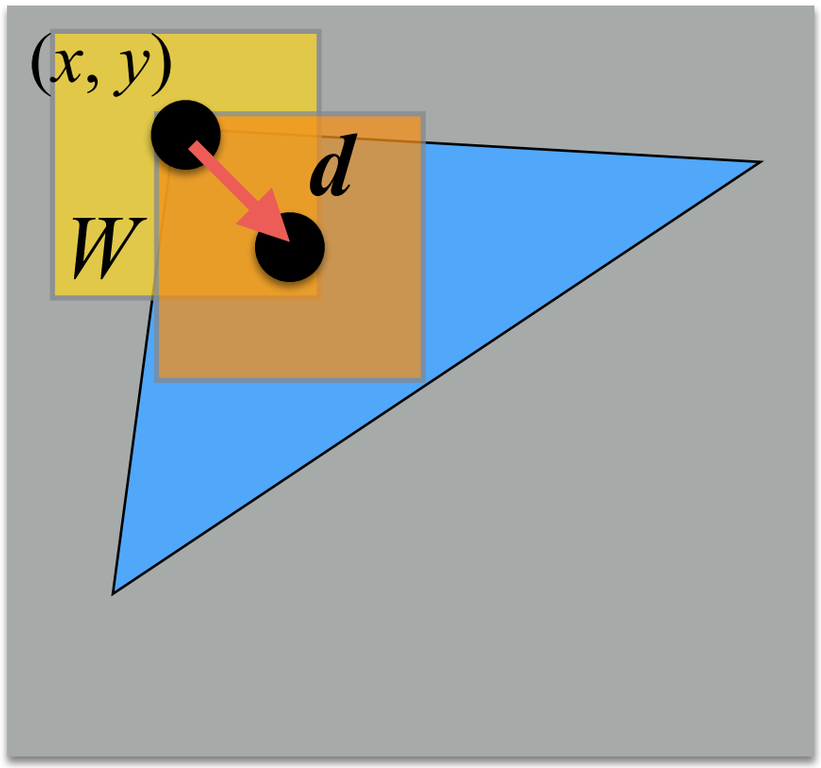
- We will focus on local features that are more robust to occlusions and variations.

# Extraction of Local Features

- We can extract different kind of features:
  - Flat regions or Blobs
  - Edges
  - Corners

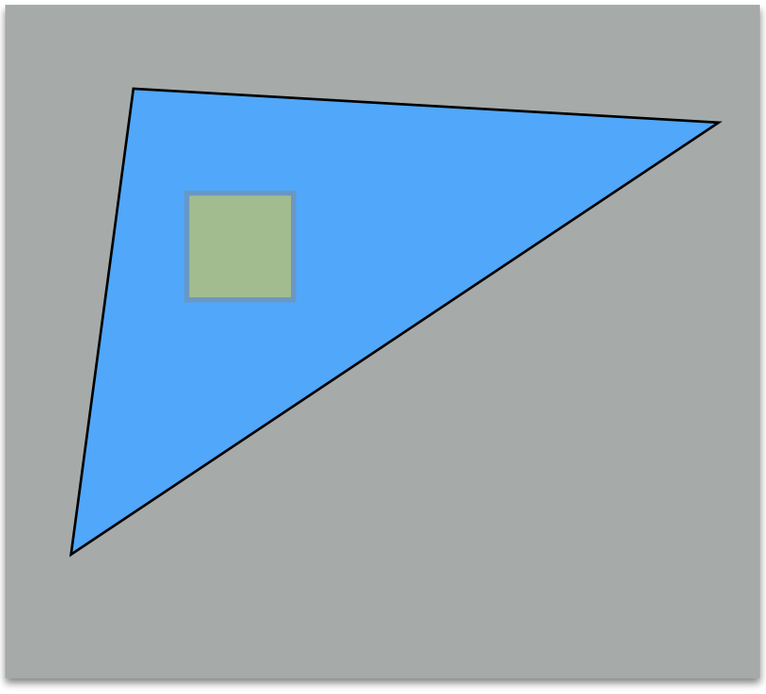
# Harris Corner Detector

- Let's consider a window $W$ centered in $(x, y)$:
  - how do pixels change from a window in $(x, y)$ to another one with a shift $\boldsymbol{d} = (u, v)$?
  - Let's compare each pixel before and after moving $W$ by $\boldsymbol{d} = (u, v)$ using the sum of squared differenced (SSD).

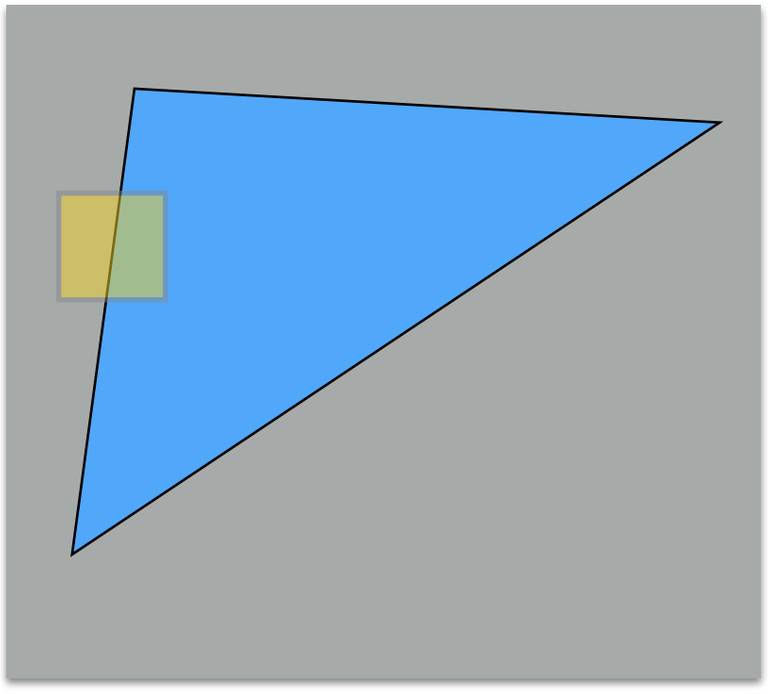



$$E(x,y) = \sum_{x_k, y_k \in W(x,y)} \Big( I(x_k + u, y_k + v) - I(x_k, y_k) \Big)^2$$

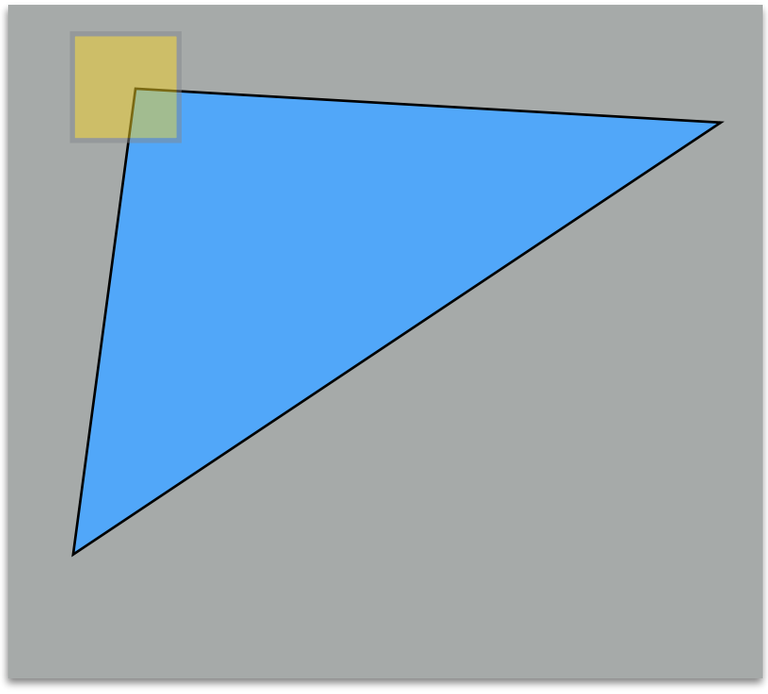
# What a Corners is



**Flat Region**:
no change
in all directions.



**Edge**:
no change
along the edge.



**Corner**:
significant change
in all directions.

# Harris Corner Detector: Small Motion Assumption

- Let's apply a first-order approximation, which provides good results for small motions:

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

# Harris Corner Detector: Small Motion Assumption

$$
\begin{aligned}
E(x,y) &= \sum_{x_k,y_k \in W(x,y)} \Big( I(x_k+u, y_k+v) - I(x_k,y_k) \Big)^2 \\
&\approx \sum_{x_k,y_k \in W(x,y)} \Big( I(x_k,y_k) + I_x(x_k,y_k)u + I_y(x_k,y_k)v - I(x_k,y_k) \Big)^2 \\
&= \sum_{x_k,y_k \in W(x,y)} \Big( I_x(x_k,y_k)u + I_y(x_k,y_k)v \Big)^2 \\
&= \sum_{x_k,y_k \in W(x,y)} \Big( I_x(x_k,y_k)^2u^2 + 2I_x(x_k,y_k)I_y(x_k,y_k) + I_y(x_k,y_k)^2v^2 \Big)
\end{aligned}
$$

# Harris Corner Detector: Small Motion Assumption

$$E(x,y) \approx \sum_{x_k,y_k \in W(x,y)} \left( I_x(x_k,y_k)^2 u^2 + 2I_x(x_k,y_k)I_y(x_k,y_k) + I_y(x_k,y_k)^2 v^2 \right)$$

$$= Au^2 + 2Buv + Cv^2$$

$$A = \sum_{x_k,y_k \in W(x,y)} I_x(x_k,y_k)^2$$

$$B = \sum_{x_k,y_k \in W(x,y)} I_x(x_k,y_k)I_y(x_k,y_k)$$

$$C = \sum_{x_k,y_k \in W(x,y)} I_y(x_k,y_k)^2$$

# Harris Corner Detector: Small Motion Assumption

- The surface at $(x, y)$ can be locally approximate by a quadratic form:

$$E(x,y) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \cdot \begin{bmatrix} A & B \\ B & C \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x_k, y_k \in W(x,y)} I_x(x_k, y_k)^2$$

$$B = \sum_{x_k, y_k \in W(x,y)} I_x(x_k, y_k) I_y(x_k, y_k)$$

$$C = \sum_{x_k, y_k \in W(x,y)} I_y(x_k, y_k)^2$$

# Harris Corner Detector: Small Motion Assumption

- $E(x,y)$ can be rewritten as

$$E(x,y) \approx \sum_{x_k,y_k \in W(x,y)} \begin{bmatrix} u & v \end{bmatrix} \cdot \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

$$= \begin{bmatrix} u & v \end{bmatrix} \cdot M \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x_k,y_k \in W(x,y)} \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix}$$

# Harris Corner Detector: Small Motion Assumption

- $E(x,y)$ can be rewritten as

$$E(x,y) \approx \sum_{x_k,y_k \in W(x,y)} \begin{bmatrix} u & v \end{bmatrix} \cdot \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

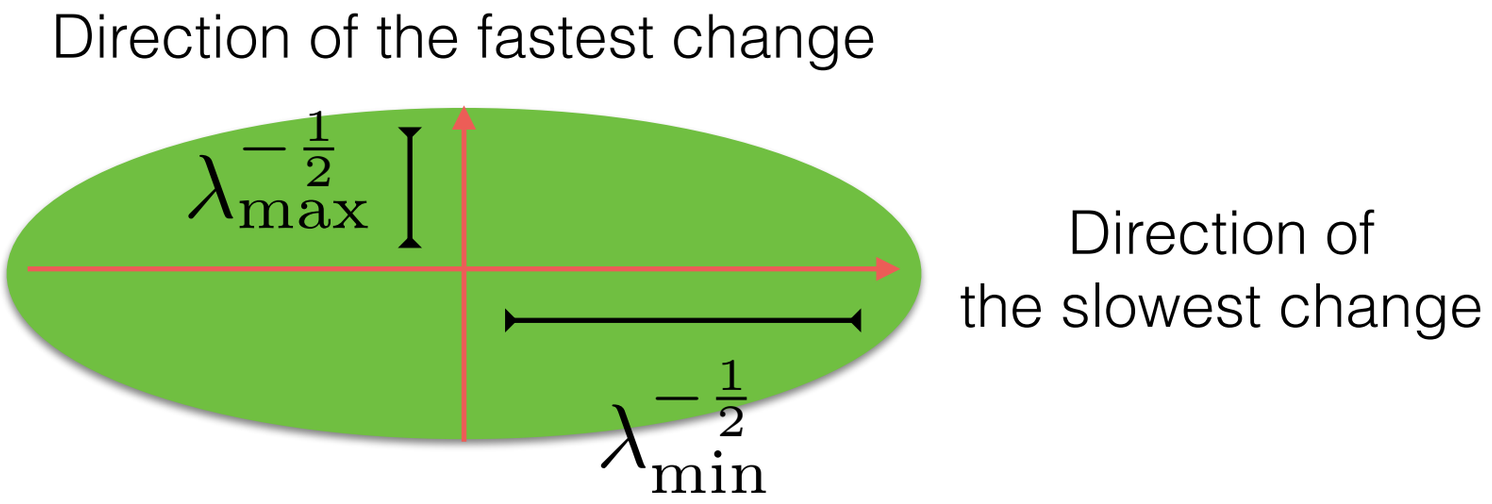
$$= \boxed{\begin{bmatrix} u & v \end{bmatrix} \cdot M \cdot \begin{bmatrix} u \\ v \end{bmatrix}}$$

Ellipse Equation:
$E(u, v) = k$

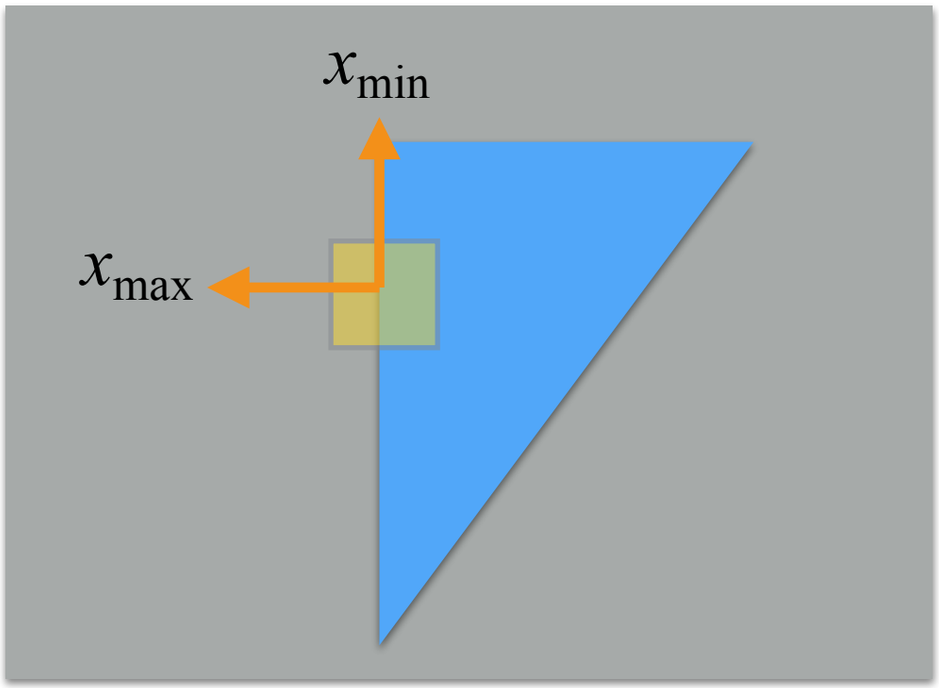
$$M = \sum_{x_k,y_k \in W(x,y)} \begin{bmatrix} I_x^2(x_k,y_k) & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y^2(x_k,y_k) \end{bmatrix}$$

# Harris Corner Detector: Second Moment Matrix

- $M$ reveals information about the distribution of gradients around a pixel.
- The eigenvectors of $M$ identify the directions of fastest and slowest change.



# Harris Corner Detector: Second Moment Matrix




Eigenvalues and eigenvectors of $M$ define shift directions with the smallest and largest change in $E$:

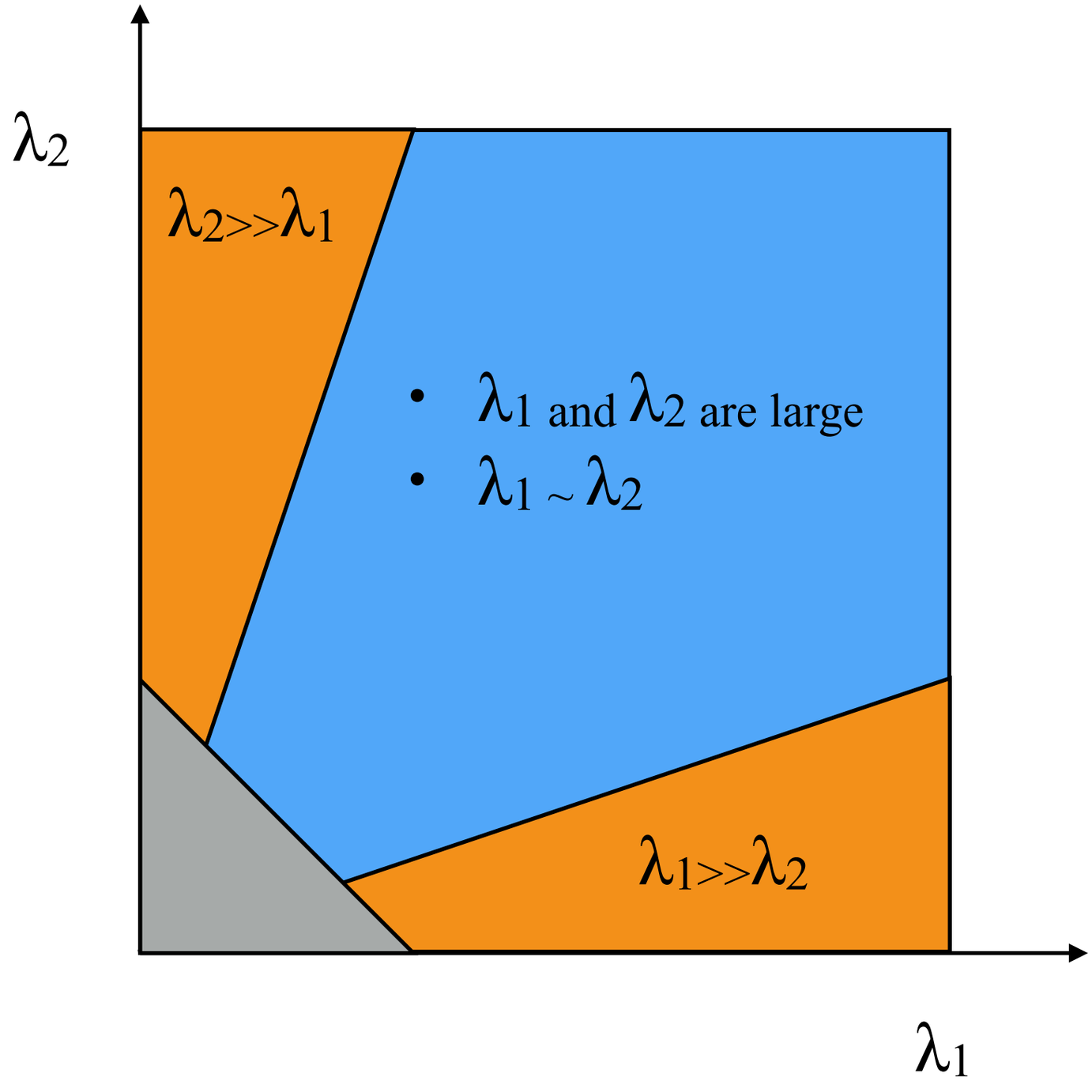
- $x_{\max}$ = direction of largest increase in $E$
- $\lambda_{\max}$ = amount of increase in direction $x_{\max}$
- $x_{\min}$ = direction of smallest increase in $E$
- $\lambda_{\min}$ = amount of increase in direction $x_{\min}$

# Classification

**Corner**

**Edge**

**Flat Region**



# Harris Corner Detector: Cornerness Measure

- Instead of directly computing the eigenvalues, we use a measure that determines the "***cornerness***" of a pixel (i.e., how close to be a corner is):

$$R = \mathrm{Det}(M) - k\mathrm{Tr}(M)^2$$

$$\mathrm{Det}(M) = \lambda_1 \lambda_2$$

$$\mathrm{Tr}(M) = \lambda_1 + \lambda_2$$

- $k$ is an empire constant with values $[0.04\ 0.06]$.

# Harris Corner Detector: Cornerness Measure

Input Image

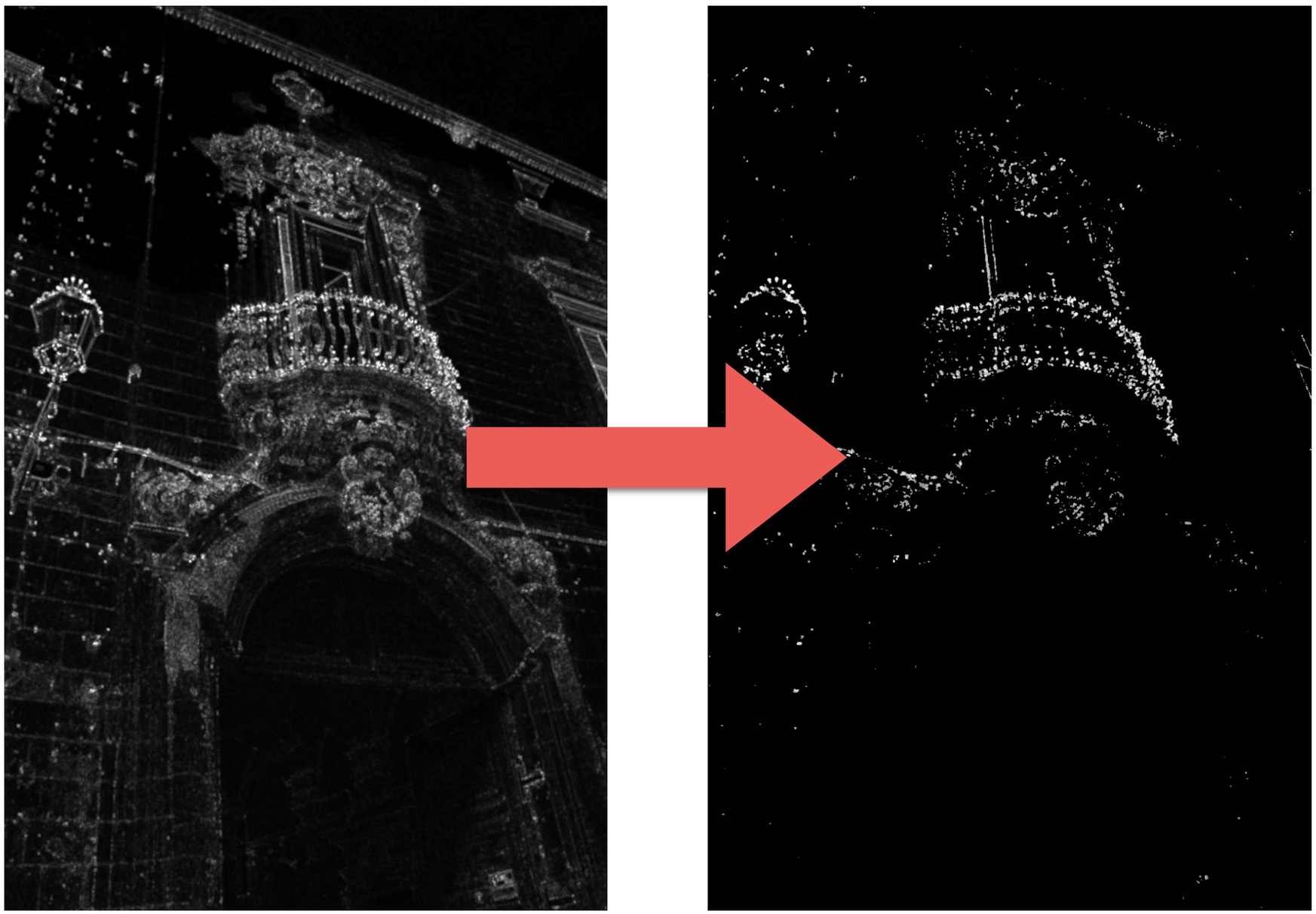
$R$

# Harris Corner Detector: Pruning Corners

- We have to find pixels with large corner response, $R$, i.e., $R > T_0$.

- Typically, $T_0$ in $[0,1]$ depends on the number of points we want to extract; a default value is 0.01.

# Harris Corner Detector: Thresholding

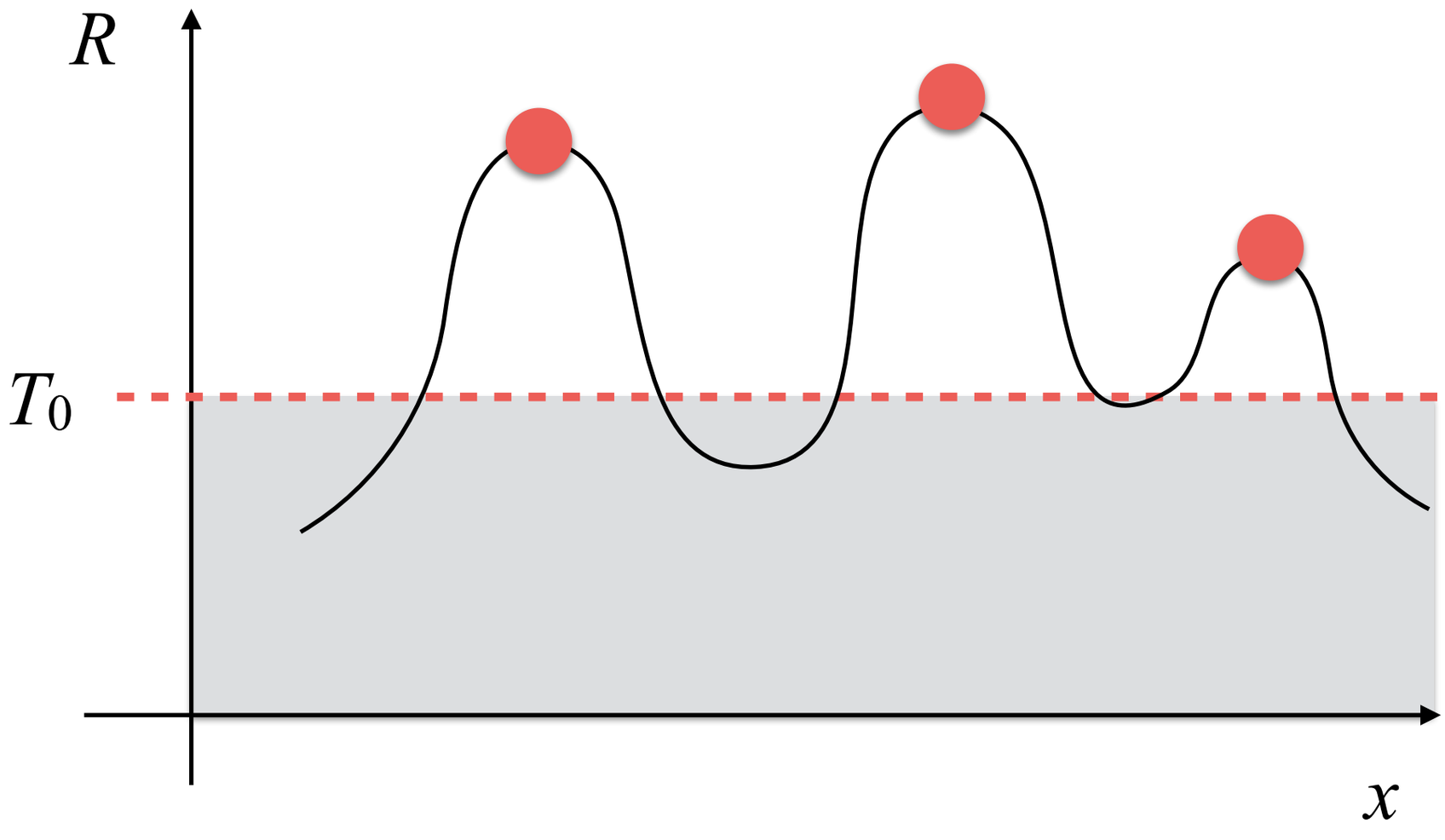


$R$

$R$ after thresholding

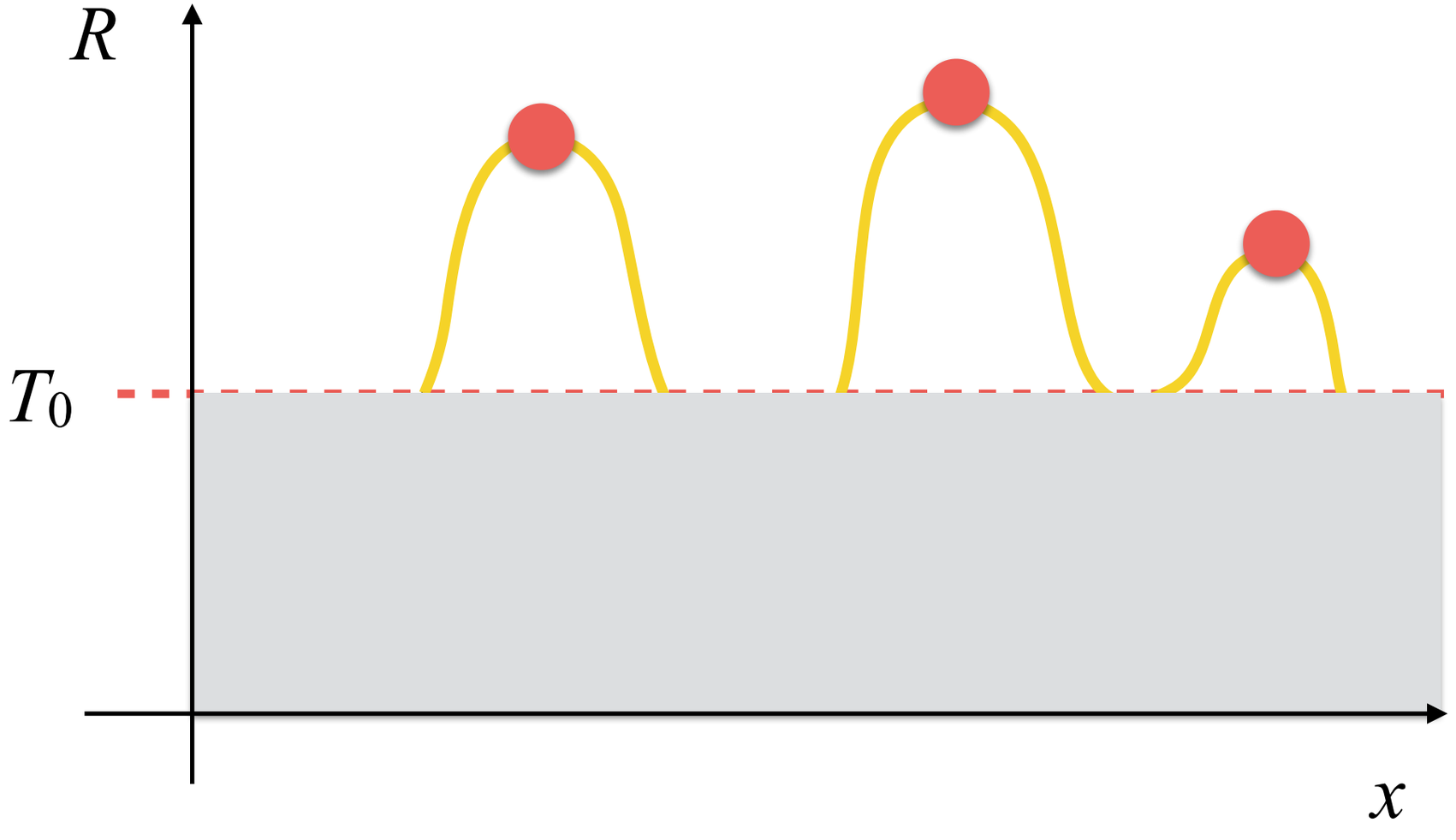
# Harris Corner Detector: Pruning Corners

- At this point, we need to suppress/remove values that are not maxima.



# Harris Corner Detector: Pruning Corners

- At this point, we need to suppress/remove values that are not maxima, but they are over the threshold (yellow pixels).



# Harris Corner Detector: Pruning Corners

- We set a radius (in pixel) for suppressing non-maxima; e.g., 3-5.

- We apply to $R$ a maximum filter; it is similar to a median filter, but it computes the maximum instead of the median. After this we obtain a filtered image called $R_{\max}$.

- A pixel at position $(x, y)$ is a local maximum if and only if:

$$R_{\max}(x, y) = R(x, y) \quad \wedge \quad R(x, y) > T_0$$

# Harris Corner Detector: Pruning Corners Example 1

| | | |
|---|---|---|
| 100 | 0 | 30 |
| 40 | **20** | 20 |
| 0 | 0 | 0 |

The current pixel that we are evaluating is the central one!

$$T_0 = 5$$

# Harris Corner Detector: Pruning Corners Example 1

| | | |
|---|---|---|
| ***100*** | 0 | 30 |
| 40 | **20** | 20 |
| 0 | 0 | 0 |

The maximum is 100!

# Harris Corner Detector: Pruning Corners Example 1

| | | |
|---|---|---|
| 100 | 0 | 30 |
| 40 | **0** | 20 |
| 0 | 0 | 0 |

20 < 100 so it has to be suppressed; i.e., set to 0!

# Harris Corner Detector: Pruning Corners Example 2

| | | |
|---|---|---|
| 20 | 0 | 30 |
| 40 | **100** | 20 |
| 0 | 0 | 0 |

The current pixel that we are evaluating is the central one!

$$T_0 = 5$$

# Harris Corner Detector: Pruning Corners Example 2

| | | |
|---|---|---|
| 20 | 0 | 30 |
| 40 | **100** | 20 |
| 0 | 0 | 0 |

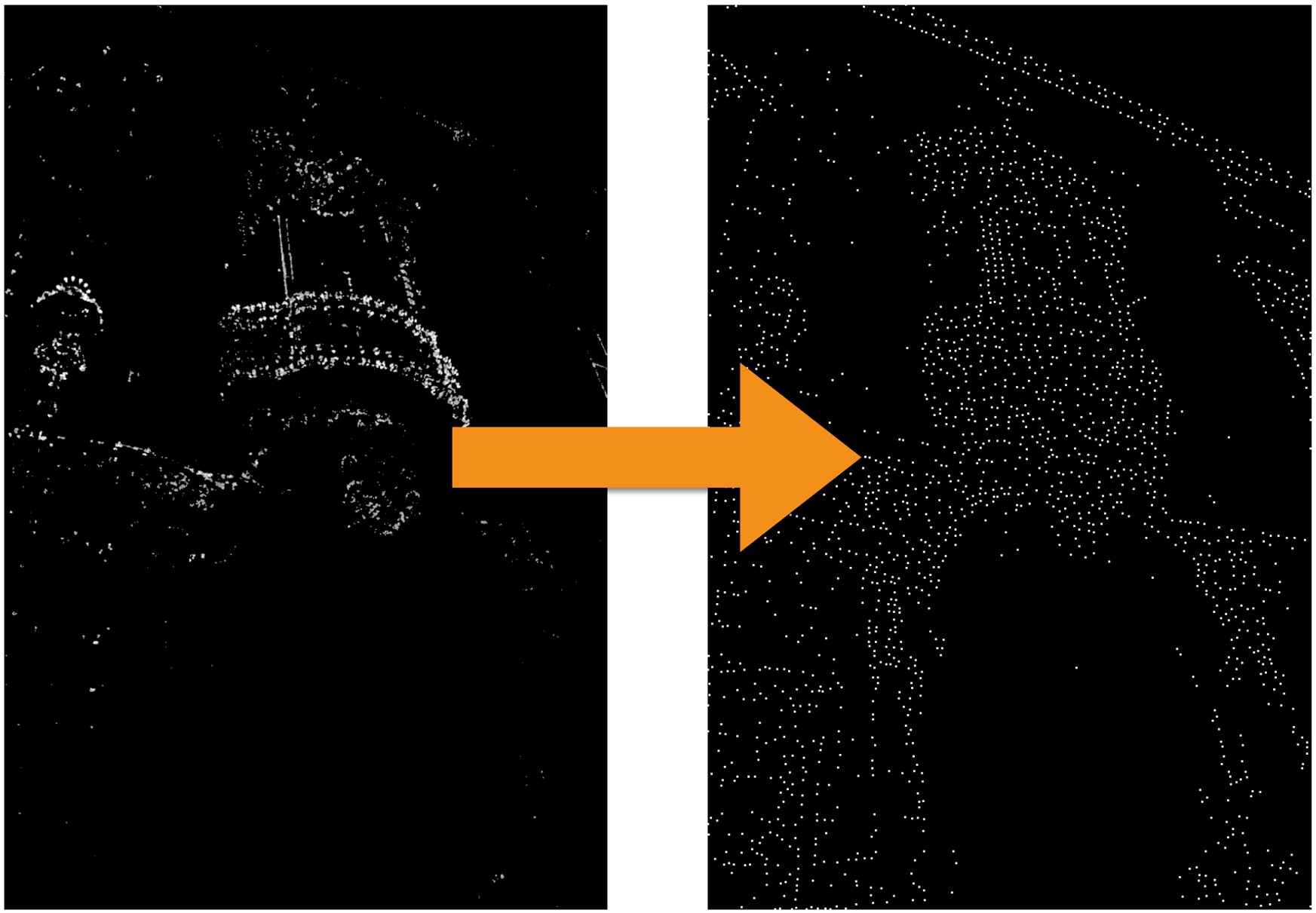
The maximum is 100!

# Harris Corner Detector: Pruning Corners Example 2

| | | |
|---|---|---|
| 20 | 0 | 30 |
| 40 | **100** | 20 |
| 0 | 0 | 0 |

100 == 100 so it has to be kept!

# Harris Corner Detector: Non-Maximal Suppression



$R$ after thresholding

Non-Maximal Suppression

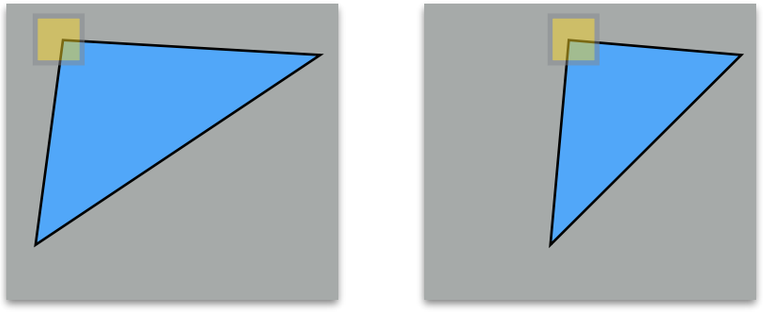
# Harris Corner Detector: Non-Maximal Suppression

# Harris Corner Detector: Non-Maximal Suppression

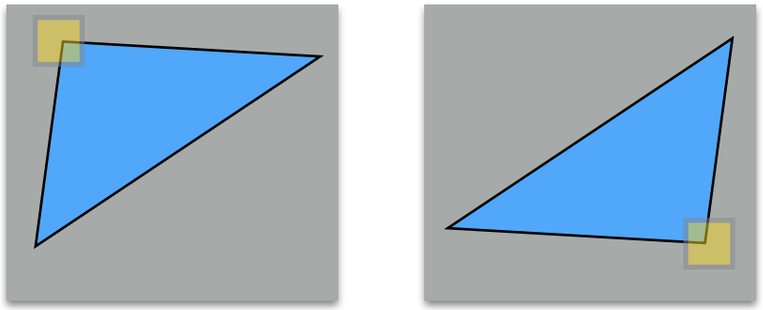
# Harris Corner Detector: Non-Maximal Suppression

# Harris Corner: Advantages

- Translational invariance:



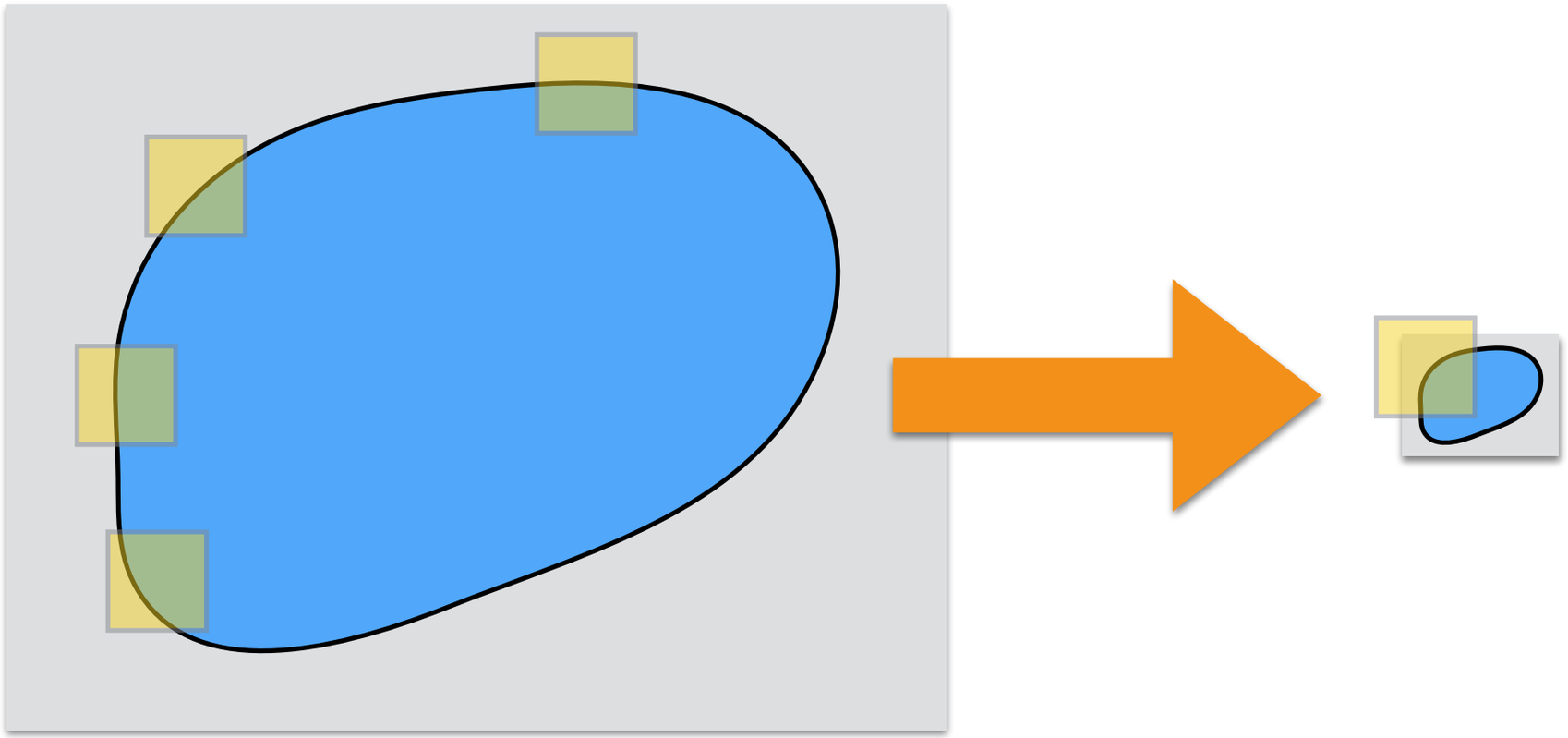
- Rotation invariance:



- Only derivatives are employed:
  - Intensity shift invariance: $I' = I + b$
  - Intensity scale invariance: $I' = I\,a$

# Harris Corner: Disadvantage

- Not scale invariant!



All points are classified as edges

It is now a corner!

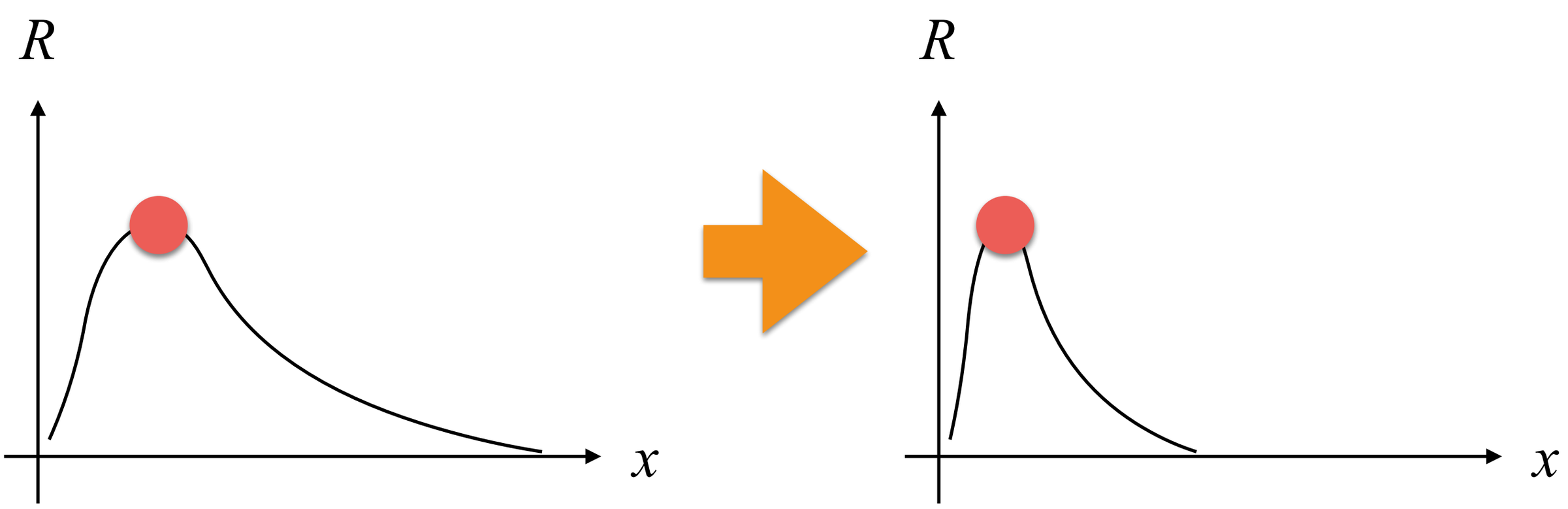
The same feature in different images can have different size!

# The Scale Problem

Near Object

Far Object

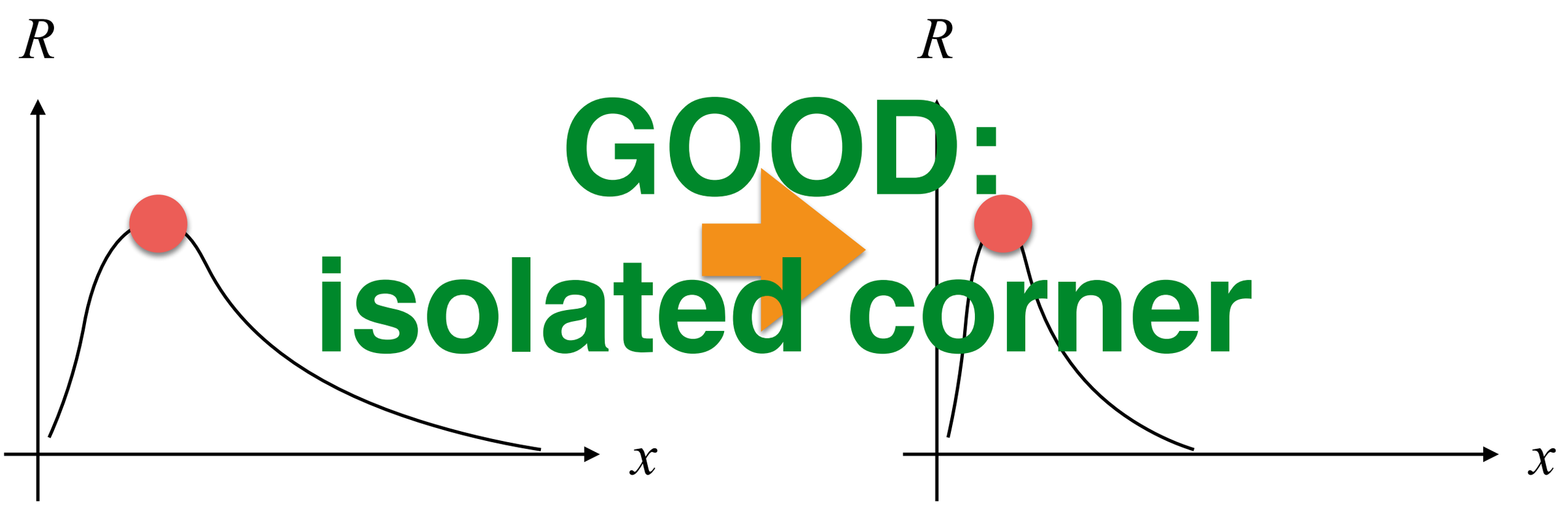
# Scale Invariant: Stable Corners




Original

1/2 scale

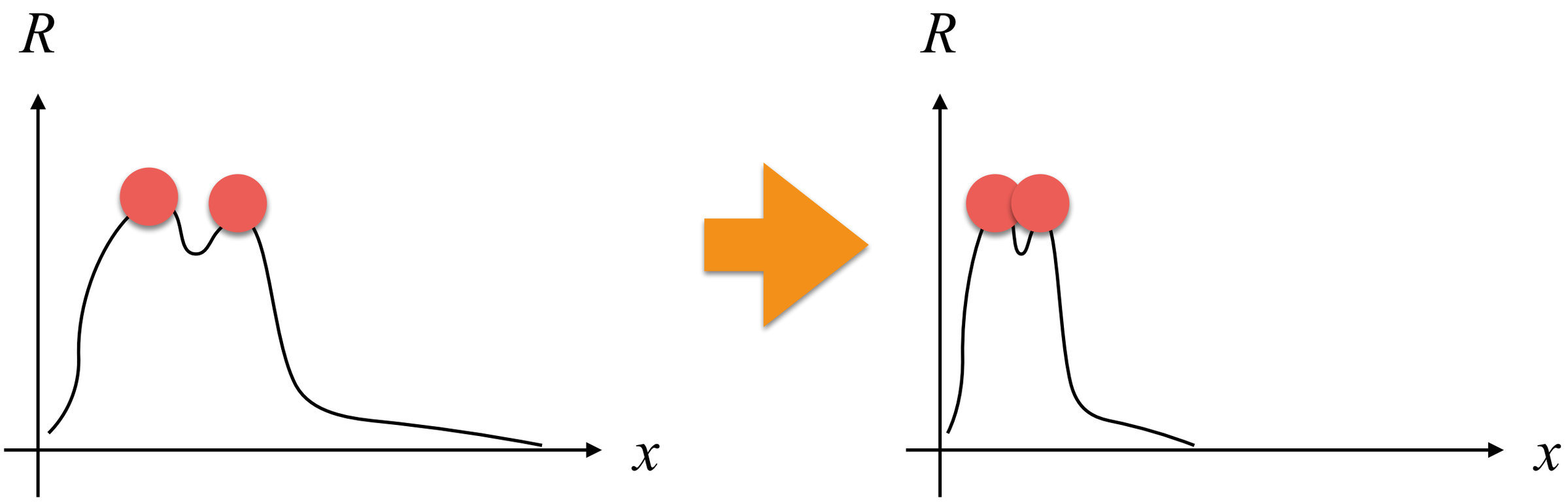
# Scale Invariant: Stable Corners




Original

1/2 scale

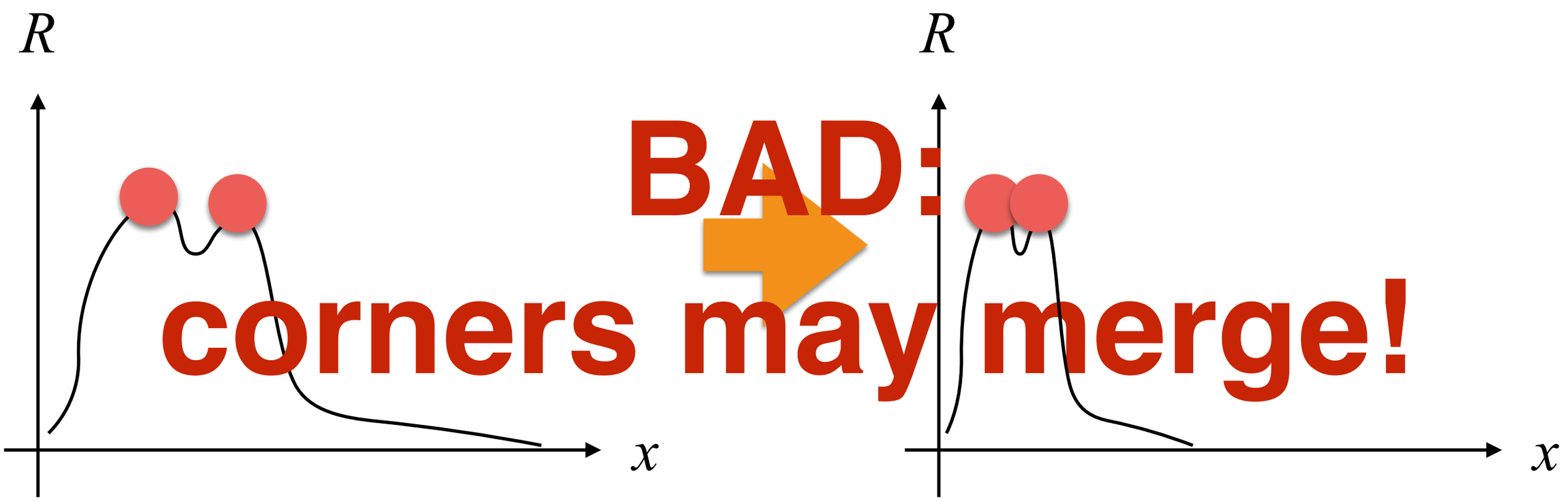
# Scale Invariant: Unstable Corners




Original

1/2 scale

# Scale Invariant: Unstable Corners




Original

1/2 scale

# Scale Invariant:
# A Multi-Scale Approach

- Depending on the content of the image:
  - We need to detect the scale of corner.
  - We need to use its scale to vary the size of the window $W$ for computing corners!

# Scale Invariant: The Signature Function

- A signature function, $s$, is a function giving us an idea of the local content of the image, $I$, around a point with coordinates $(x, y)$ at a given scale $\sigma$.

- An example of signature function is the Difference of Gaussians (DoG):

$$s(I, x, y, \sigma) = [I \otimes G(\sigma)](x, y) - [I \otimes G(\sigma \cdot 2)](x, y)$$

- where $G$ is a Gaussian kernel.

# Scale Invariant:
# The Signature Function


-


DoG

# Scale Invariant: The Approach

We need to find the right scale for resizing $W$ for each image!

# Scale Invariant: The Approach

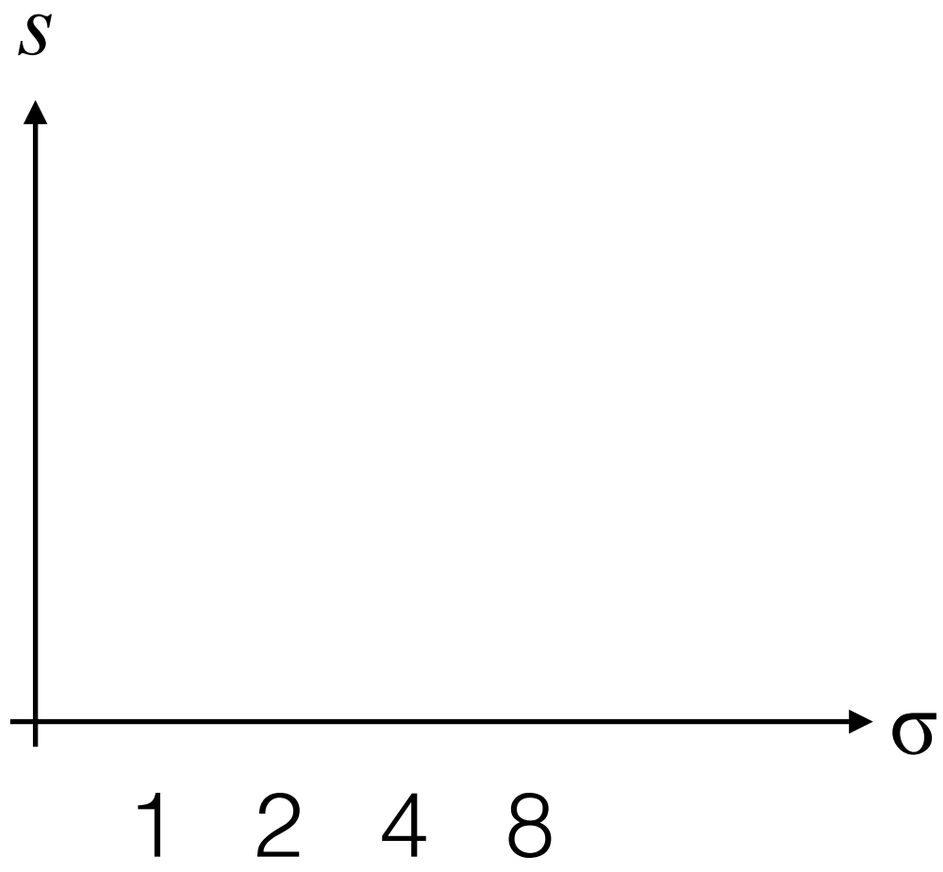
- The signature function, $s$, can give us an idea of the content of the image.

- Therefore, we need to find a maximum point of $s$ for pixel of an input image!

# Scale Invariant: The Approach

Let's build $s$ at the red point!

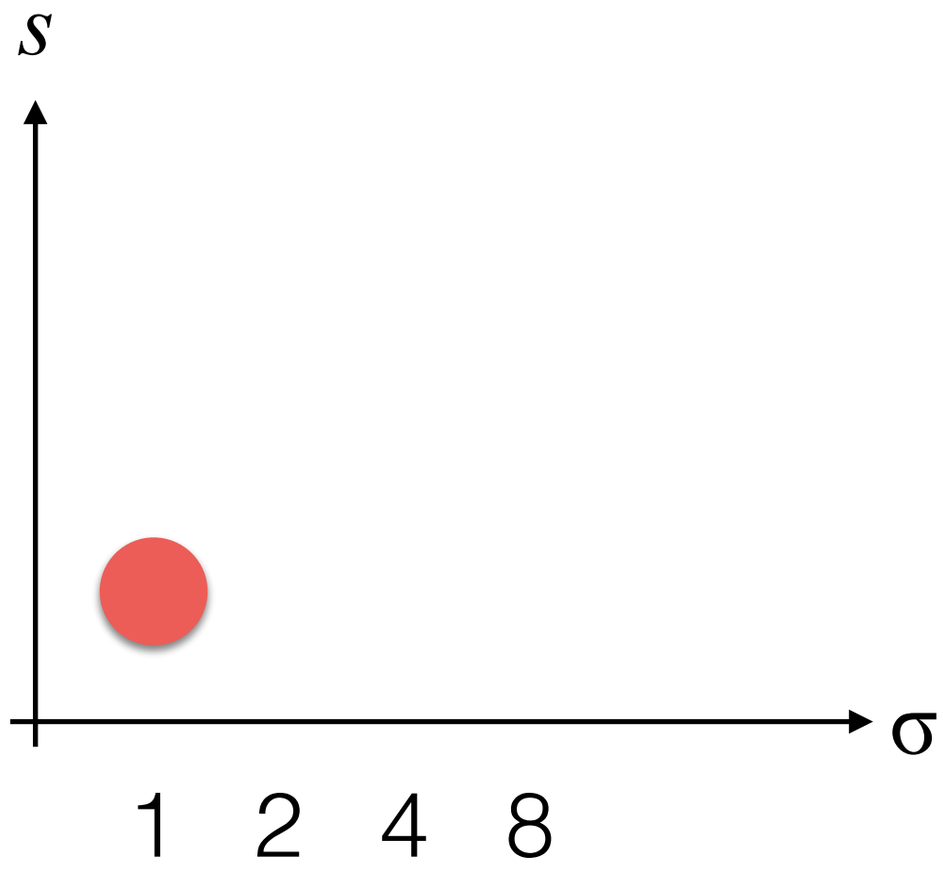
# Scale Invariant: The Approach




This is our start!

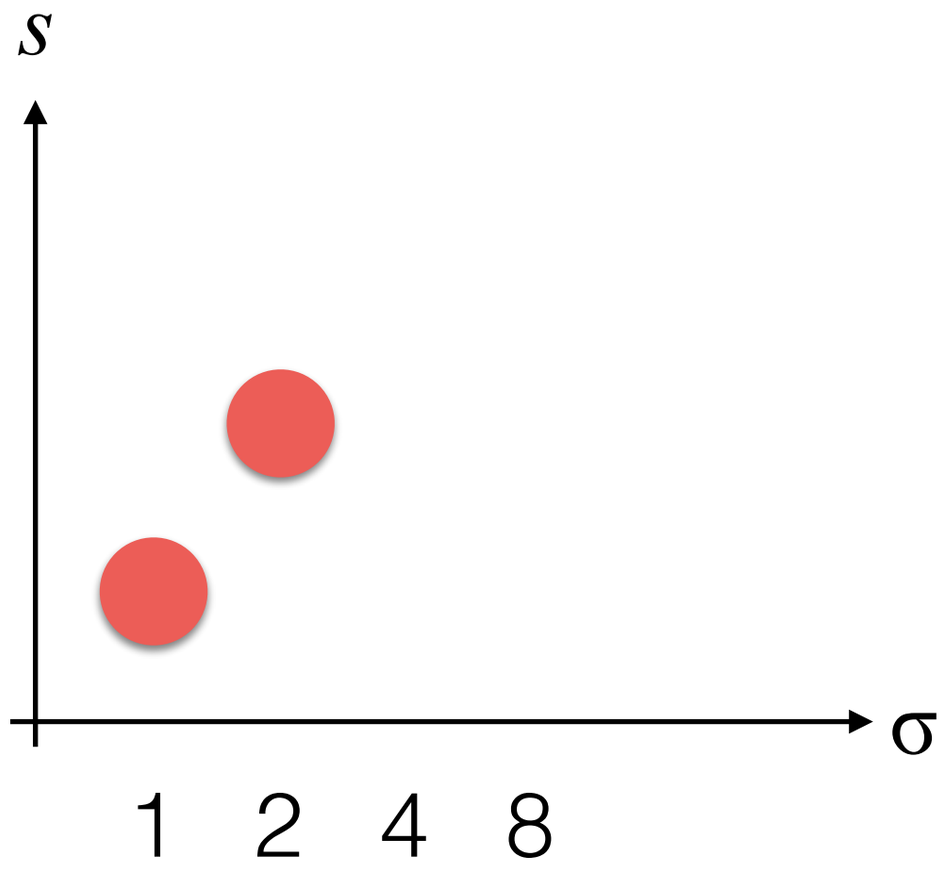
# Scale Invariant: The Approach

$\sigma = 1$



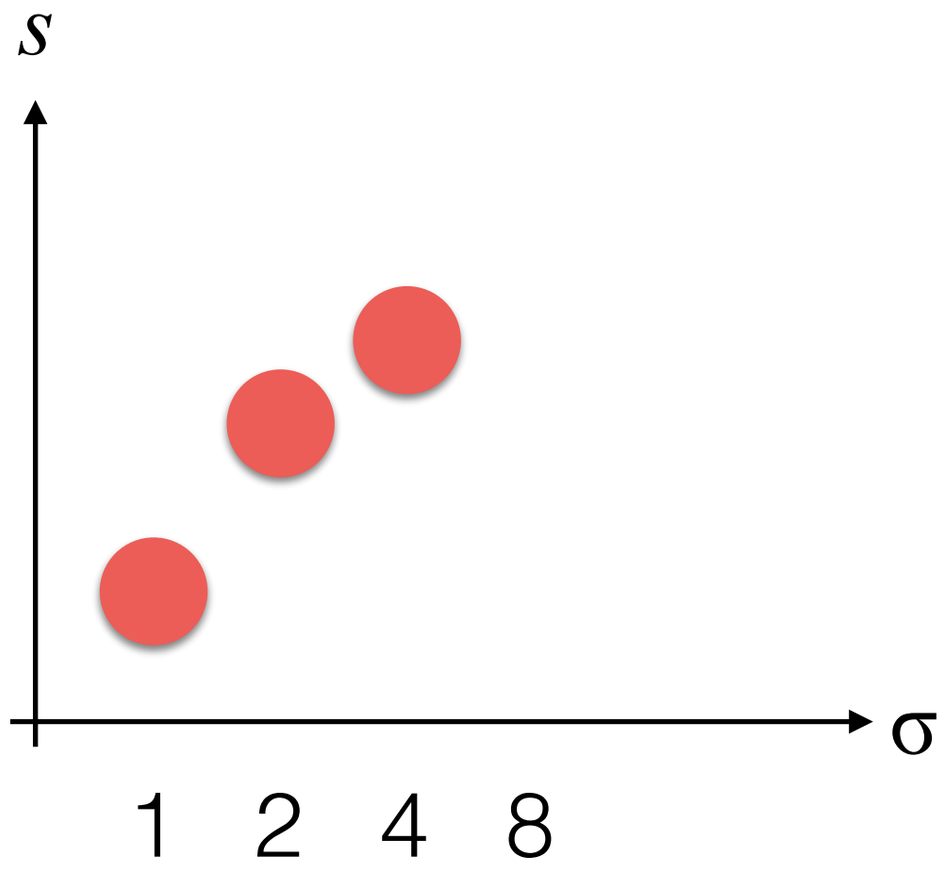
# Scale Invariant: The Approach

$\sigma = 2$

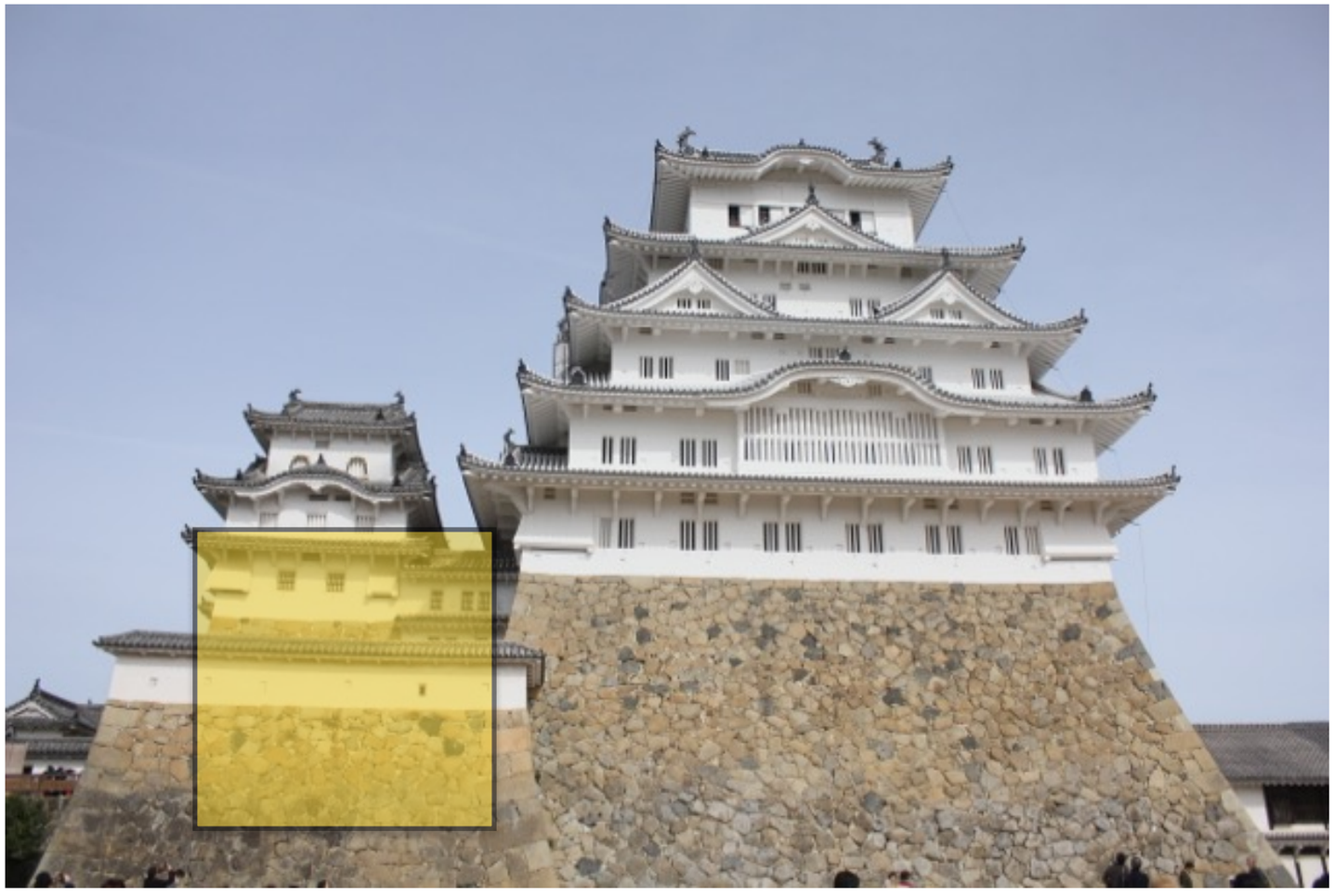


# Scale Invariant: The Approach

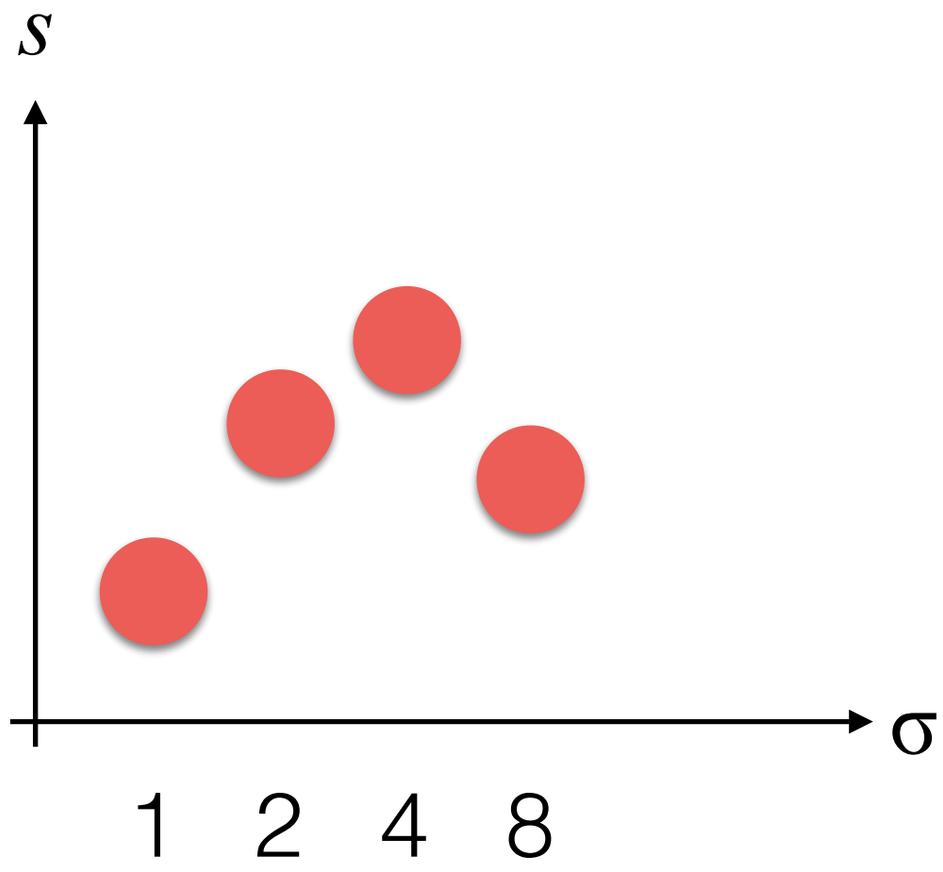
$\sigma = 4$



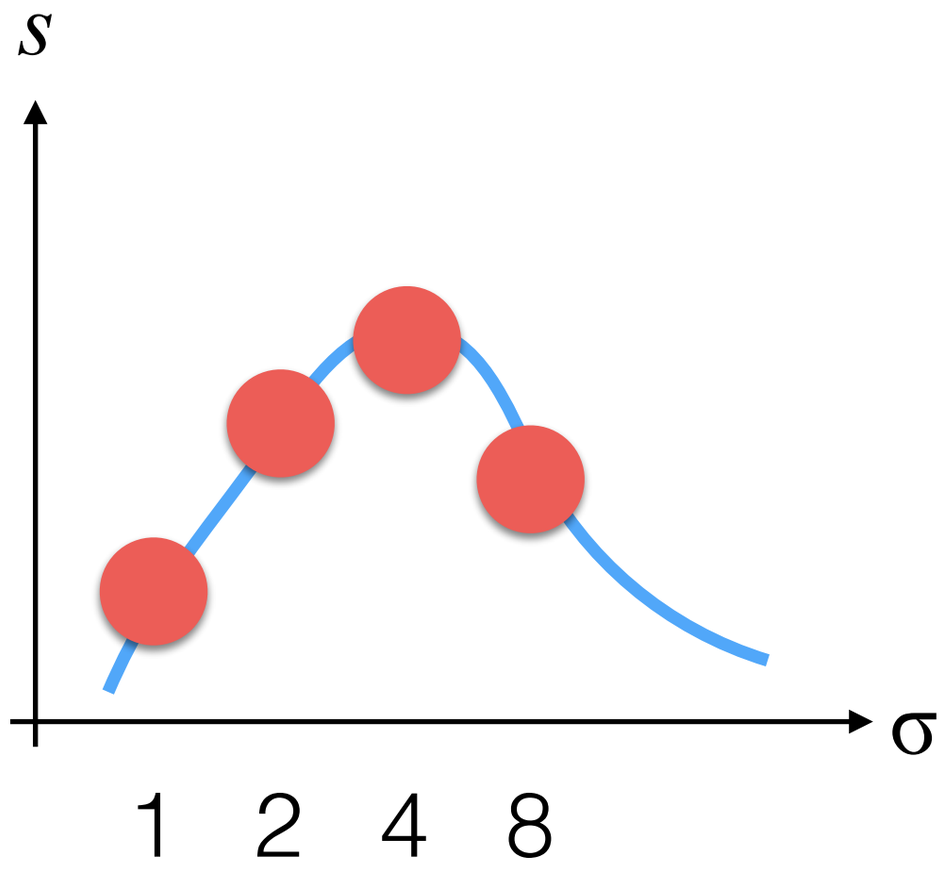
# Scale Invariant: The Approach



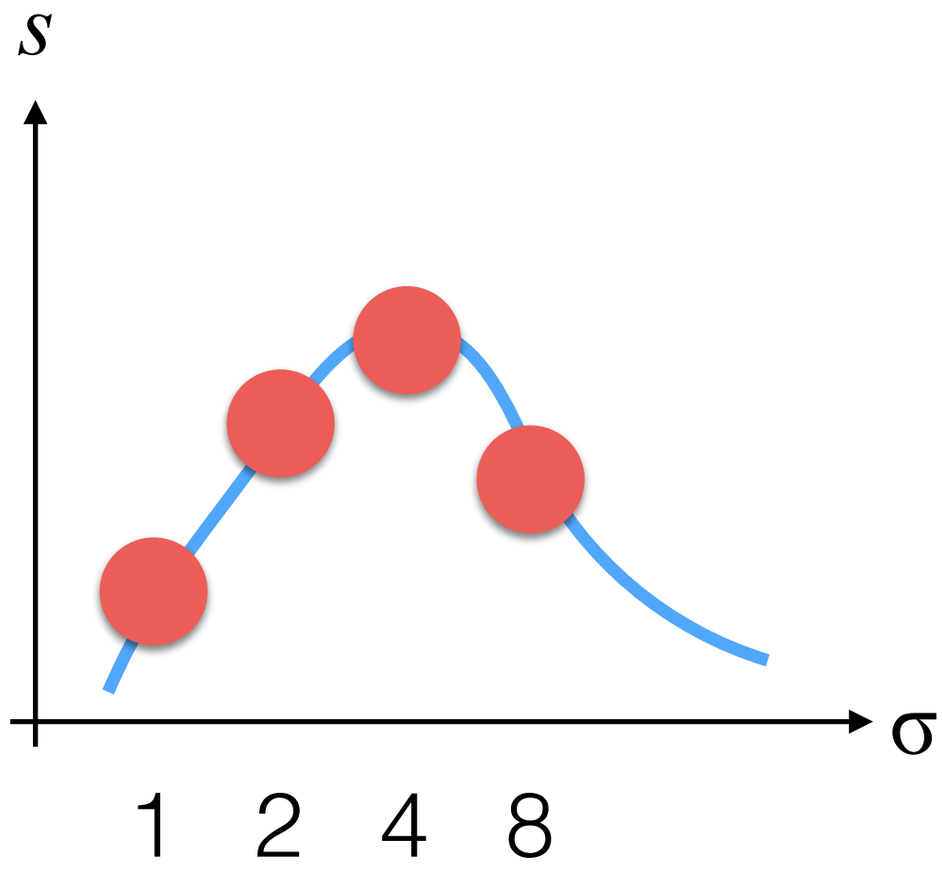
$\sigma = 8$



# Scale Invariant: The Approach

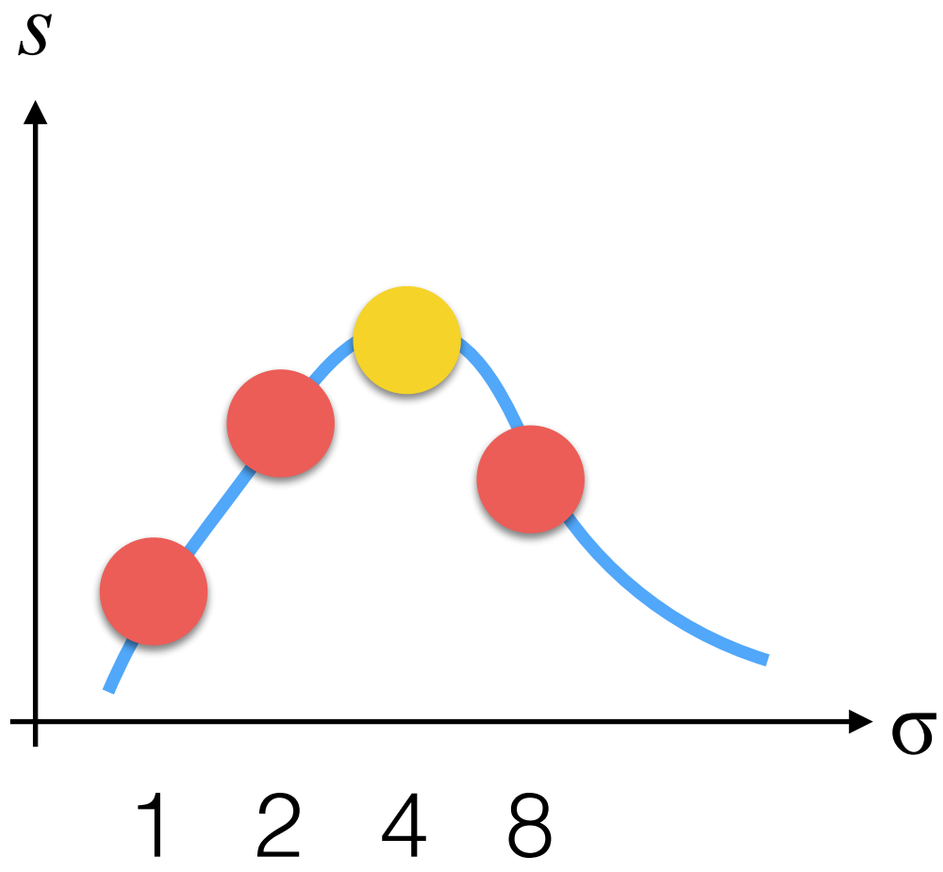


# Scale Invariant: The Approach



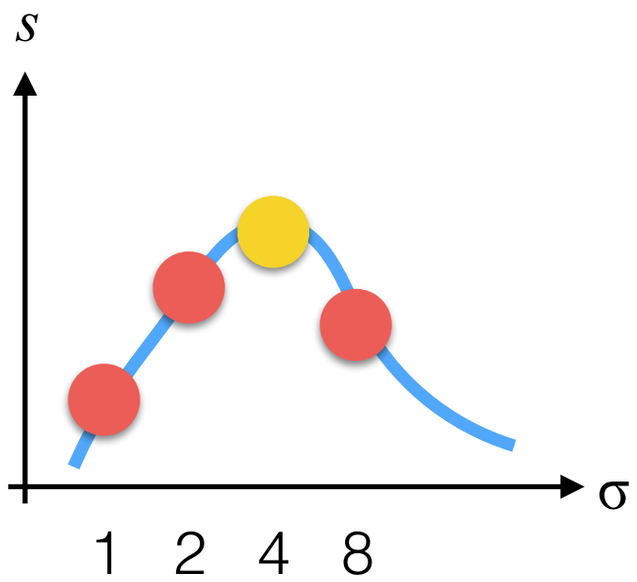

Which is σ for which $s$ is the maximum?

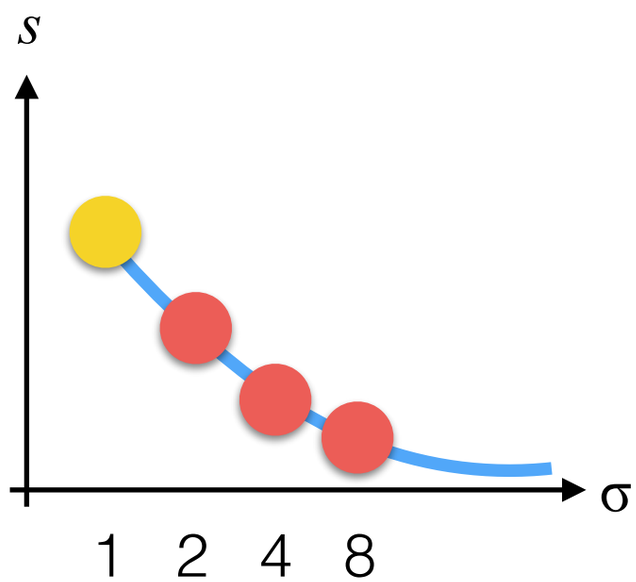
# Scale Invariant: The Approach




It is σ = 4

# Scale Invariant: The Approach



# Extraction of Features

- General overview:
  - We compute the scale for each pixel using the sigma value at which we have the maximum value of the signature function.
  - We compute the Harris Corner using the scale to increase the size of the local window; i.e., the scale of the window will be multiplied by the sigma value.

# Feature Descriptors

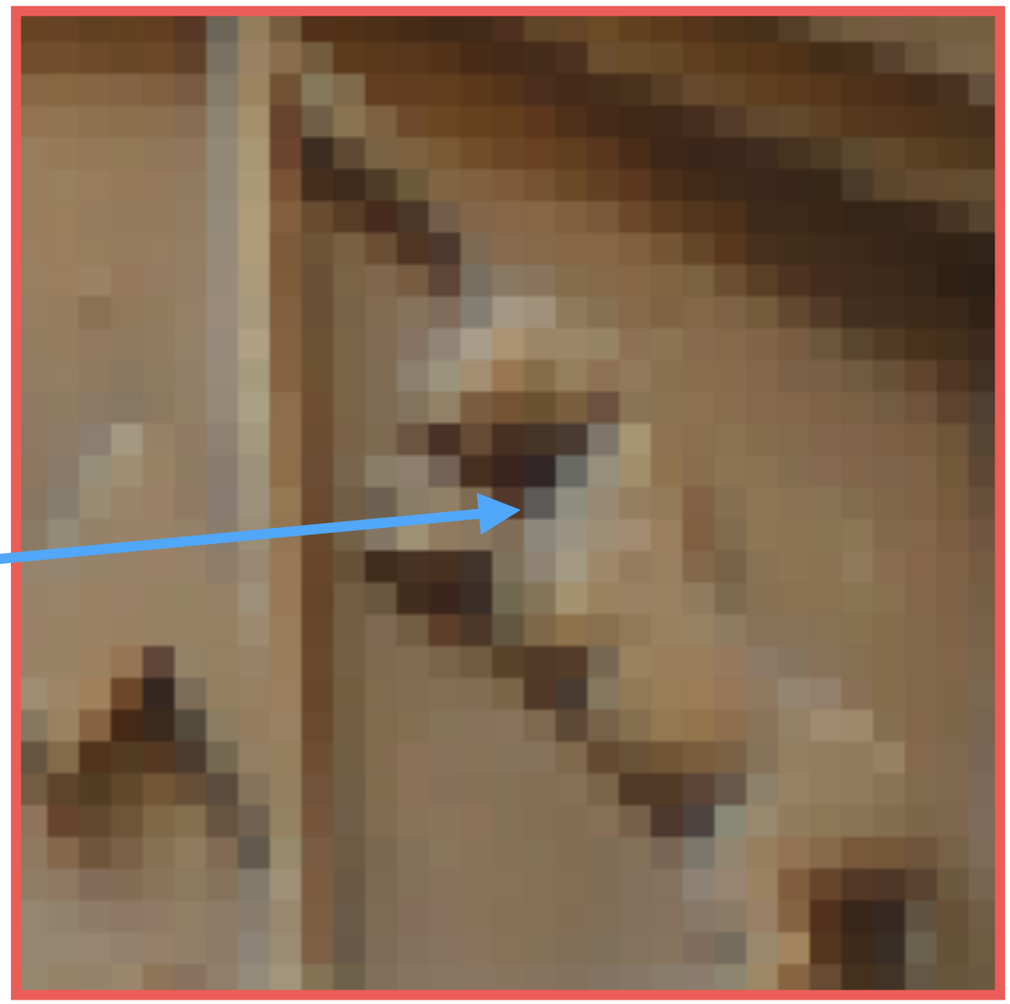
# Feature Descriptors

- Once we found our features (i.e., corners), we need to describe in a meaningful and possibly unique way.

- Why?

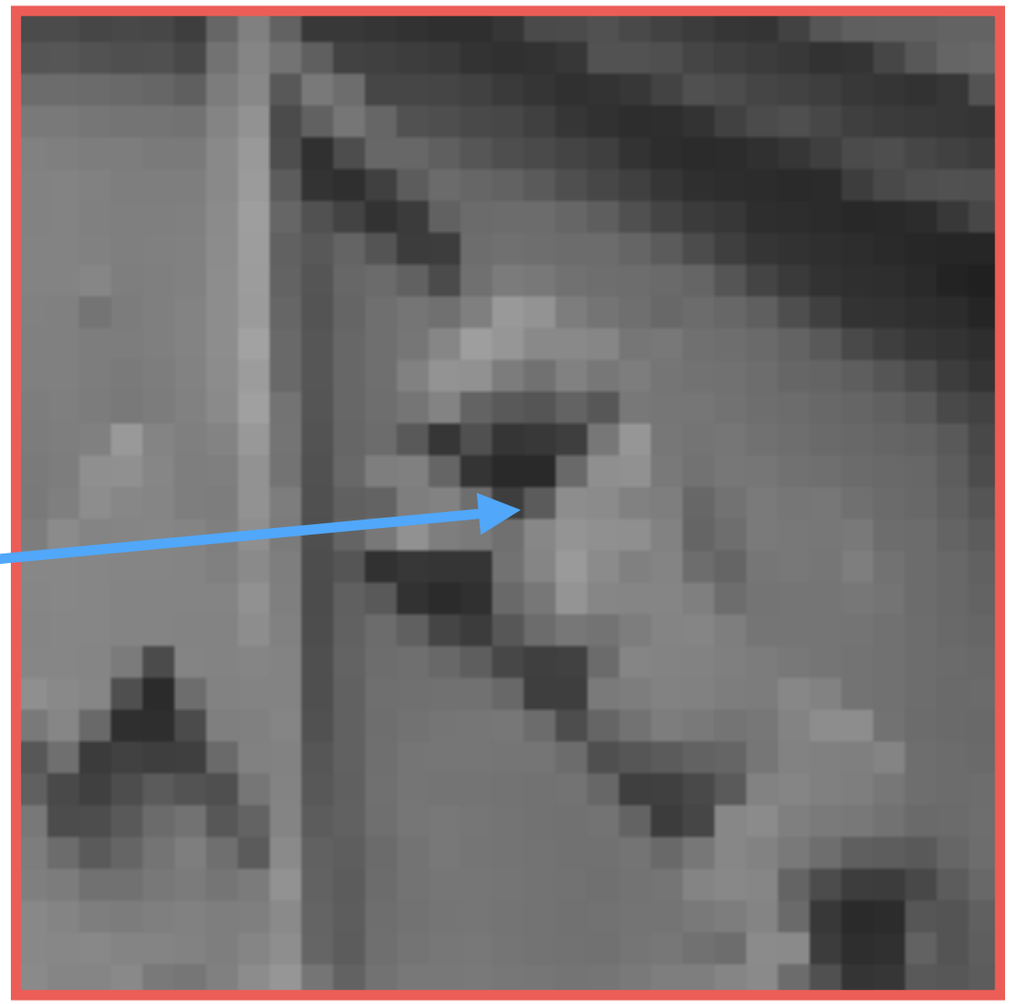
    - We want compare corners between images in order to find correspondences between images.

# Feature Descriptors



A patch, $P$, is a  sub-image centered in a given point $(u, v)$.

# Feature Descriptors



A patch, $P$, is a  sub-image centered in a given point $(u, v)$.

# Feature Descriptors

- There are many local features descriptors in literature:
  - BRIEF/ORB descriptor.
  - SIFT descriptor.
  - SURF descriptor.
  - etc.

# Feature Descriptors

- Good properties that we want are invariance to:
  - Illumination changes.
  - Rotation.

# BRIEF Descriptor

- The descriptor creates a vector of $n$ binary values:

$$\mathrm{BRIEF}(P) = \mathbf{b} = [0, 1, 0, 0, \ldots, 1]^\top$$

- For efficiency, it is encoded as a number:

$$n_{\mathbf{b}} = \sum_{I=1}^{n} 2^{i-1} b_i$$

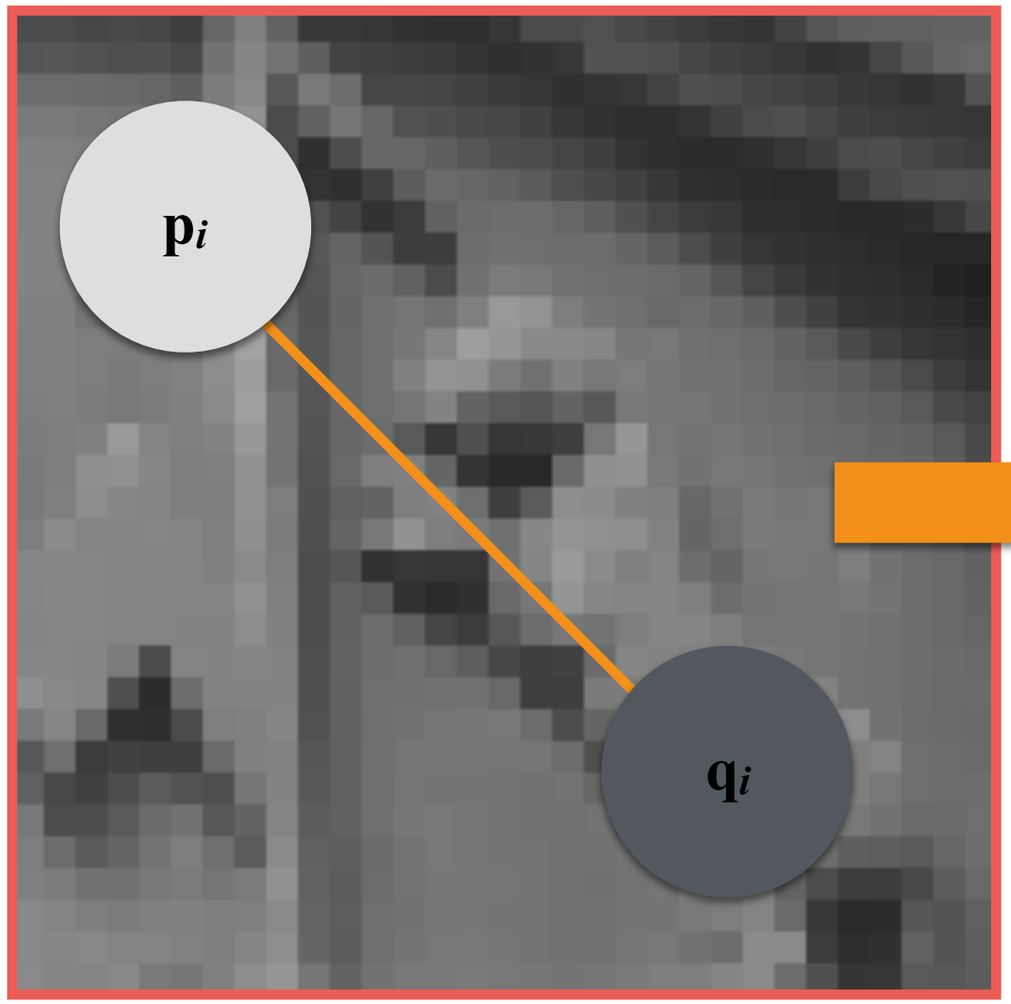
# BRIEF Descriptor

- Given a patch, $P$, of size $S$×$S$ an element of $\mathbf{b}$ is defined as

$$b_i(\mathbf{q}_i, \mathbf{p}_i) = \begin{cases} 1 & \text{if } P(\mathbf{p}_i) < P(\mathbf{q}_i), \\ 0 & \text{otherwise} \end{cases}$$

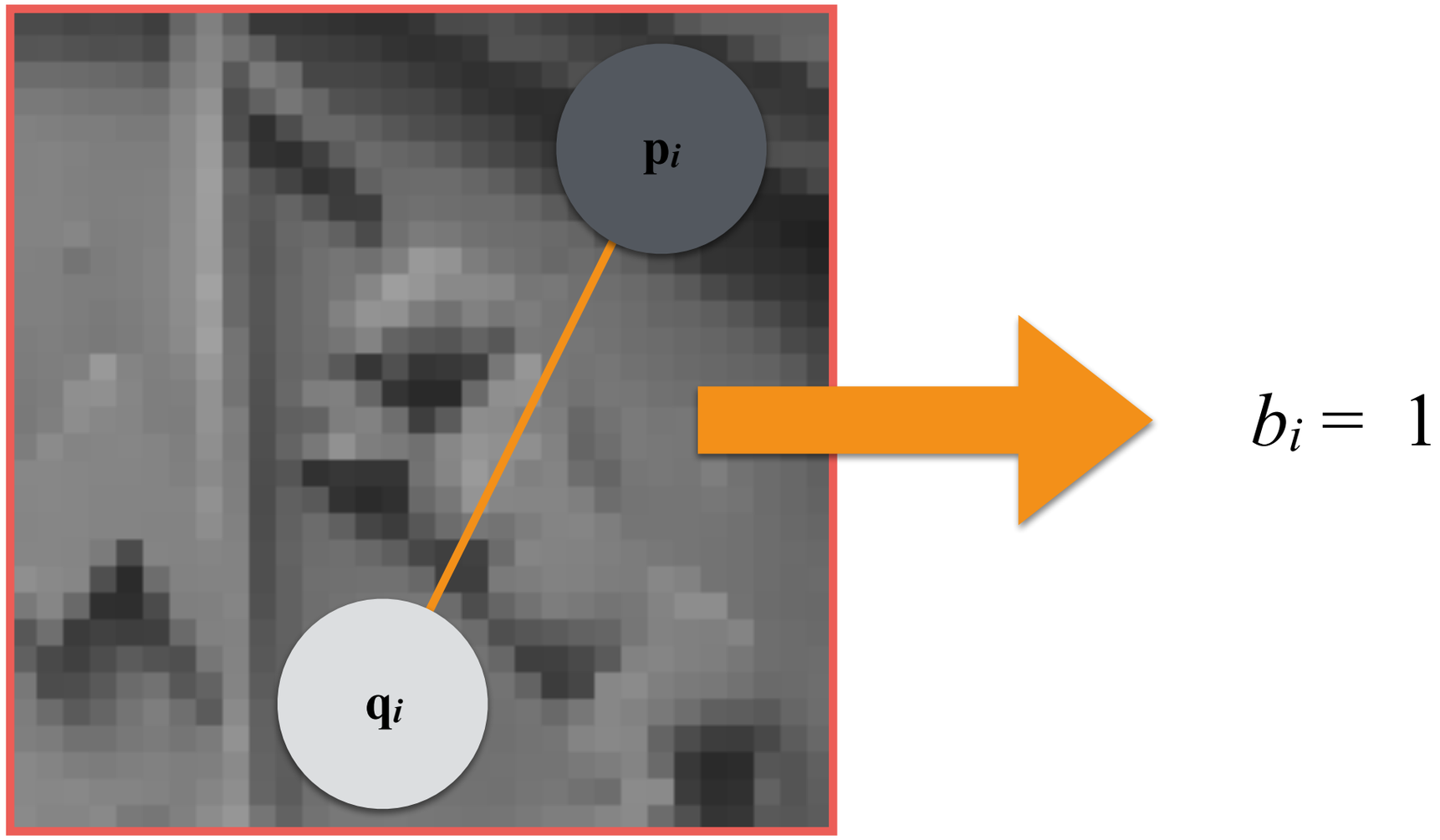
- where $\mathbf{p}_i$ and $\mathbf{q}_i$ are the coordinates $(x, y)$ of two random points in $P$.

# BRIEF Descriptor: Example




$$b_i = 0$$

# BRIEF Descriptor: Example



# BRIEF Descriptor: Test

- Let's say we have two descriptor $\mathbf{b}^1$ and $\mathbf{b}^2$. How do we check if they are describing the same corner?

- We count the number of different bits in the two vectors (Hamming distance):

$$D_H(\mathbf{b}^1, \mathbf{b}^2) = \sum_{i=1}^{n} \neg\text{xor}(b_i^1, b_i^2)$$

- **Higher the closer**:

  - This is a very computationally efficient distance function.

# BRIEF Descriptor: Test

| A | B | A XOR B<br>=<br>[(NOT A) AND B] OR<br>[(NOT B) AND A] | NOT (A XOR B) |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

# BRIEF Descriptor: Point-Set

- The optimal number of points' couple (i.e., the size of the descriptor; $n$) is **256**.

- This value was computed from experiments testing different lengths: 16, 32, 64, 128, 256, and 512.

- Points can be generated in different ways:

  - Uniform distribution in the patch

  - $(\mathbf{p}_i, \mathbf{q}_i) \sim$ i.i.d. $\text{Gaussian}\left(0, \frac{S^2}{25}\right)$

# BRIEF Descriptor: Point-Set

- Points are pre-computed, ***only once***, generating a set:

$$A = \begin{bmatrix} \mathbf{p}_0, & \mathbf{p}_1, & \cdots & \mathbf{p}_n \\ \mathbf{q}_0, & \mathbf{q}_1, & \cdots & \mathbf{q}_n \end{bmatrix}$$

  - This set is ***always*** used for the extraction of all descriptors in all photos!

    - If this is not done, we cannot do comparisons because we are comparing different tests (e.g., comparing apples and organges):

      - We need to keep ***consistency***

# BRIEF Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Patent free.
- Disadvantage:
  - Rotation is an issue:
    - The method can handle rotations up to 10-15 degrees only.

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# ORB Descriptor

- The descriptor is a modified version of BRIEF and it can handle rotations!

- The first step of the algorithm is to compute the orientation of the current patch $P$.

- **Idea**: we determine the “center of mass” of the image, and we compute the angle between this “center of mass” and the center of the patch. This is a hint for the orientation of the patch.

# ORB Descriptor: Patch Orientation

- We compute the patch orientation using Rosin moments of a patch:

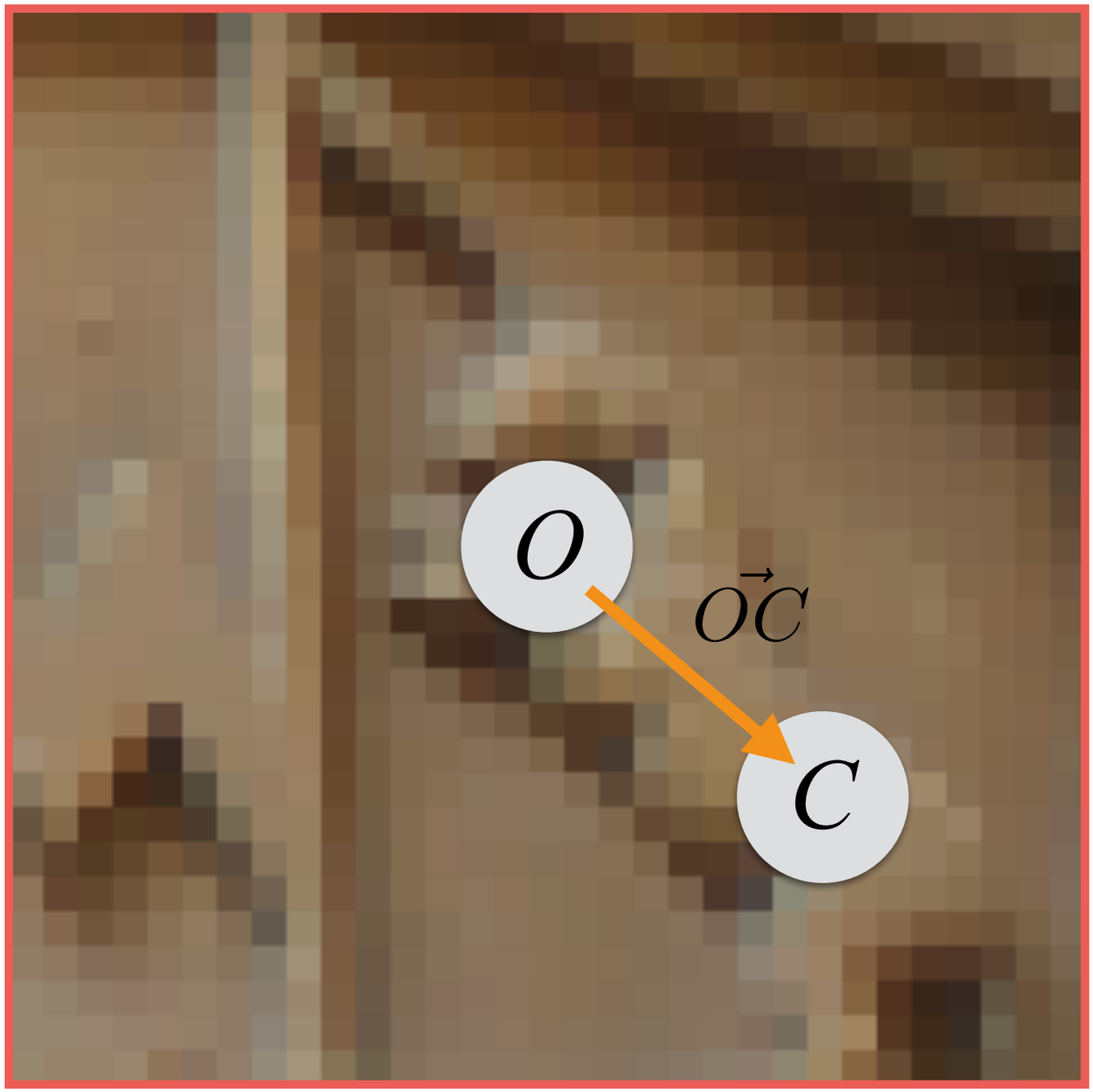
$$m_{a,b} = \sum_{x,y \in P} x^a y^b P(x,y)$$

- From this, we define the centroid, $C$, as

$$C = \left( \frac{m_{1,0}}{m_{0,0}}, \frac{m_{0,1}}{m_{0,0}} \right)$$

- Now, we can create a vector from corner's center, $O$, to the centroid, $C$. This allows us to calculate the angle of rotation.

# ORB Descriptor: Patch Orientation



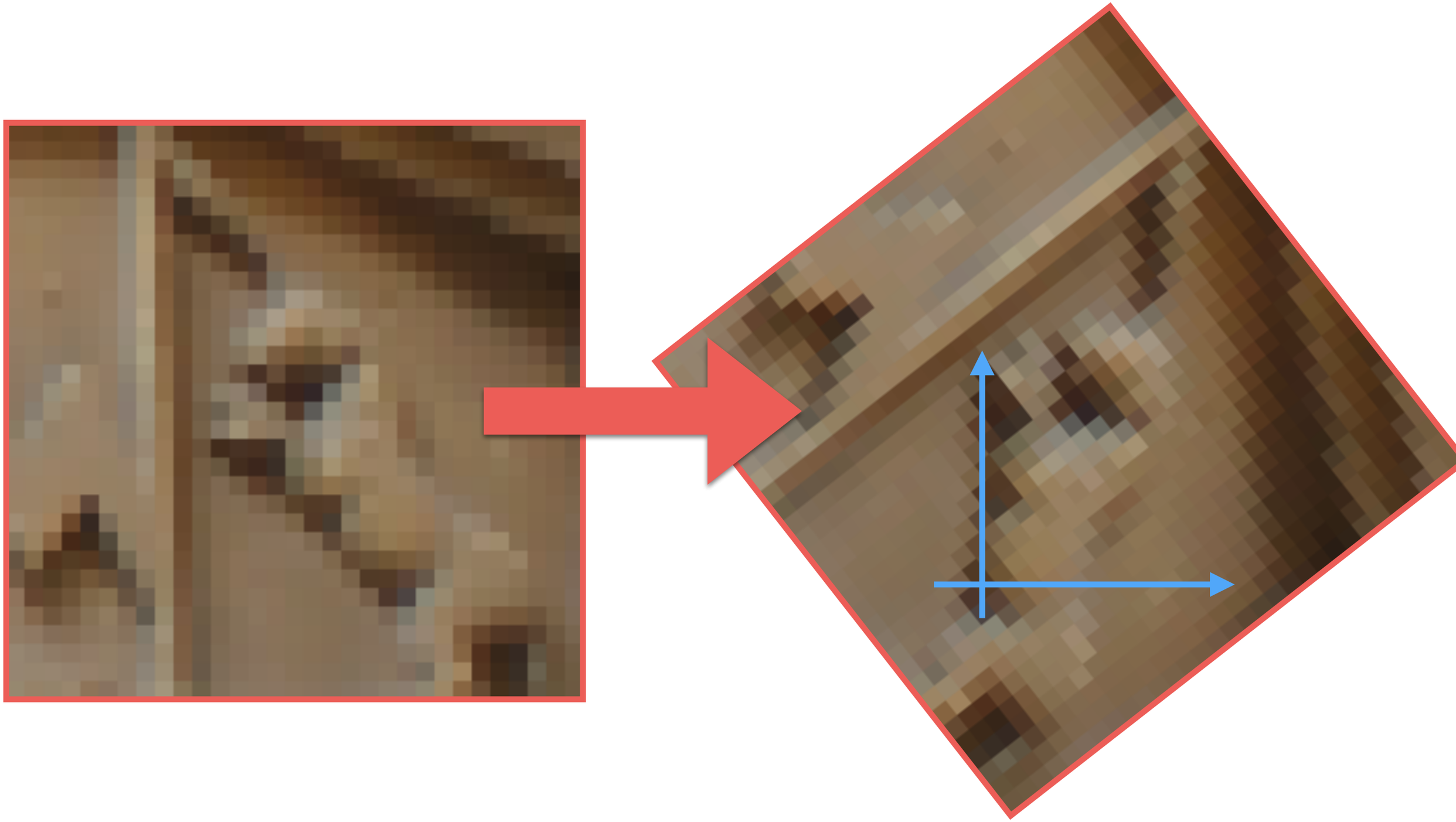
# ORB Descriptor: Patch Orientation

- From this vector, the orientation of the patch can be computed simply as

$$\theta = \mathrm{atan2}(m_{0,1}, m_{1,0})$$

- From this, we can rotate the patch $P$, but this operation is very computationally expensive:
  - We need to rotate each single point in the patch!

# ORB Descriptor



# ORB Descriptor: Patch Orientation

- Instead of rotating the whole patch, we can rotate only the points stored in $A$ as

$$A_\theta = R_\theta \cdot A$$

- where $R_\theta$ is a 2D rotation matrix.

- **NOTE**: we need to rotate less points!

# ORB Descriptor

- Advantages:
  - Computationally fast.
  - Invariant to illumination changes.
  - Compact!
  - Invariant to rotation.
  - Patent free.
- Disadvantage:
  - Not robust as SIFT.

# SIFT Descriptor

- It is the state-of-the-art descriptor.

- It was introduced in 1999, but it is still the king.

# SIFT Descriptor: Patch Orientation

- The first step is to compute the orientation of $P$.

- We compute the horizontal ($P_x$) and vertical ($P_y$) gradients of the $P$.

- For each pixel at coordinates $(i, j)$ in the patch we compute its orientation and magnitude:

$$m(i,j) = \sqrt{P_x(i,j)^2 + P_y(i,j)^2}$$

$$\theta(i,j) = \mathrm{atan2}\Big( P_y(i,j), P_x(i,j) \Big)$$

# SIFT Descriptor: Patch Orientation

- A histogram, $H$, of directions is created for each orientation taking into account its magnitude.

- We repeat this process for all gradients in the patch!

  - Note that $H$ is initialized as a vector of zeros.

# SIFT Descriptor: Patch Orientation

- Let's say, we have a histogram $H$ with 18 bins ($b = 18$).
  - This means each bin has a size ($k$) in degree of 20° ($k = 360 / b = 360 / 18$).
- Now, we have to insert a gradient ($m = 10$ and $\theta = 45°$) from our patch in $H$ we need to process a gradient in the patch.
  - First, we compute the index of the bin to update:

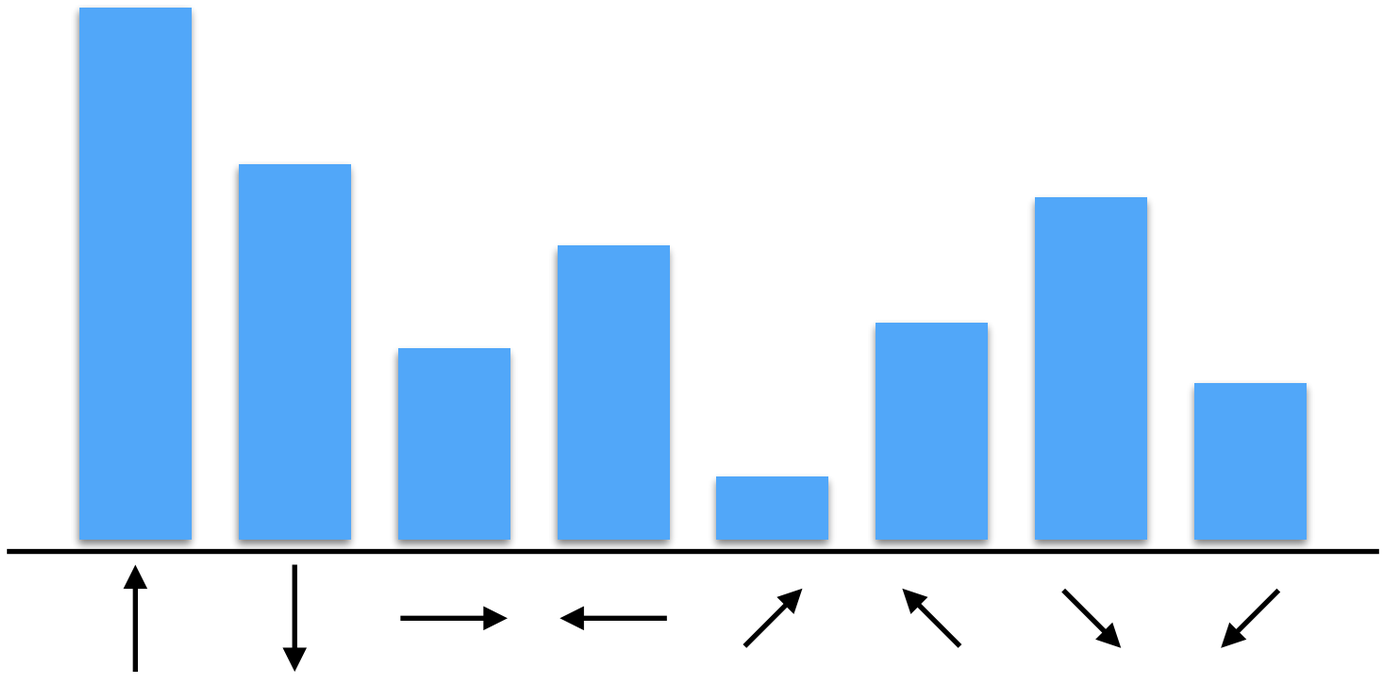
$$i = \left\lfloor \frac{\theta}{k} \right\rfloor = \left\lfloor \frac{45}{20} \right\rfloor$$

  - Then, we update $H$ as

$$H(i) = H(i) + m = H(i) + 10$$

# SIFT Descriptor: Patch Orientation

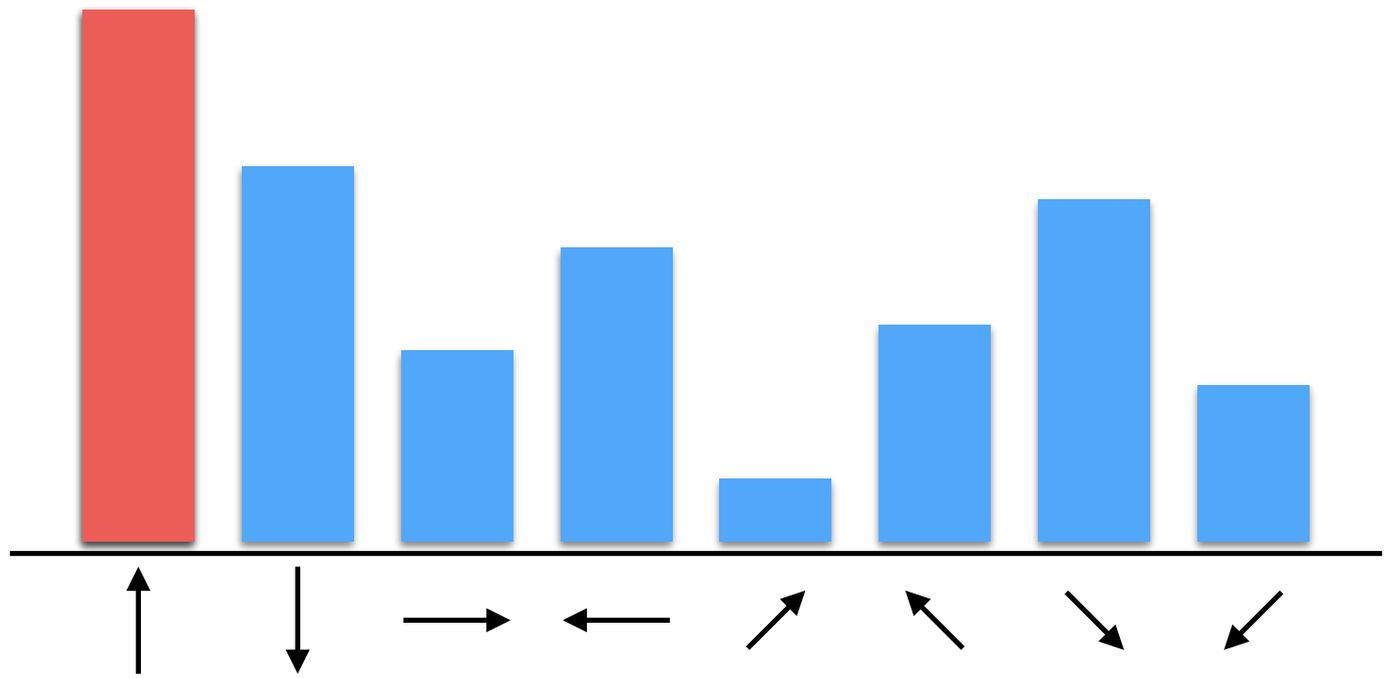
- Finally, we get this (an example with 8 bins; i.e., 8 directions):



- The patch orientation, $\alpha$, is given by the highest peak:
  - If we have two equal peaks, we take the as winner the first one in histogram.

# SIFT Descriptor: Patch Orientation

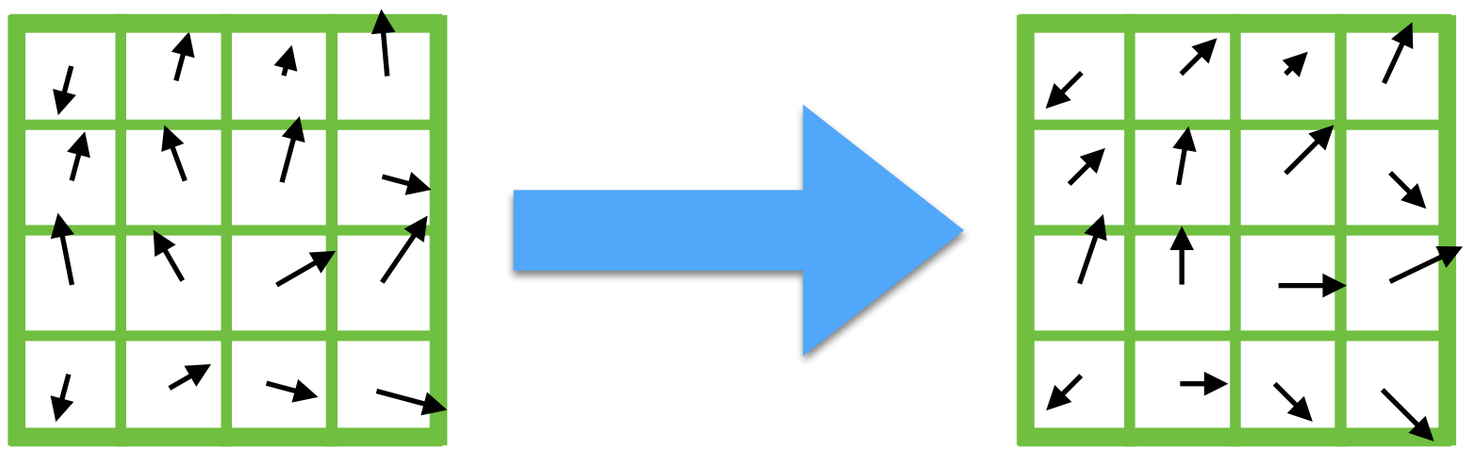
- Finally, we get this (an example with 8 bins; i.e., 8 directions):



- The patch orientation, $\alpha$, is given by the highest peak:
  - If we have two equal peaks, we take the as winner the first one in histogram.

# SIFT Descriptor

- Once we have $\alpha$, we can rotate all gradients in the patch using it.
  - This ensures to be invariant to rotations!



# SIFT Descriptor

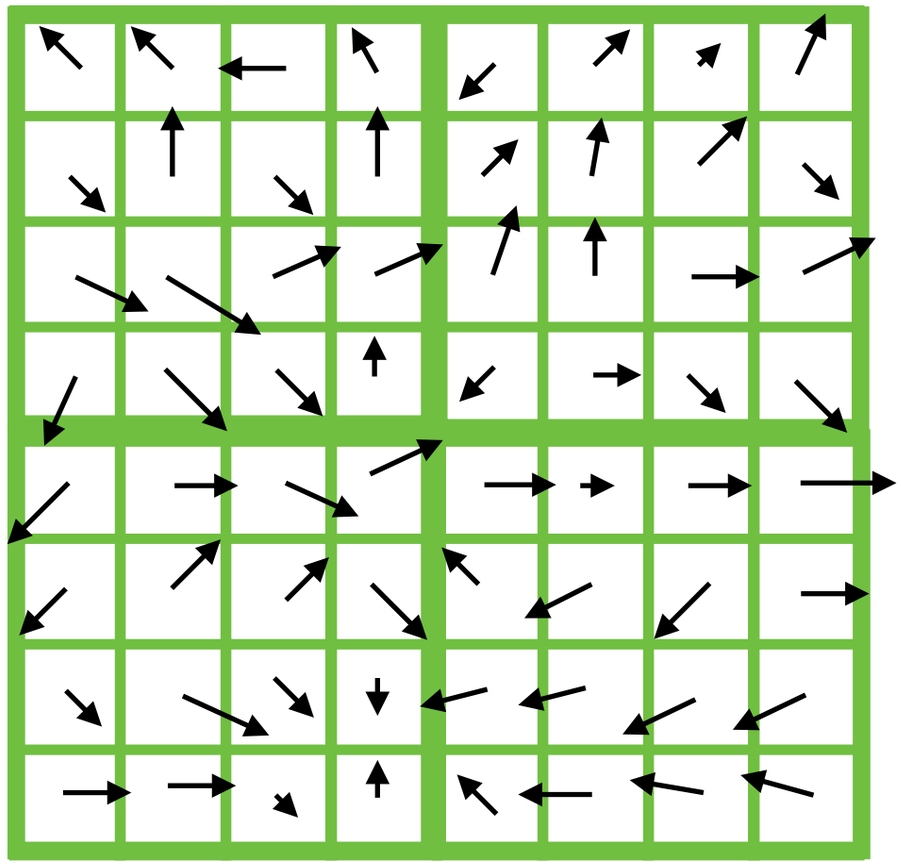
- Why do we rotate the gradients? It is computationally faster:
  - In theory, we should rotate the patch and then recompute the gradients.
  - This is computationally expensive!

# SIFT Descriptor

- At this point, we divide the patch into 4x4 blocks. For each block, we compute a new histogram of directions.

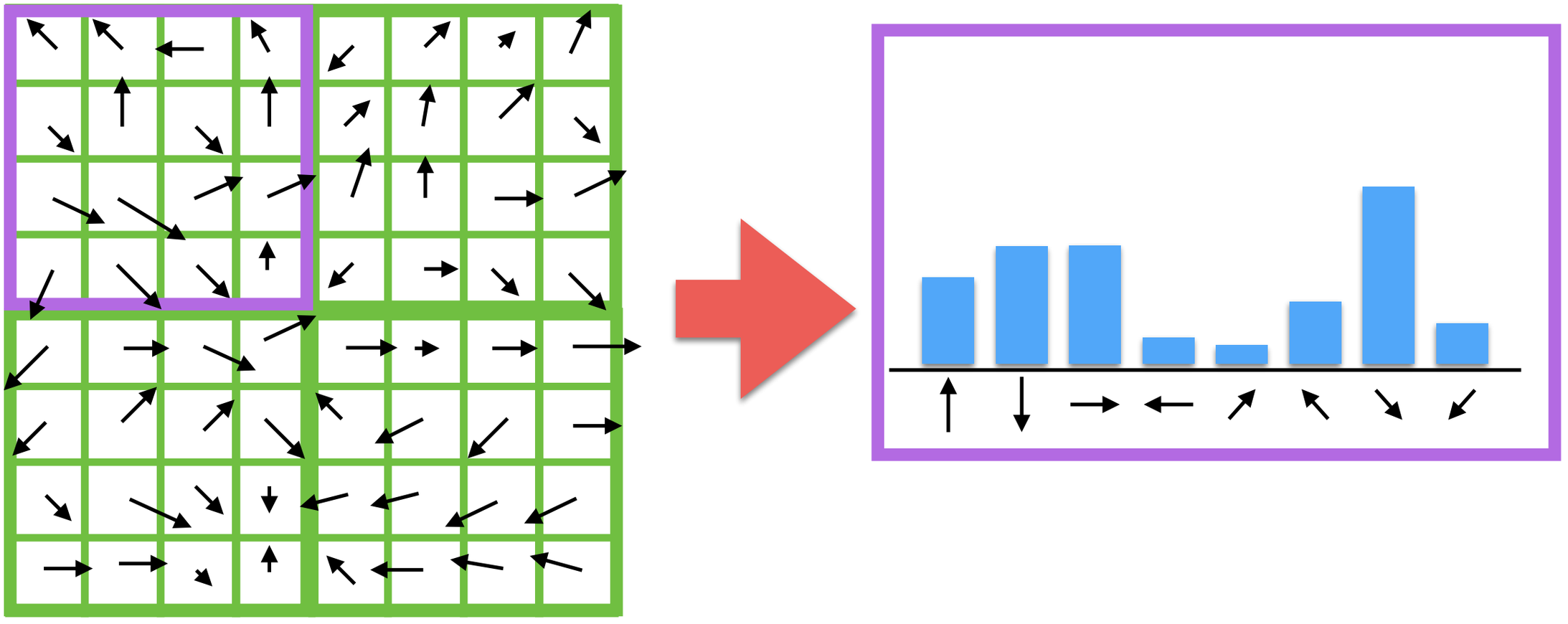
- The final SIFT descriptor is the concatenation (flattening) of all these histograms.

# SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

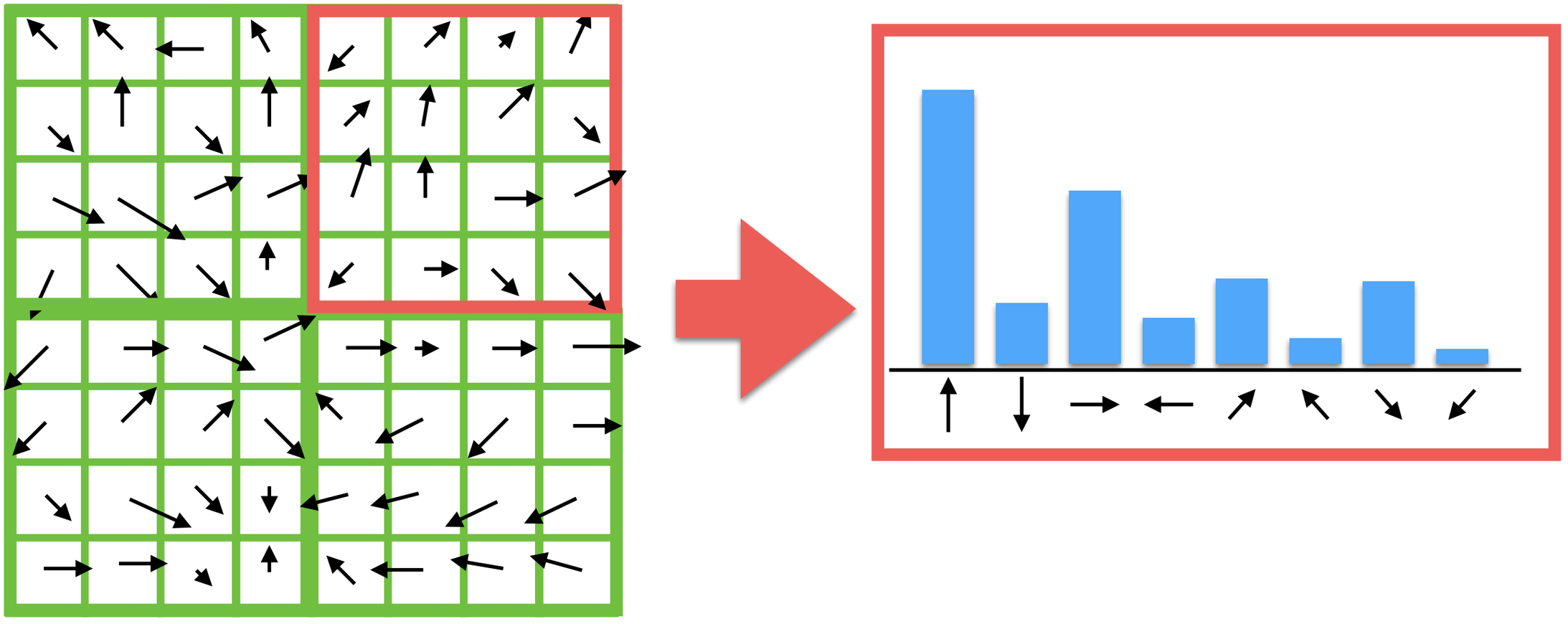
# SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the first block in violet

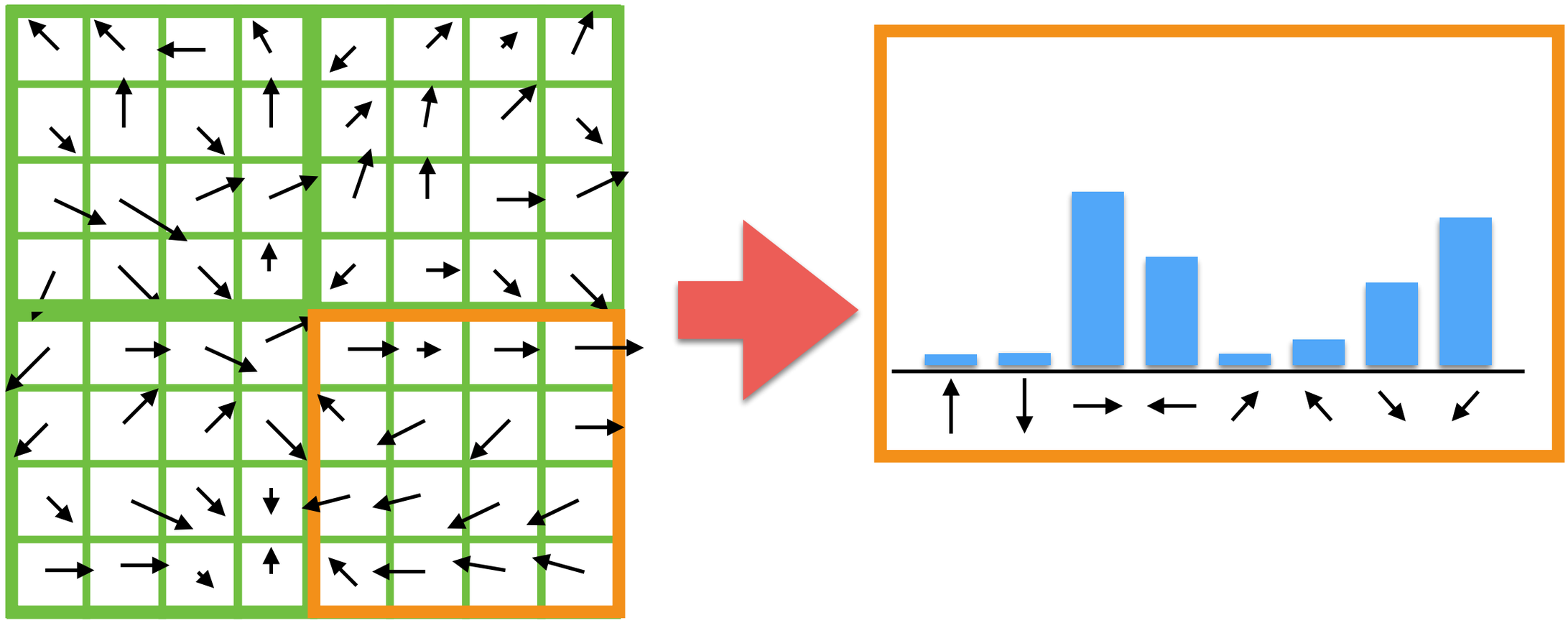
# SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the second block in red

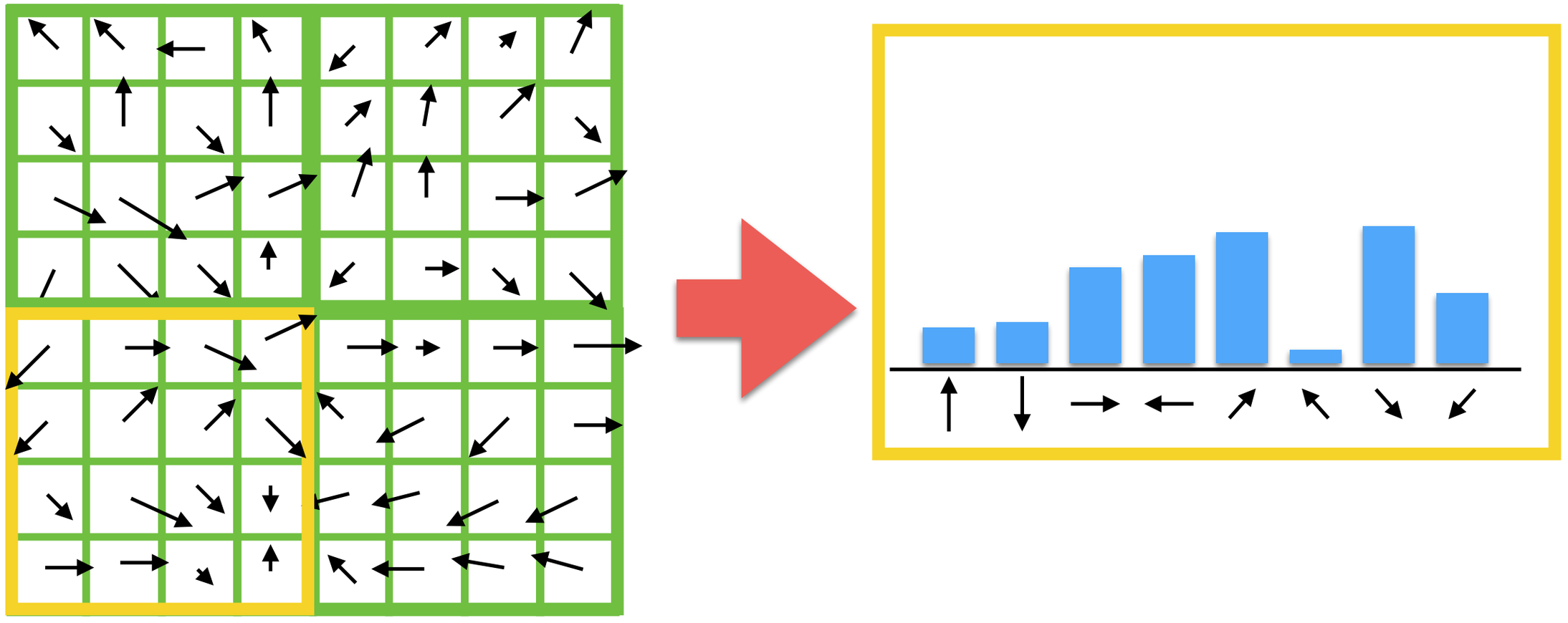
# SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the third block in orange

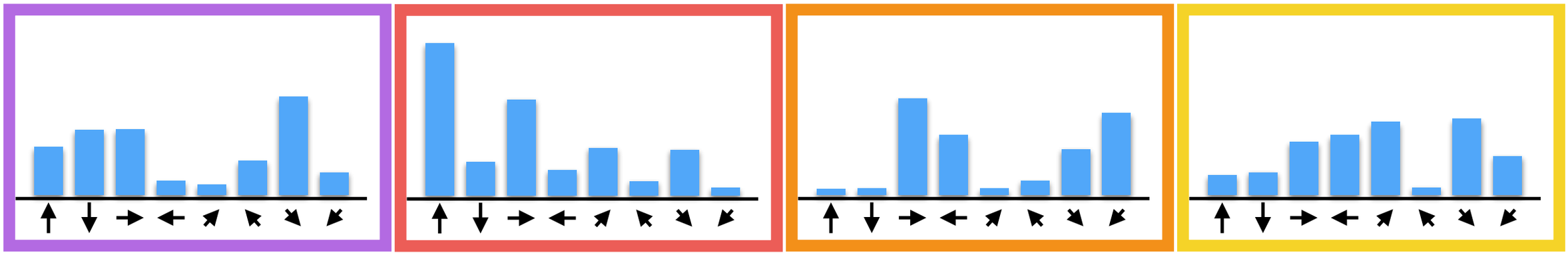
# SIFT Descriptor: Example with 2x2 Blocks



Patch and its gradients

We compute the histogram for the fourth block in yellow

# SIFT Descriptor:
# Example with 2x2 Blocks



The ***final descriptor*** is the concatenation of the histogram of all blocks.
Note that this can be encoded as a vector; in this example the vector has size equal to:

$$4x8 = \mathbf{32}$$

4 because we have 2x2 Blocks
8 because we have 8 direction for each histogram.

# SIFT Descriptor: Test

- We test the differences as distance between histograms:

$$D_2(\mathbf{h}^1, \mathbf{h}^2) = \sqrt{\sum_{i=1}^{n} (h_i^1 - h_i^2)^2}$$

- **The lower the closer**:
  - This is the opposite compared to BRIEF/ORB.

# SIFT Descriptor

- Advantages:
  - Invariant to illumination changes.
  - Invariant to rotation.
- Disadvantages:
  - Slower than BRIEF/ORB.
  - More memory than binary methods.
  - ~~Patented!~~ It is patent-free from 12th of Aprile 2020!

# Matching Images

# Matching:
# An Image Against Another One

- **Input**: two descriptor lists (*they can be of equal or different size*), $\mathbf{desc}_1$ and $\mathbf{desc}_2$, respectively of image $I_1$ and $I_2$.

- **Output**: a vector with indices of matches for each list:

  - The output is called $\mathbf{M}_{12}$ if we match $I_1$ against $I_2$

  - The output is called $\mathbf{M}_{21}$ if we match $I_2$ against $I_1$

# Matching:
# How the Output is Encoded Example 1

- Let's say we have 4 descriptors in $\mathbf{desc}_1$

- Let's say we have 3 descriptors in $\mathbf{desc}_2$

- Let's say that we want to match $I_1$ against $I_2$, this means that we want to compute $\mathbf{M}_{12}$.

# Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ \\ d_2^1 \\ \\ d_3^1 \\ \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ \\ d_2^2 \\ \\ d_3^2 \end{bmatrix}$$

$$\mathbf{M}_{12} = [\ ]$$

# Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the first descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [\ ]$$

# Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the first descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2,]$$

# Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the second descriptor of $\mathbf{desc}_1$ matches with the third descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2,]$$

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We find out that the third descriptor of $\mathbf{desc}_1$ matches with the first descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3, 1,]$$

# Matching: Example 1

$$\mathbf{desc}_1 = \begin{bmatrix} d_1^1 \\ d_2^1 \\ d_3^1 \\ d_4^1 \end{bmatrix} \qquad \mathbf{desc}_2 = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

We find out that the fourth descriptor of $\mathbf{desc}_1$ matches with the second descriptor of $\mathbf{desc}_2$.

$$\mathbf{M}_{12} = [2, 3, 1, 2]$$

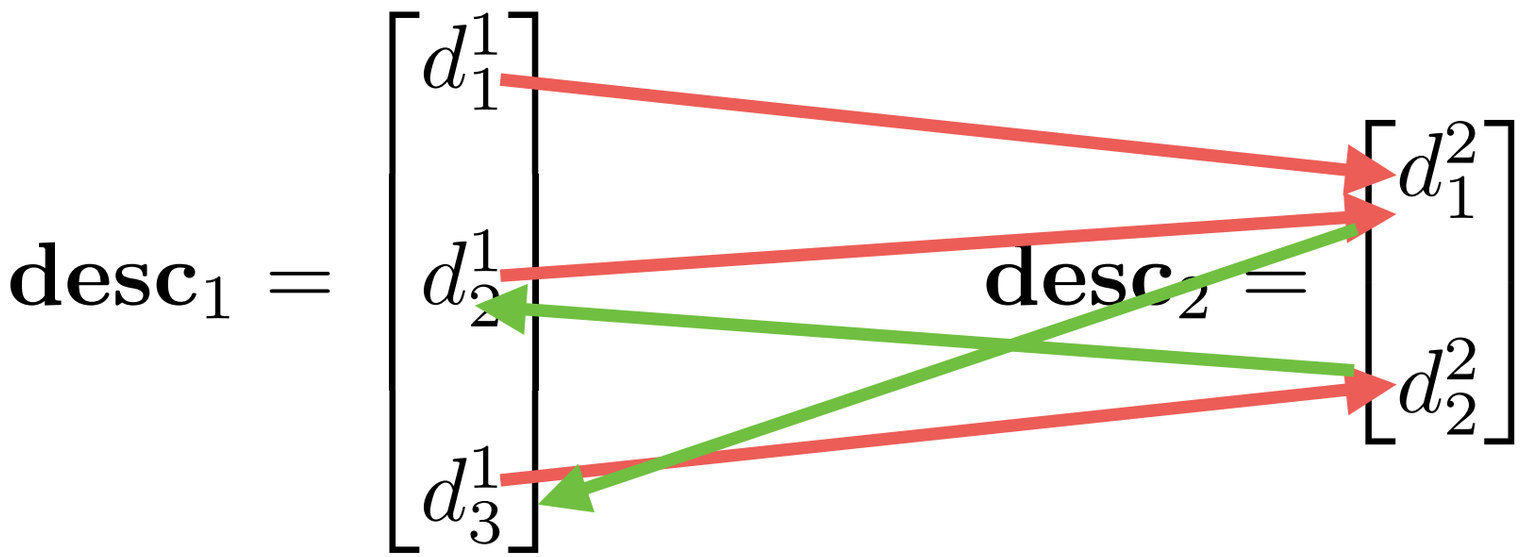
# Matching:
# How the Output is Encoded Example 2

- Let's say we have 3 descriptors in $\mathbf{desc}_1$

- Let's say we have 2 descriptors in $\mathbf{desc}_2$

- Let's say that we match $I_1$ against $I_2$, obtaining $\mathbf{M}_{12}$. Then, we match $I_2$ against $I_1$ obtaining $\mathbf{M}_{21}$.

# Matching: Example 2




$\mathbf{M}_{12} = [1,1,2]$ $\mathbf{M}_{21} = [3,2]$

# Matching: Example 2

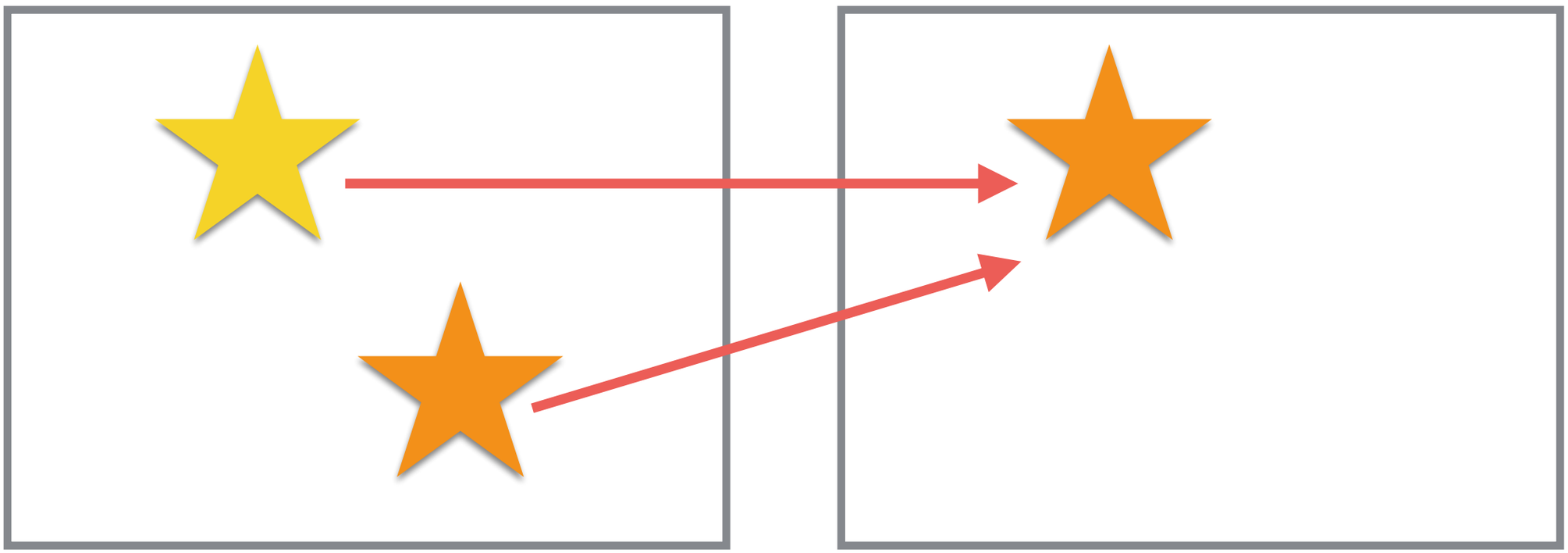
- From this example, we can notice that:
  - The matching operator is ***NOT*** an invertible function:
    - Therefore, $\mathbf{M}_{12}$ and $\mathbf{M}_{21}$ can be very different!
- Why? Let's see it!

# Matching: Example 2

$I_1$ $I_2$

Let’s say that stars in this example are feature-point.

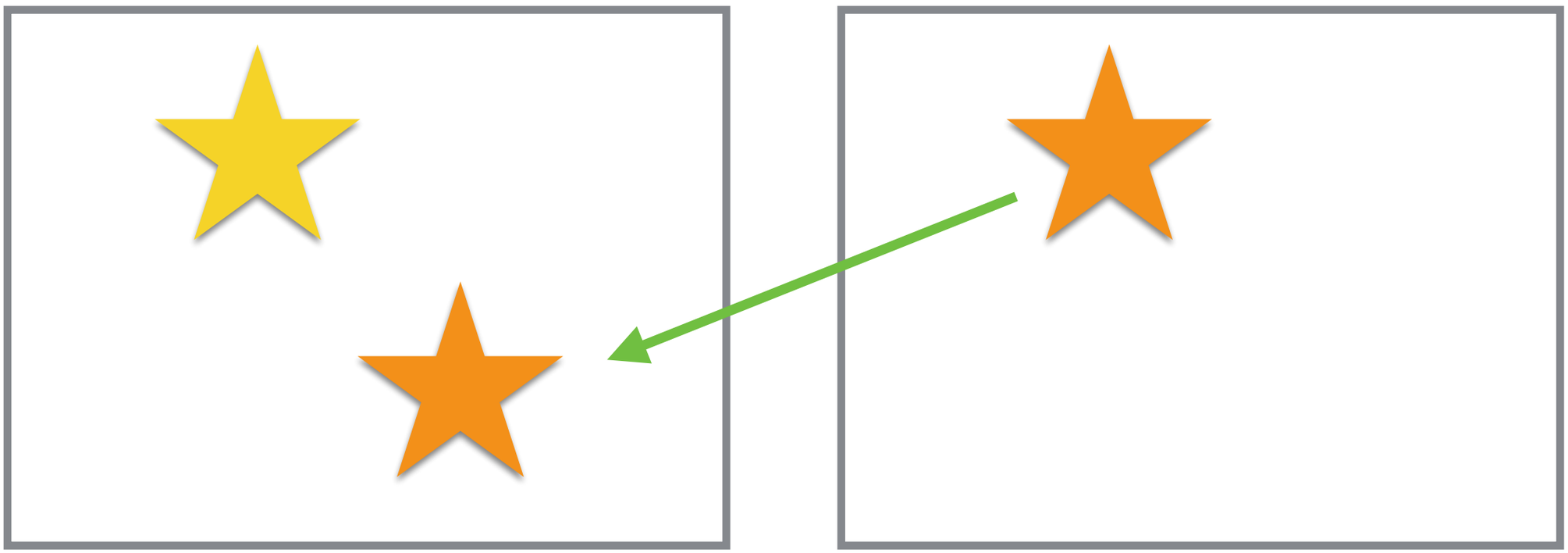
# Matching: Example 2



$I_1$ $I_2$

When we match $I_1$ against $I_2$, we have two matches; between the two stars in $I_1$ and the one in $I_2$. This is because we need to match the **yellow** star with something in the other image no matter what.

# Matching: Example 2



$I_1$ $I_2$

When we match $I_2$ against $I_1$, we do not have a match between the **yellow** star in $I_1$ and the **orange** one in $I_2$ because the other **orange** star in $I_1$ is closer than the **yellow** star!

# Matching:
# Brute Force Algorithm

- A simple method to find a ***matched descriptor*** in $\mathbf{desc}_2$ for each descriptor in $\mathbf{desc}_1$:

- For each descriptor in $\mathbf{desc}_1$, we test it against all descriptors in $\mathbf{desc}_2$, and we keep as matched one the ***closest*** (in terms of distance; either Hamming or Euclidean).

# Matching:
# Brute Force Algorithm

For each descriptor $\mathbf{d}^1_i$ in $\mathbf{desc}_1$:

matched($i$) = -1;

matched_dist = **BOTTOM**;

For each descriptor $\mathbf{d}^2_j$ in $\mathbf{desc}_2$:

if Closer( $D(\mathbf{d}^1_i, \mathbf{d}^2_j)$, matched_dist)

matched($i$) = $j$;

matched_dist = $D(\mathbf{d}^1_i, \mathbf{d}^2_j)$;

endif

$D(,)$ is a distance function; it can be Hamming, Euclidean, etc.

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matched_dist = $D(\mathbf{d}^1_i, \mathbf{d}^2_j)$;

endif

**BOTTOM** = +Inf for SIFT
**BOTTOM** = 0 for BRIEF/ORB

$D(,)$ is a distance function; it can be Hamming, Euclidean, etc.

# Matching:
# Brute Force Algorithm

- Advantage:
  - It is exhaustive and finds the ***best solution***!
- Disadvantage:
  - This method is very slow:
    - Let's say we have $n$ descriptors in $\mathbf{desc}_2$ and $n$ in $\mathbf{desc}_2$. In the worst case, we need to compare roughly $n^2$ descriptors. This becomes an issue when we have more than 100 descriptors per image!

# Matching: Improving Efficiency

- How can we improve (approximating results)?

- **Hashing**: the idea is to group similar descriptors in $k$ groups or buckets that have a constant size.

# Matching: Improving Efficiency

- We create $k$ bucket.

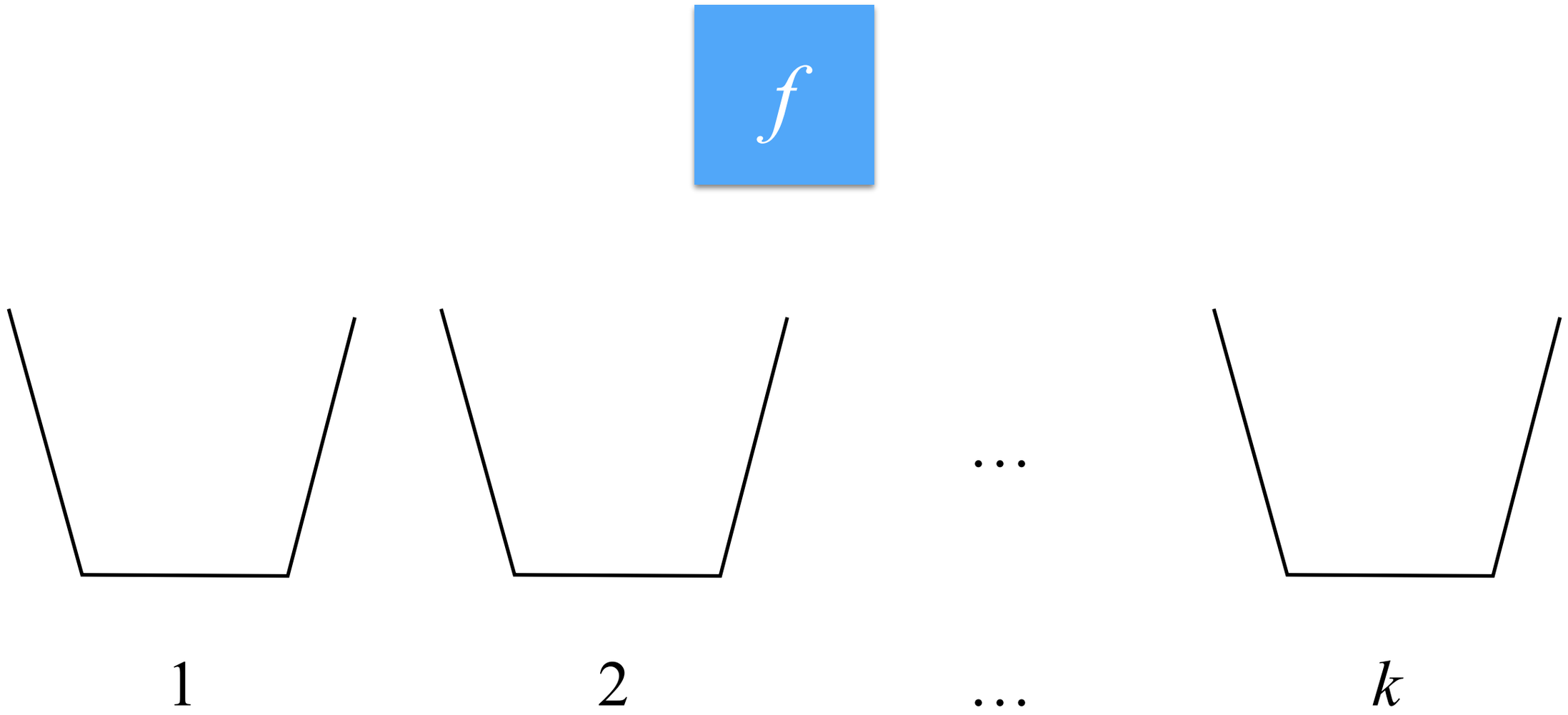
  - Each ***descriptor*** in $\mathbf{desc}_2$ of $I_2$ is assigned to a bucket using a function $f$, called hash function. This is defined as:

$$f\text{: } \textbf{\textit{descriptor}} \text{ —> } [1, k] \text{ (positive integer numbers!)}$$

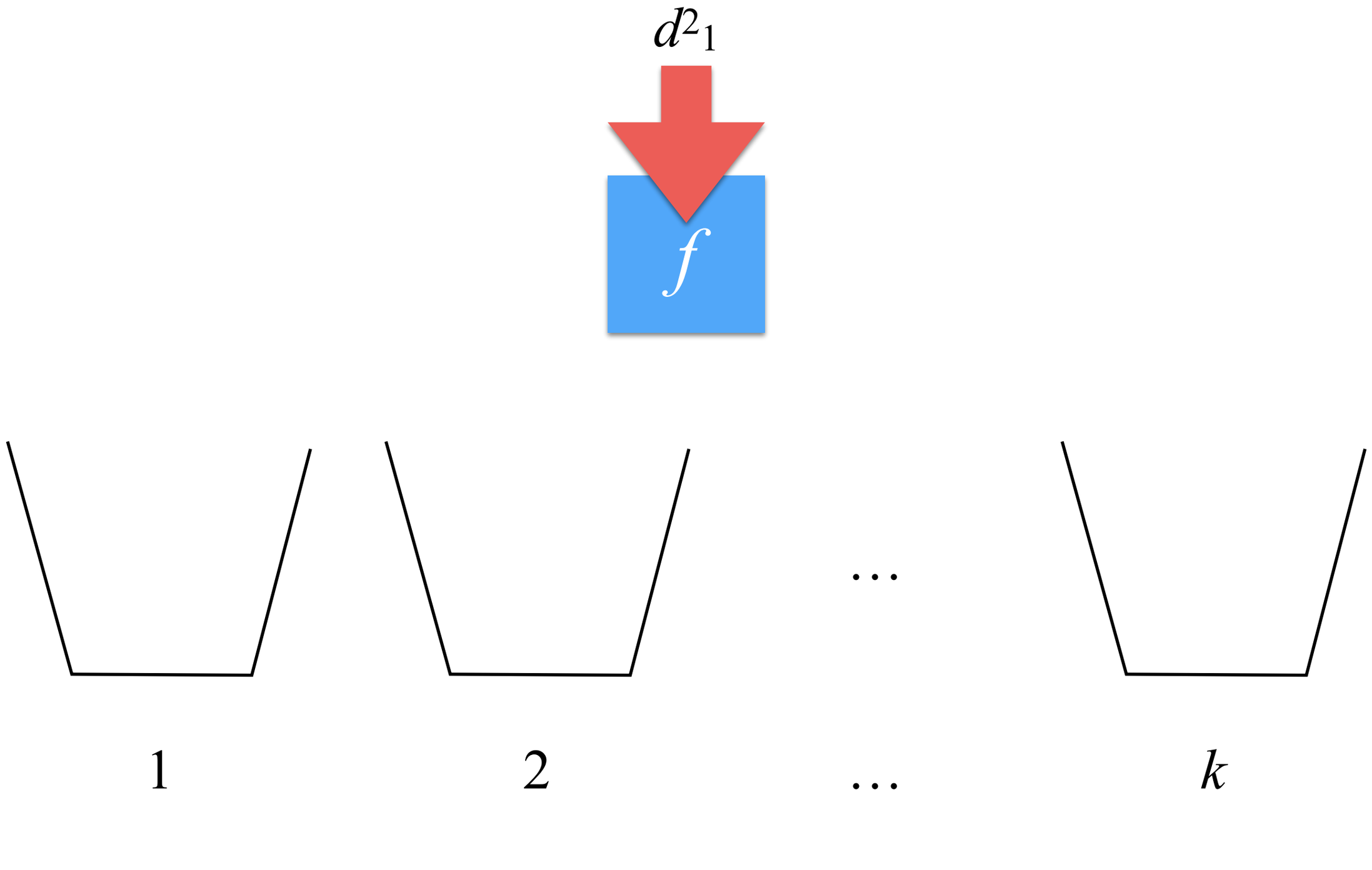
- This means that f cover generates a number in $[1, k]$ given a descriptor.

  - For example, an $f$ for BRIEF/ORB, where the descriptor is a 256-bit number, is the modulo operation.

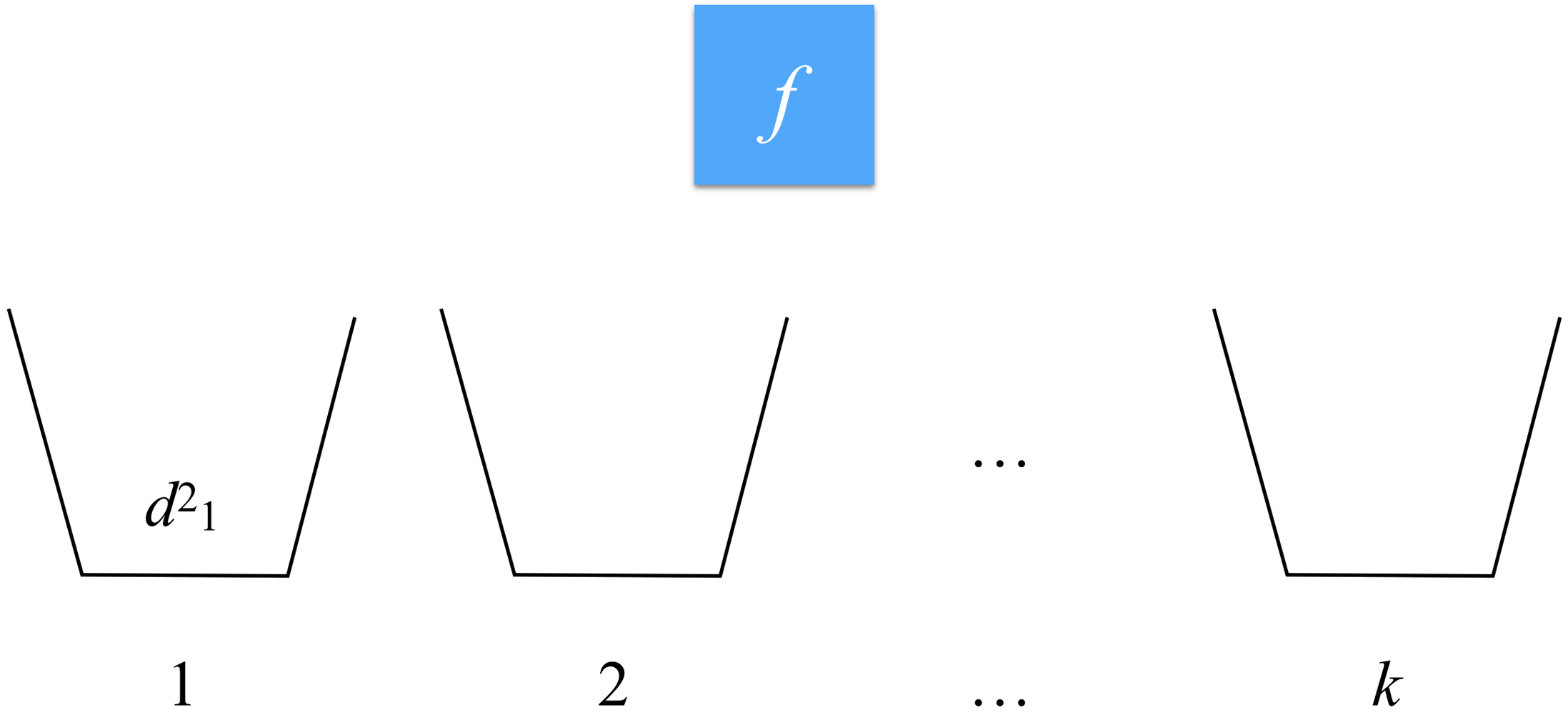
# Matching:
# Improving Efficiency



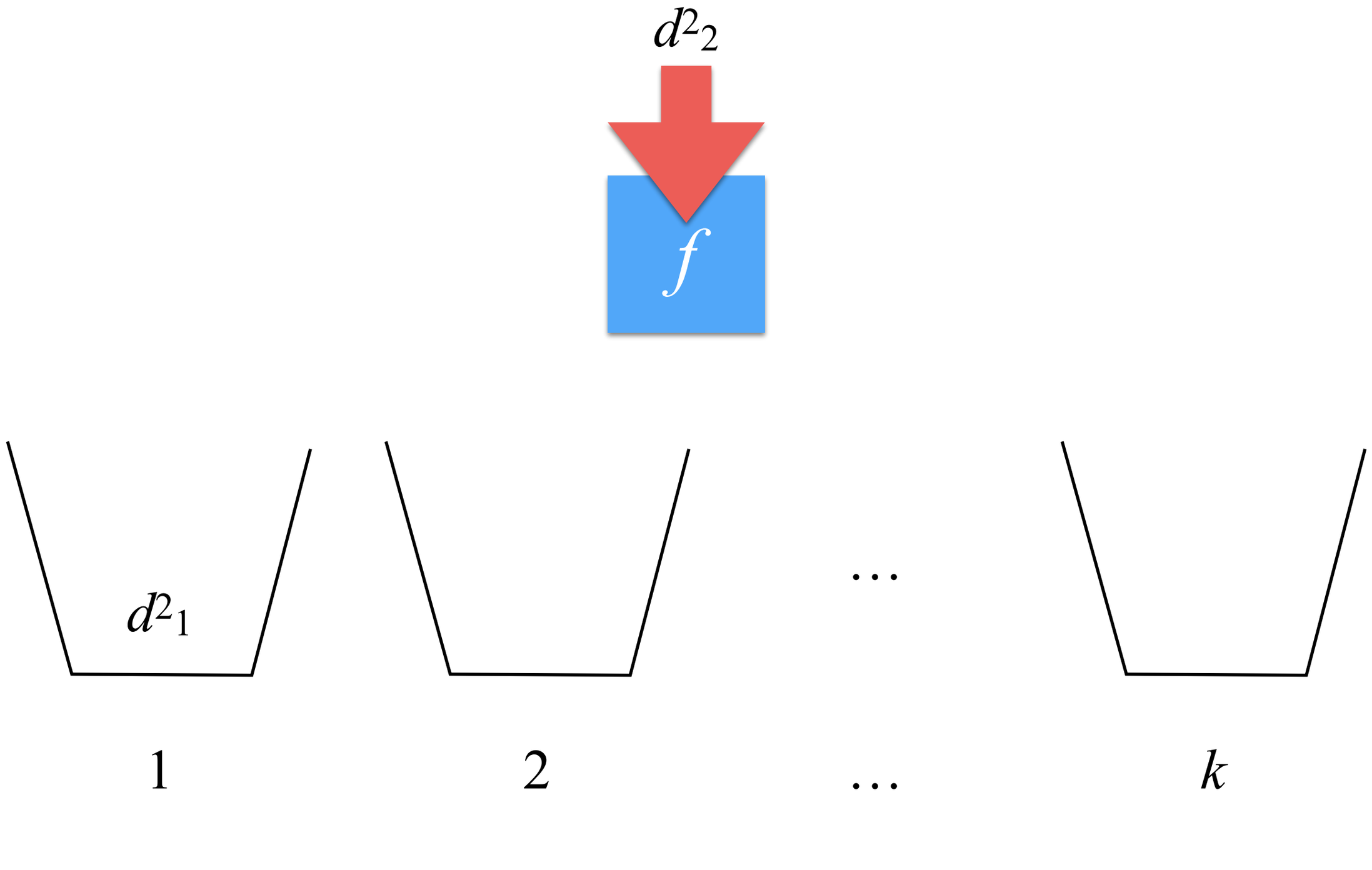
# Matching: Improving Efficiency



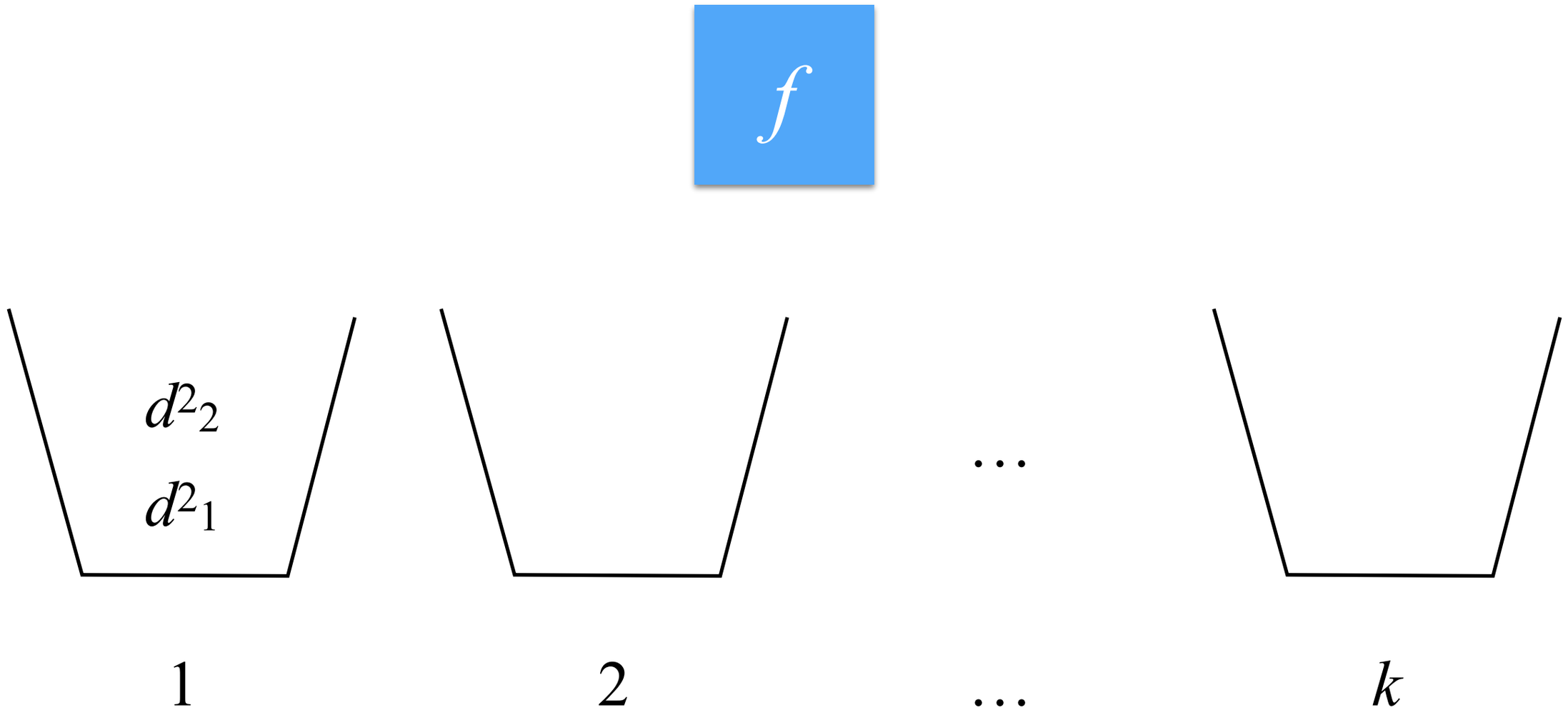
# Matching:
# Improving Efficiency



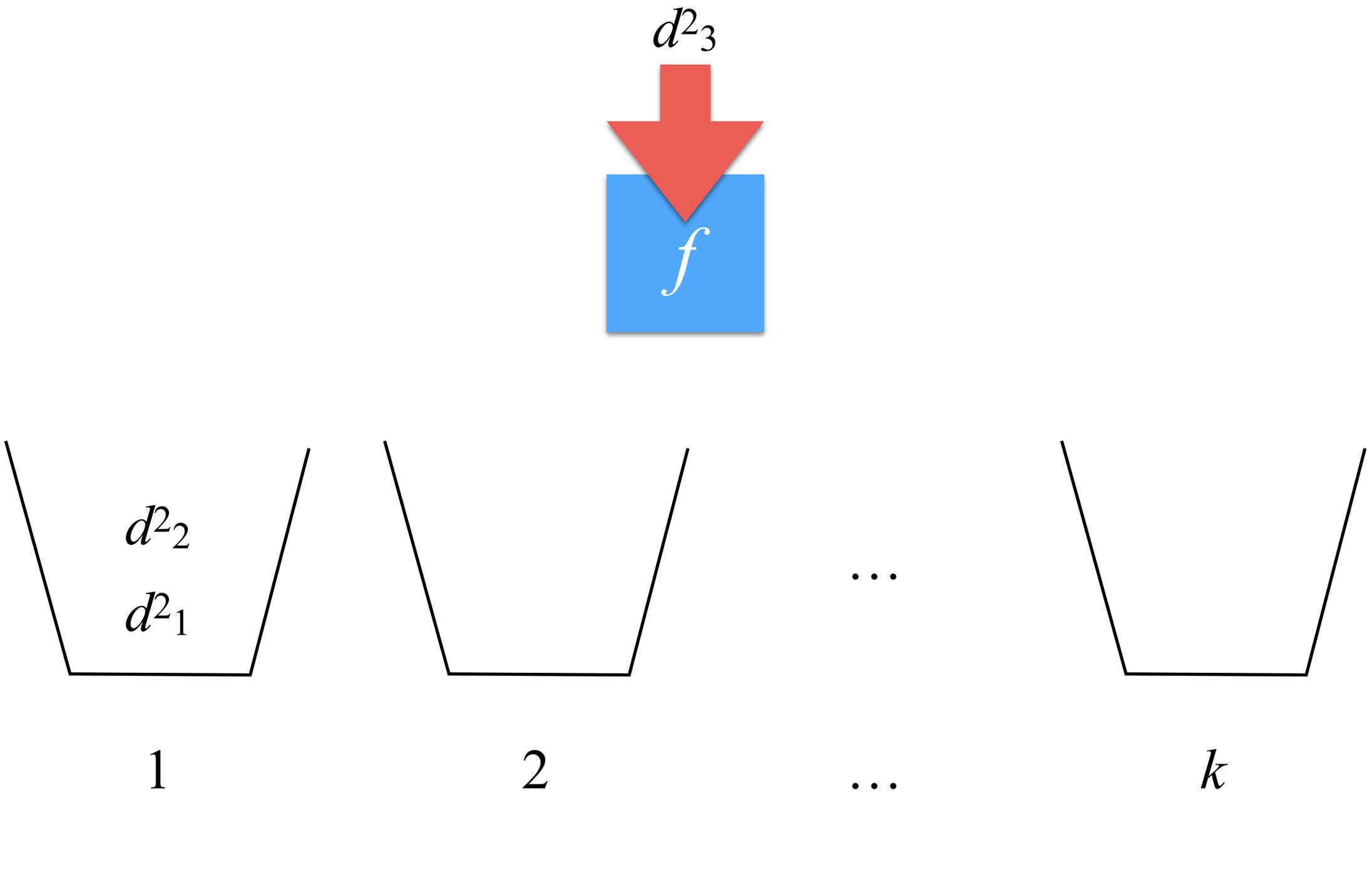
# Matching: Improving Efficiency



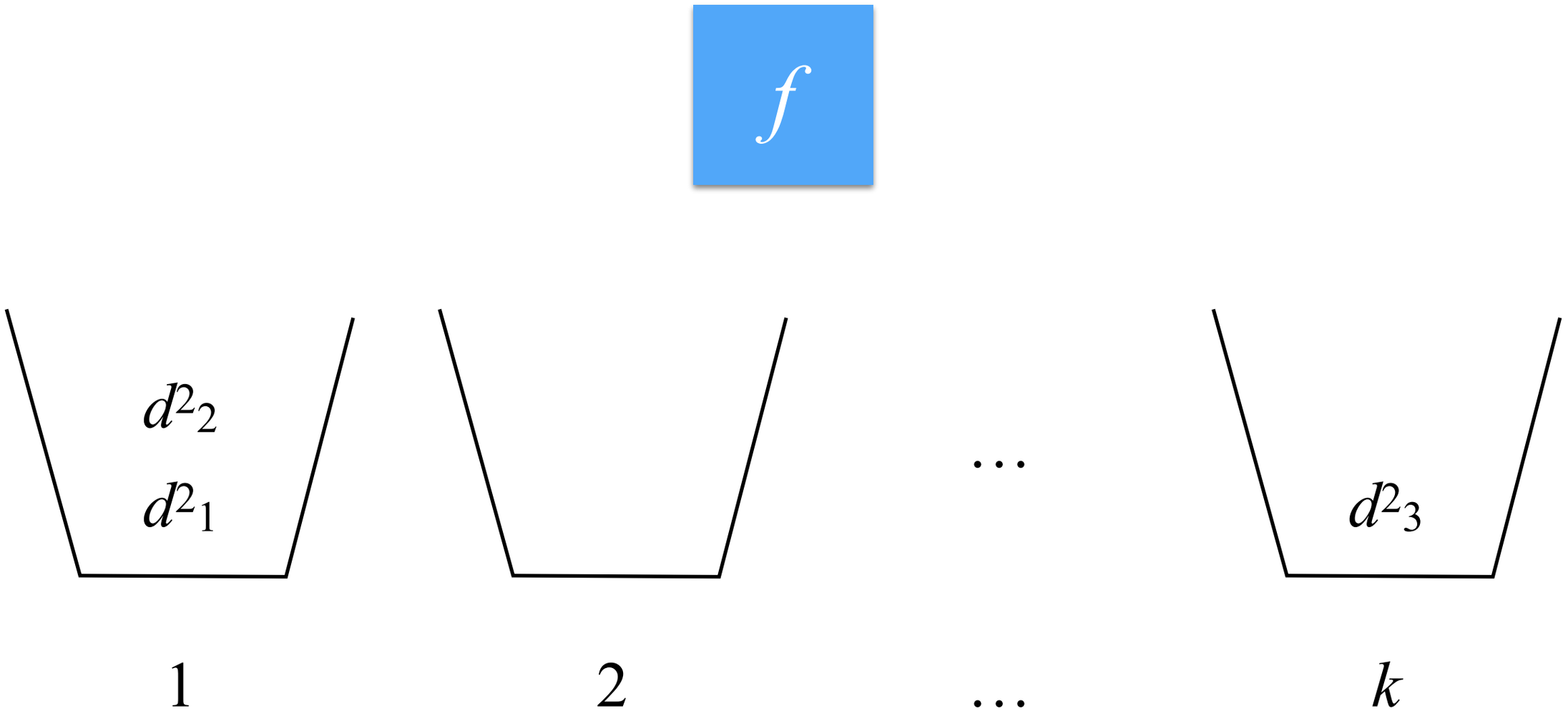
# Matching:
# Improving Efficiency



# Matching: Improving Efficiency



# Matching:
# Improving Efficiency



etc.

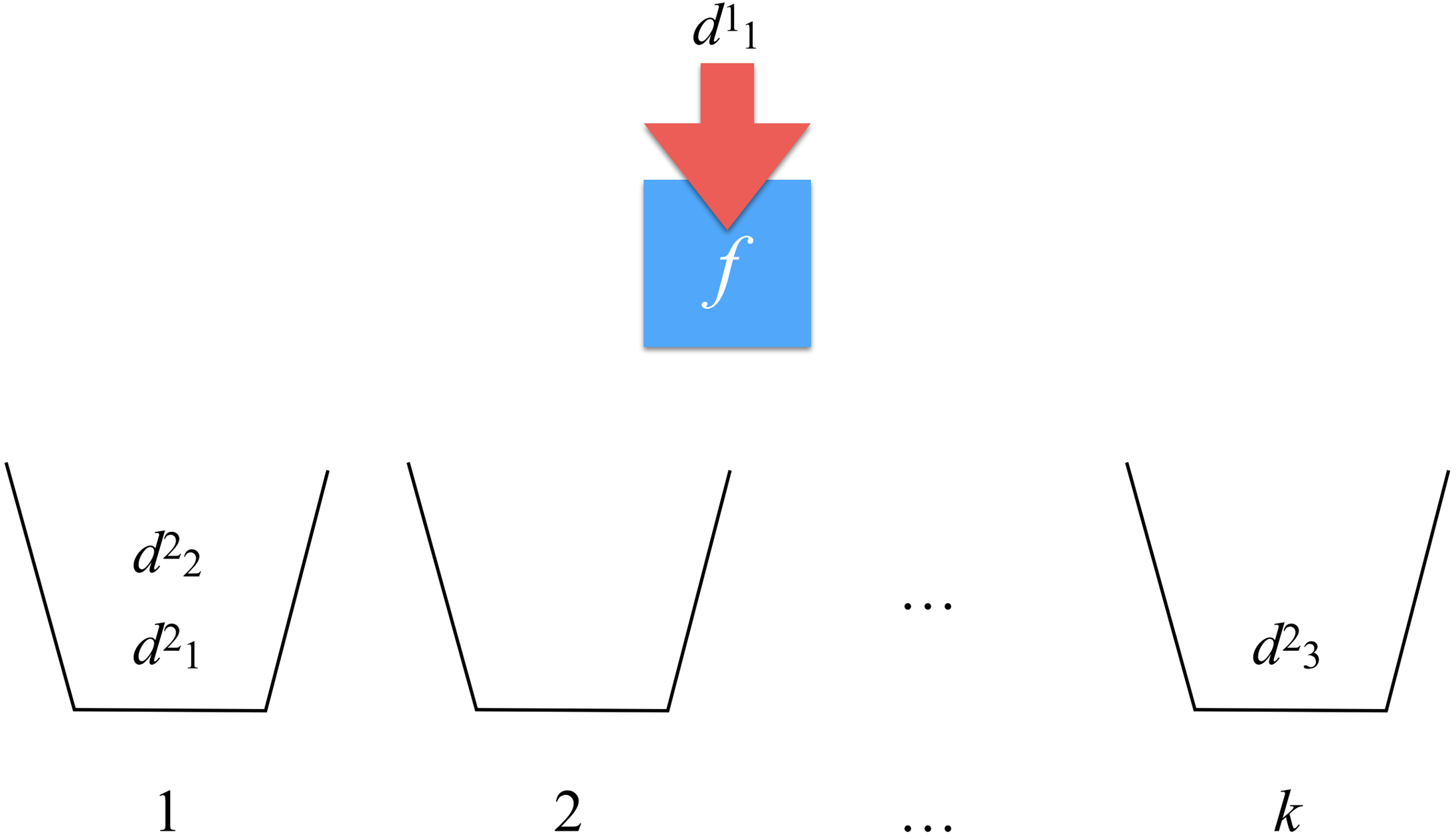
# Matching: Improving Efficiency

- Now, we have all descriptors of $I_2$ into buckets.

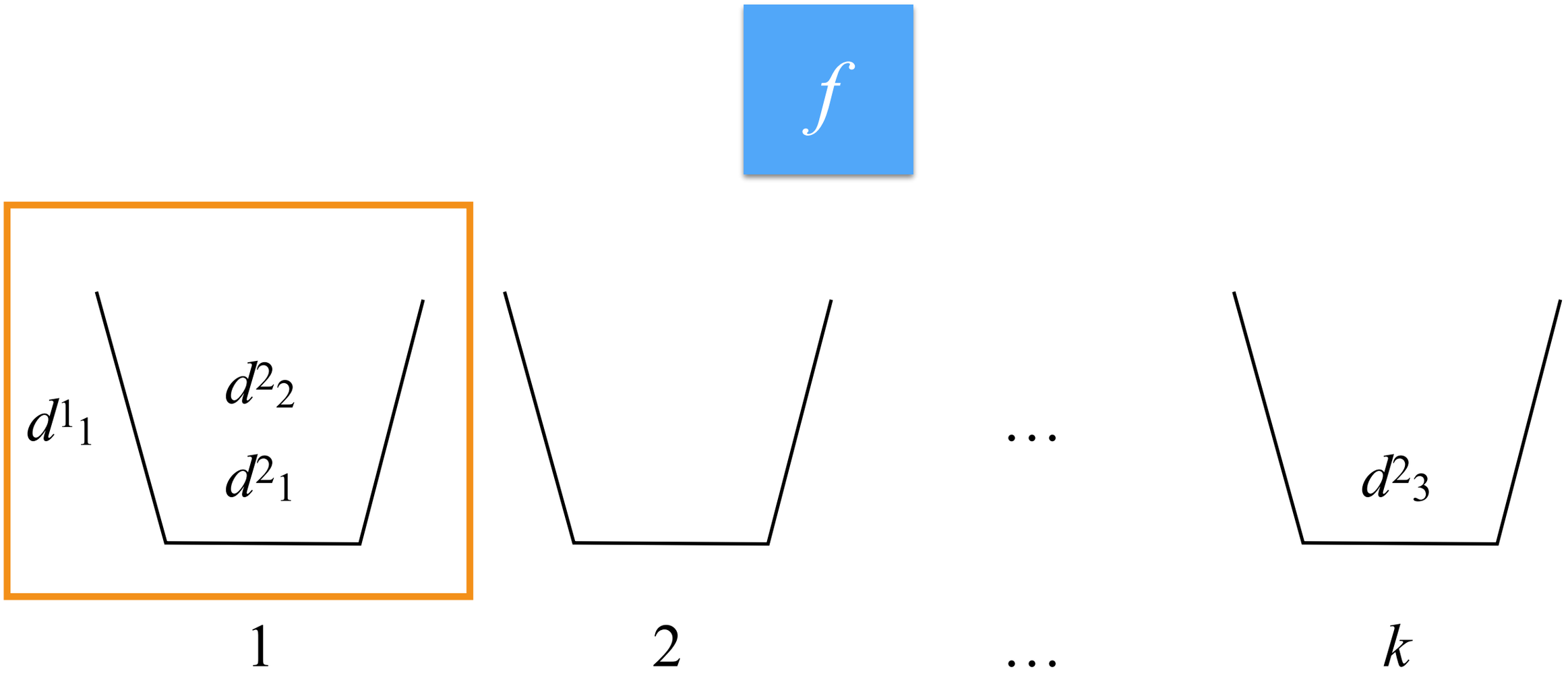
- To find a match for a descriptor $d^1{}_i$ of $I_1$, we apply $f$ to $d^1{}_i$. In this way, we obtain a bucket number, let's call it $r$.

- We run the brute force method between $d^1{}_i$ and all the descriptors that are in $r$.

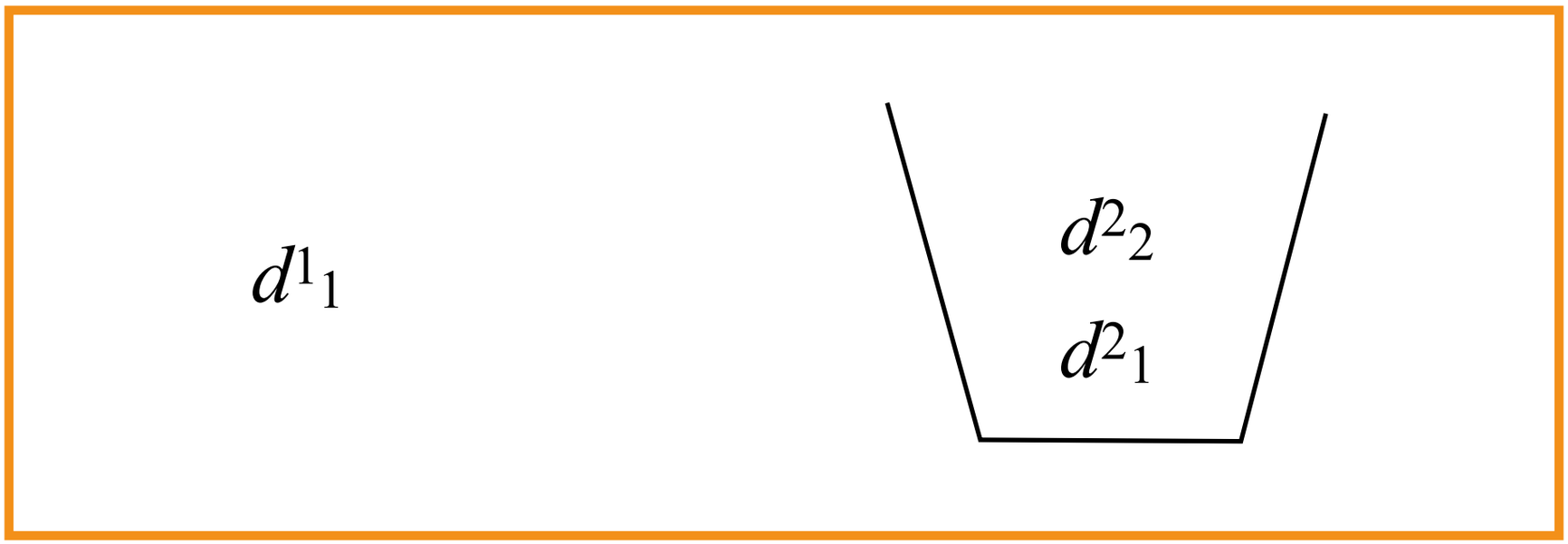
# Matching: Improving Efficiency



# Matching:
# Improving Efficiency



# Matching:
# Improving Efficiency

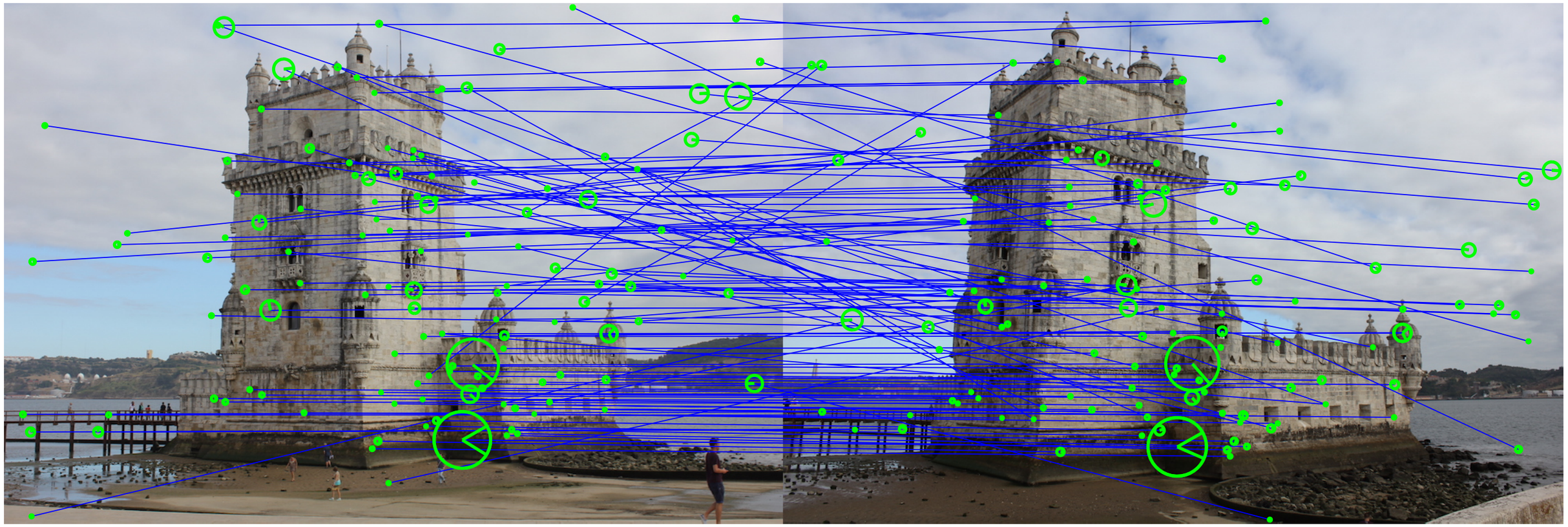



We run brute-force: we compare $d^1_1$ with descriptor in the bucket.

# Matching: Improving Efficiency

- Advantages:
  - It is faster, we run the brute force method for a subset of descriptors.
- Disadvantages:
  - It is not exact, it is ***approximate***; i.e., we test only a sub-set of descriptors.
  - If $f$ is not well crafted, we may have distant descriptors in the same bucket.

# Matching: Example



# Matching

- Once we have know matches between images, we can understand which images are near each others!

- This is important for stable algorithms for triangulation of points and determining cameras' poses!

that’s all folks!