Surface Registration

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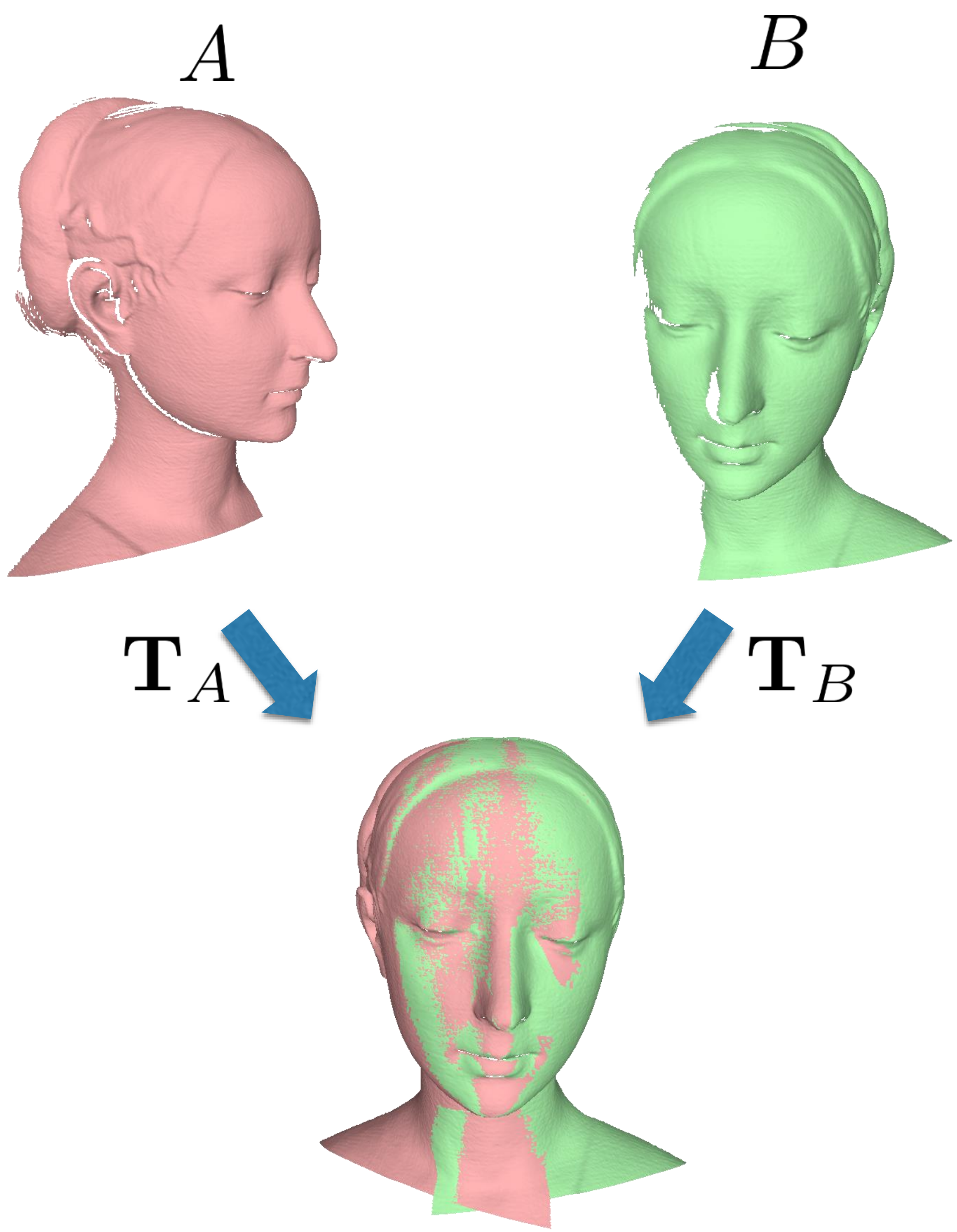
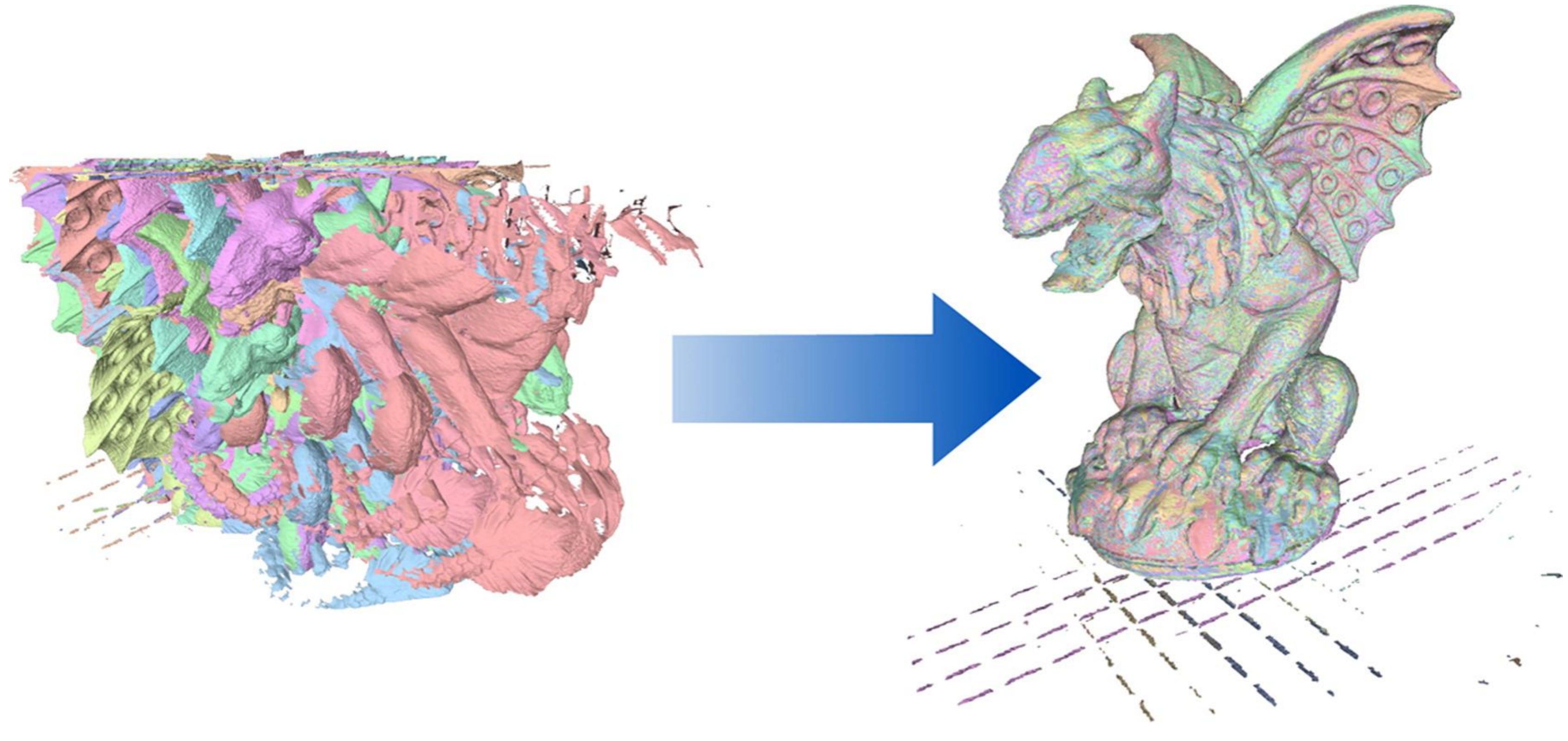
The problem

- 3D scanning generates multiple range images
- Each contain 3D points for different parts of the model in the local coordinates of the scanner
- Find a rigid transformation (rotation + translation) for each scan to align them in the same reference system



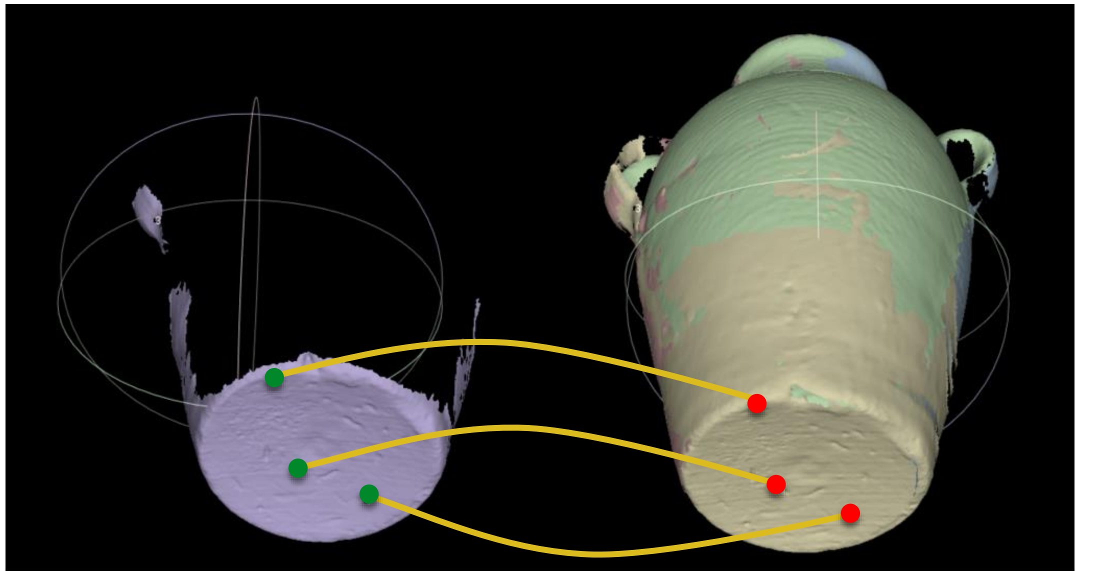
Registration

1. Rough alignment (manual or automatic)
2. Pair-wise refinement by ICP (Iterative Closest Point)
3. Global registration



Rough Alignment

- Different solutions to find corresponding points among the range scans
  - Manually by point picking (MeshLab)
  - Some scanner automatically during the acquisition using markers
  - Automatically after the acquisition
- Compute the best align matrix between the correspondence point



Best-fitting Rigid Transformation

- Given the point set p and q, to compute the rotation matrix and the translation vector that minimize the point-to-point error function E

$$\mathbf{p} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\} \qquad \mathbf{q} = \{\mathbf{q}_1, \ldots, \mathbf{q}_n\}$$

$$E = \sum_{i=1}^{n} \left\| \mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i \right\|^2$$

$$\min_{\mathbf{R},\mathbf{t}} E \quad \text{with} \quad \mathbf{R} \in \mathbb{R}^{3\times 3},\ \mathbf{t} \in \mathbb{R}^3$$

Best-fitting Rigid Transformation

1. Compute centroid

$$\bar{\mathbf{p}} = \frac{1}{N}\sum_{i=1}^{n}\mathbf{p}_i \qquad \bar{\mathbf{q}} = \frac{1}{N}\sum_{i=1}^{n}\mathbf{q}_i$$

2. Compute bary-centered point set

$$\hat{\mathbf{p}}_i = \mathbf{p}_i - \bar{\mathbf{p}} \qquad \hat{\mathbf{q}}_i = \mathbf{q}_i - \bar{\mathbf{q}}$$

3. Compute covariance matrix

$$\mathbf{S} = \mathbf{P}\mathbf{Q}^T$$

$$\mathbf{P} = (\hat{p}_0 \ldots \hat{p}_n) \in \mathbb{R}^{n\times 3}, \mathbf{Q} = (\hat{q}_0 \ldots \hat{q}_n) \in \mathbb{R}^{n\times 3}$$

Best-fitting Rigid Transformation

4. Compute Singular Value Decomposition of covariance matrix

$$\mathbf{S} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

5. Compute Rotation

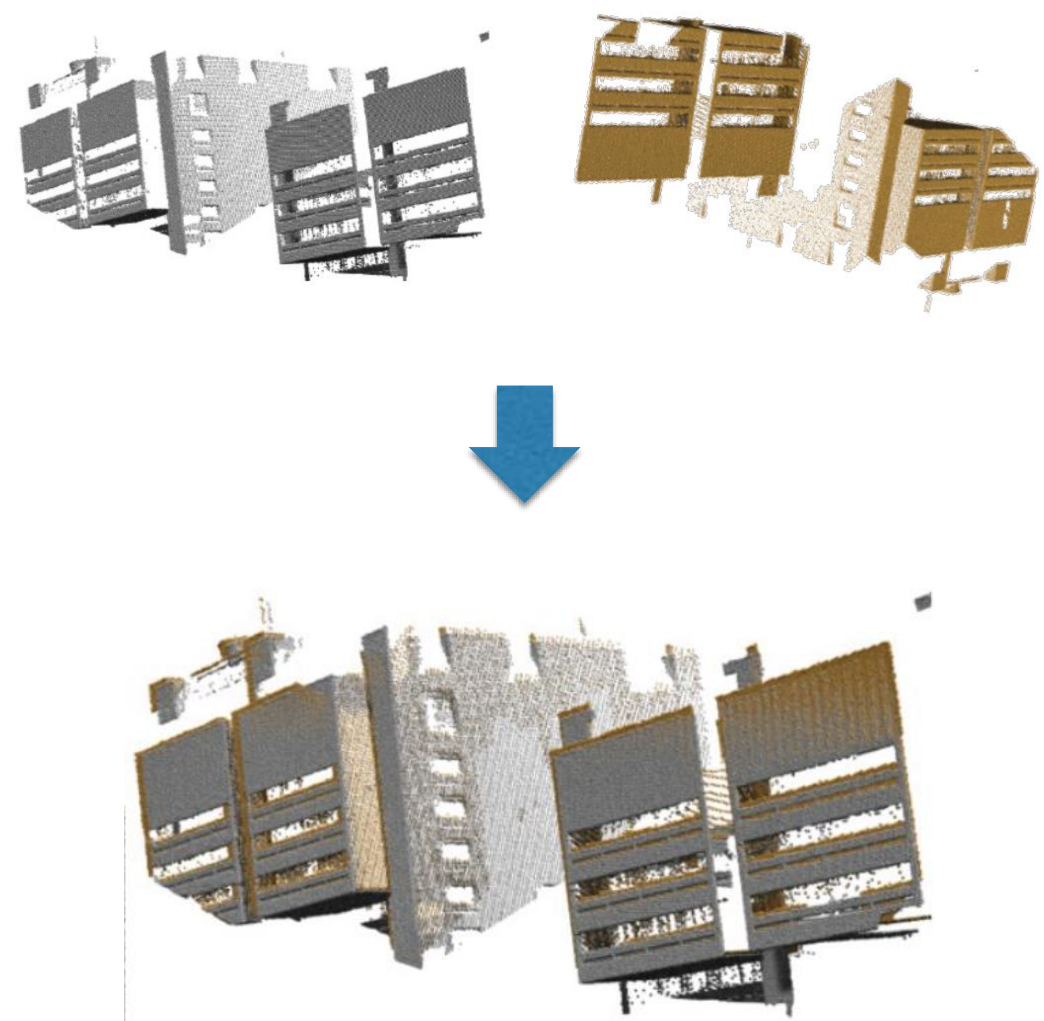
$$\mathbf{R} = \mathbf{U}\mathbf{V}^T$$

6. Compute Translation

$$\mathbf{t} = \bar{p} - \mathbf{R}\bar{q}$$

4 Point Congruent Sets

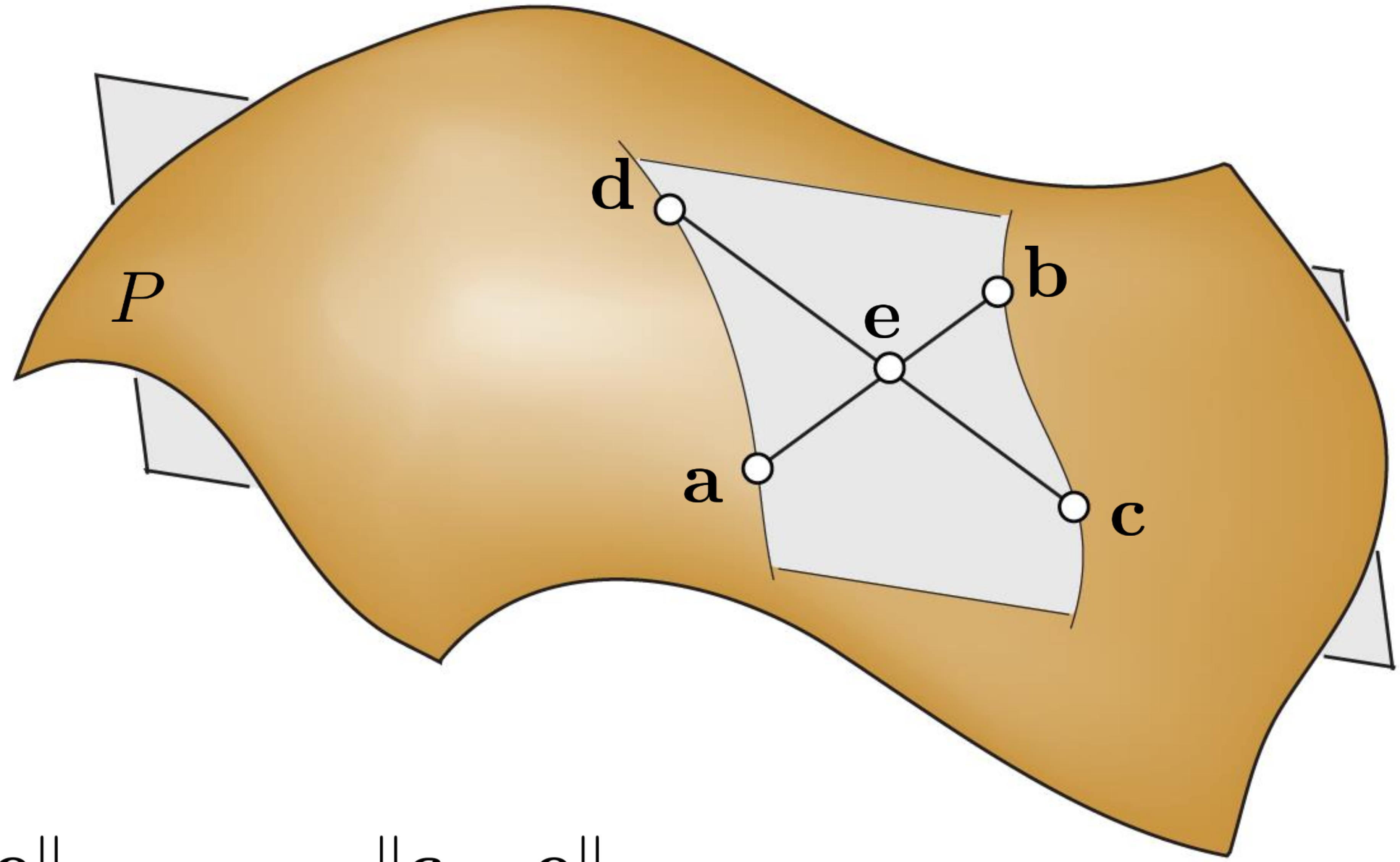
- Input: two point clouds P and Q in arbitrary initial poses, with unknown percentage of overlap regions. The point clouds can present significant amount of noise and outliers
- Output: a transformation T aligning P to Q with a probabilistic approach
- The algorithm is completely automatic



4 Point Congruent Sets

- Observations
  - A pair of triplet (from P and Q) is enough to uniquely define a rigid transformation
  - A special set of 4 points, congruent sets, makes the problem simpler
  - Affine transformations preserve collinearity and ratios of distances

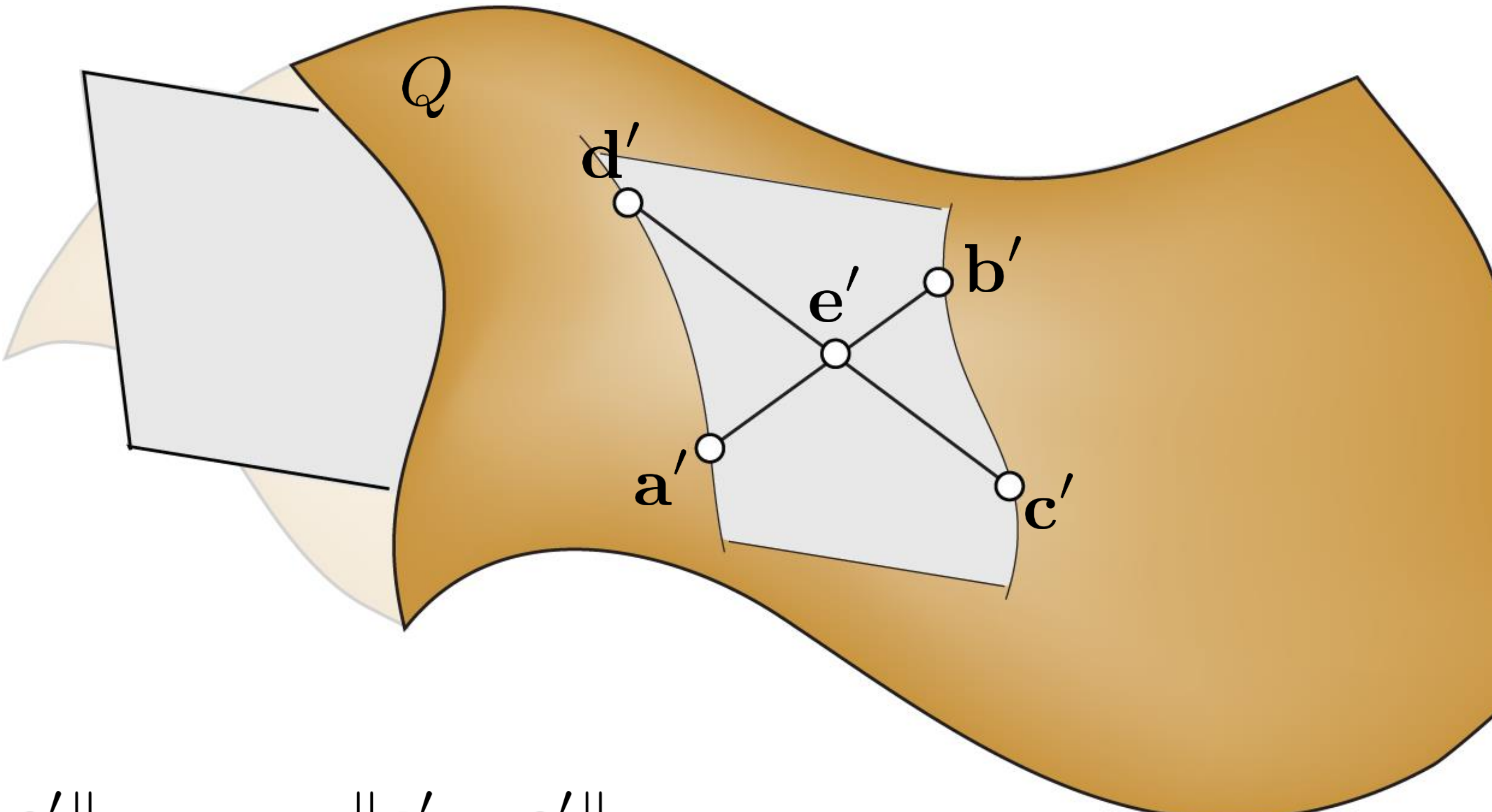
4 Point Congruent Sets



$$\mathbf{r}_1 = \frac{\|\mathbf{a} - \mathbf{e}\|}{\|\mathbf{a} - \mathbf{b}\|} \qquad \mathbf{r}_2 = \frac{\|\mathbf{c} - \mathbf{e}\|}{\|\mathbf{c} - \mathbf{d}\|}$$

Slides by [Aiger et al., SIGGRAPH 08]

4 Point Congruent Sets



$$\mathbf{r}'_1 = \frac{\|\mathbf{a}' - \mathbf{e}'\|}{\|\mathbf{a}' - \mathbf{b}'\|} \quad \mathbf{r}'_2 = \frac{\|\mathbf{c}' - \mathbf{e}'\|}{\|\mathbf{c}' - \mathbf{d}'\|}$$

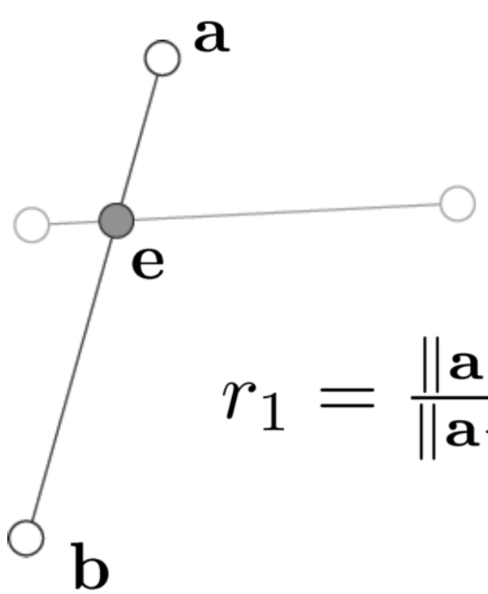
Slides by [Aiger et al., SIGGRAPH 08]

4 Point Congruent Sets

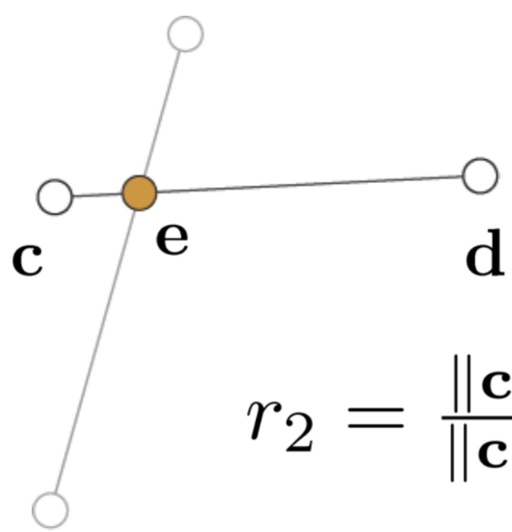
- Proposed approach
  1. Select coplanar base (4 points from P)
     - Select 3 point from P at random and find the 4th point to ensure coplanarity
     - Initial guess of the overlaps between P and Q to limit the maximum distance among the points of the coplanar base
  2. Find congruent point in Q
  3. Estimate rigid transformation

4 Point Congruent Sets

- Extracting Congruent 4-points



$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$

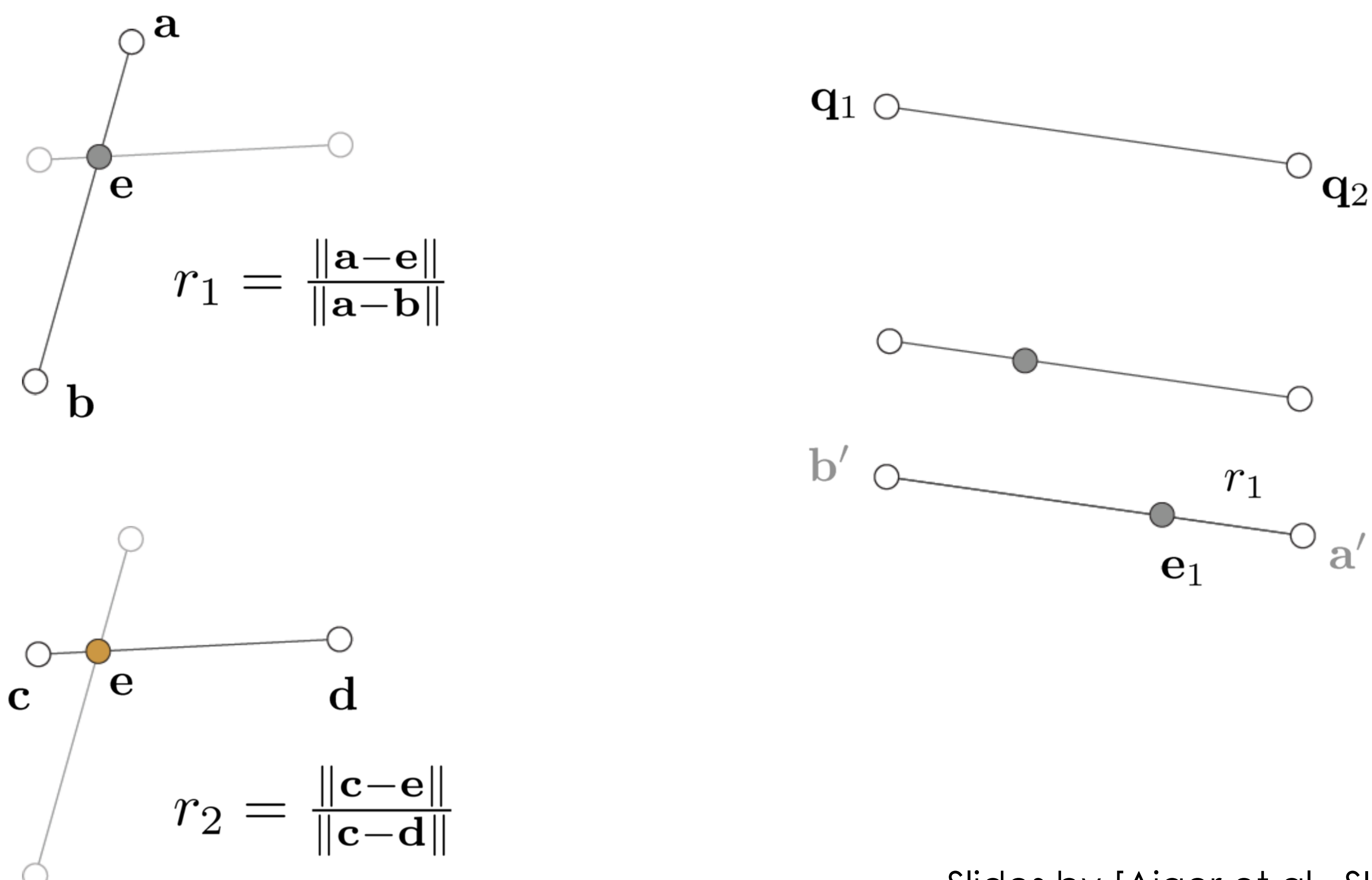


$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

Slides by [Aiger et al., SIGGRAPH 08]

4 Point Congruent Sets

- Extracting Congruent 4-points



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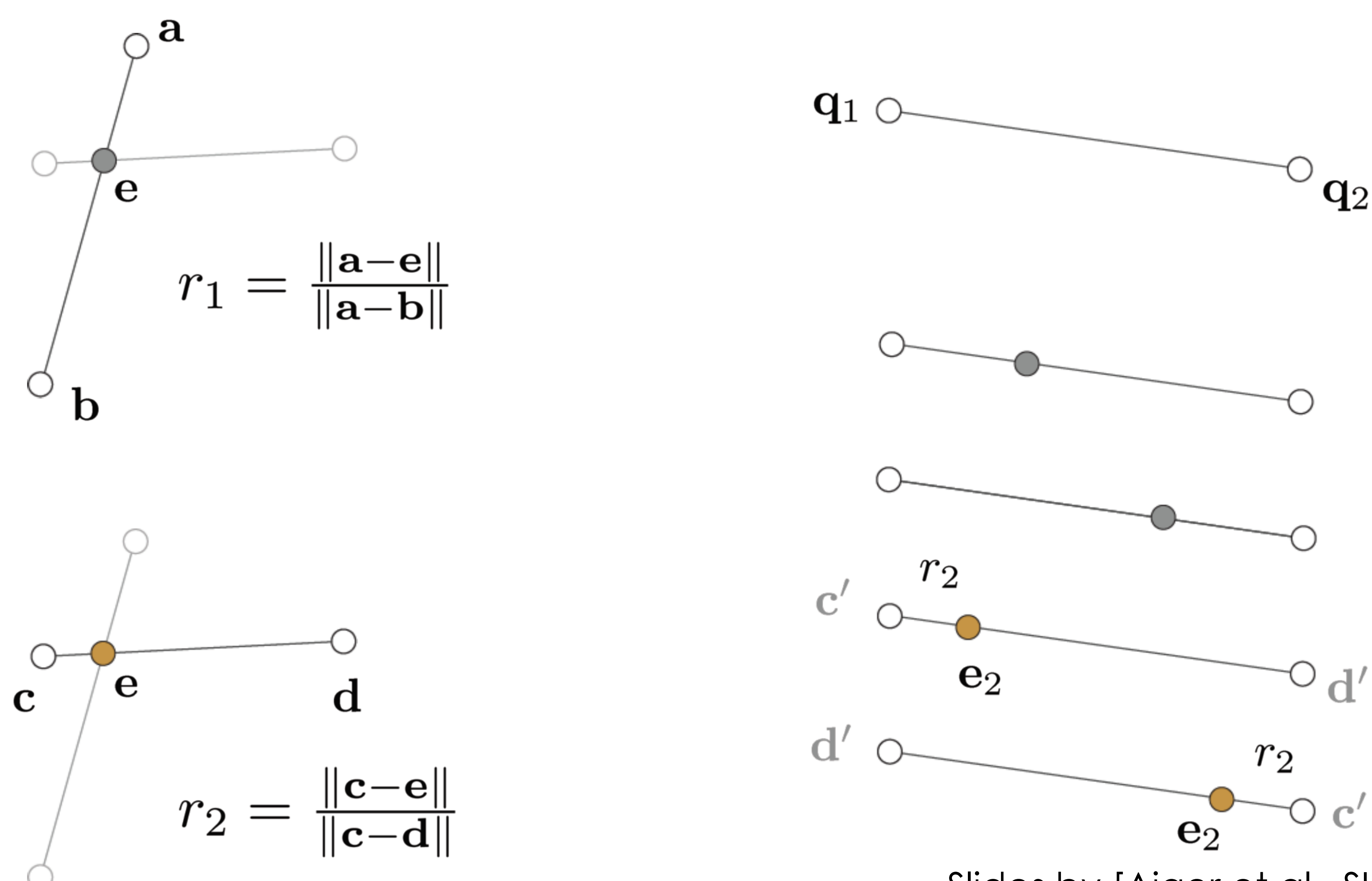

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4 Point Congruent Sets

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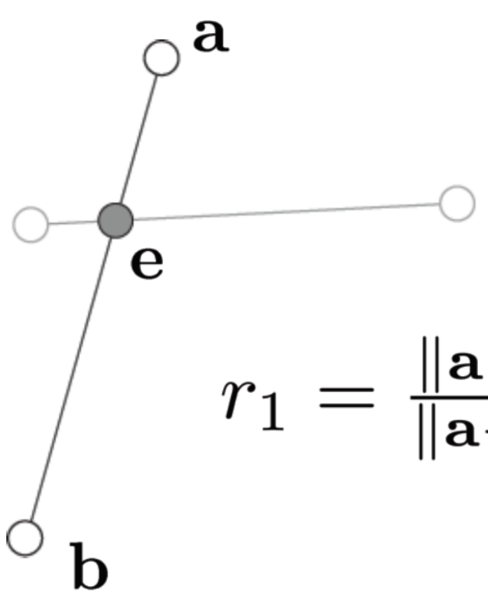
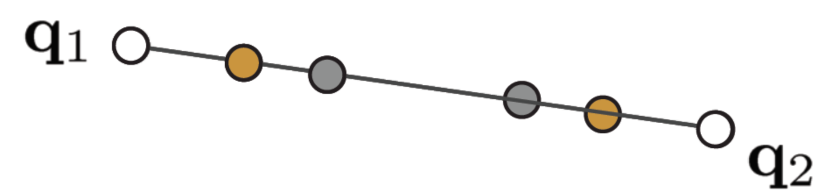
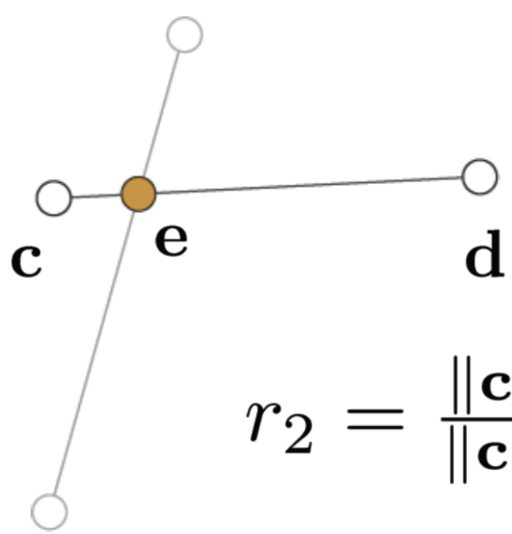


$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$



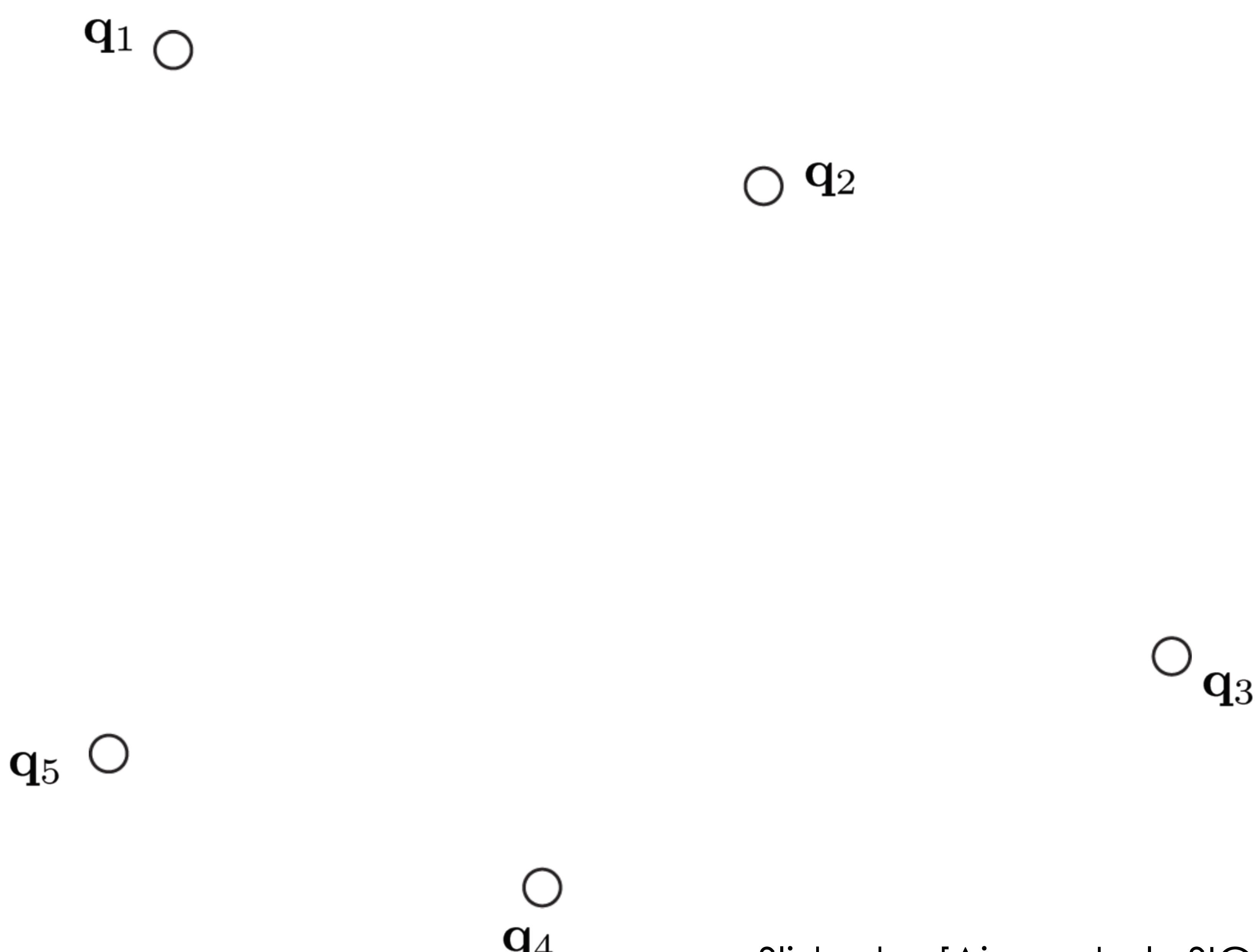


$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

Slides by [Aiger et al., SIGGRAPH 08]

4 Point Congruent Sets

- Extracting Congruent 4-points



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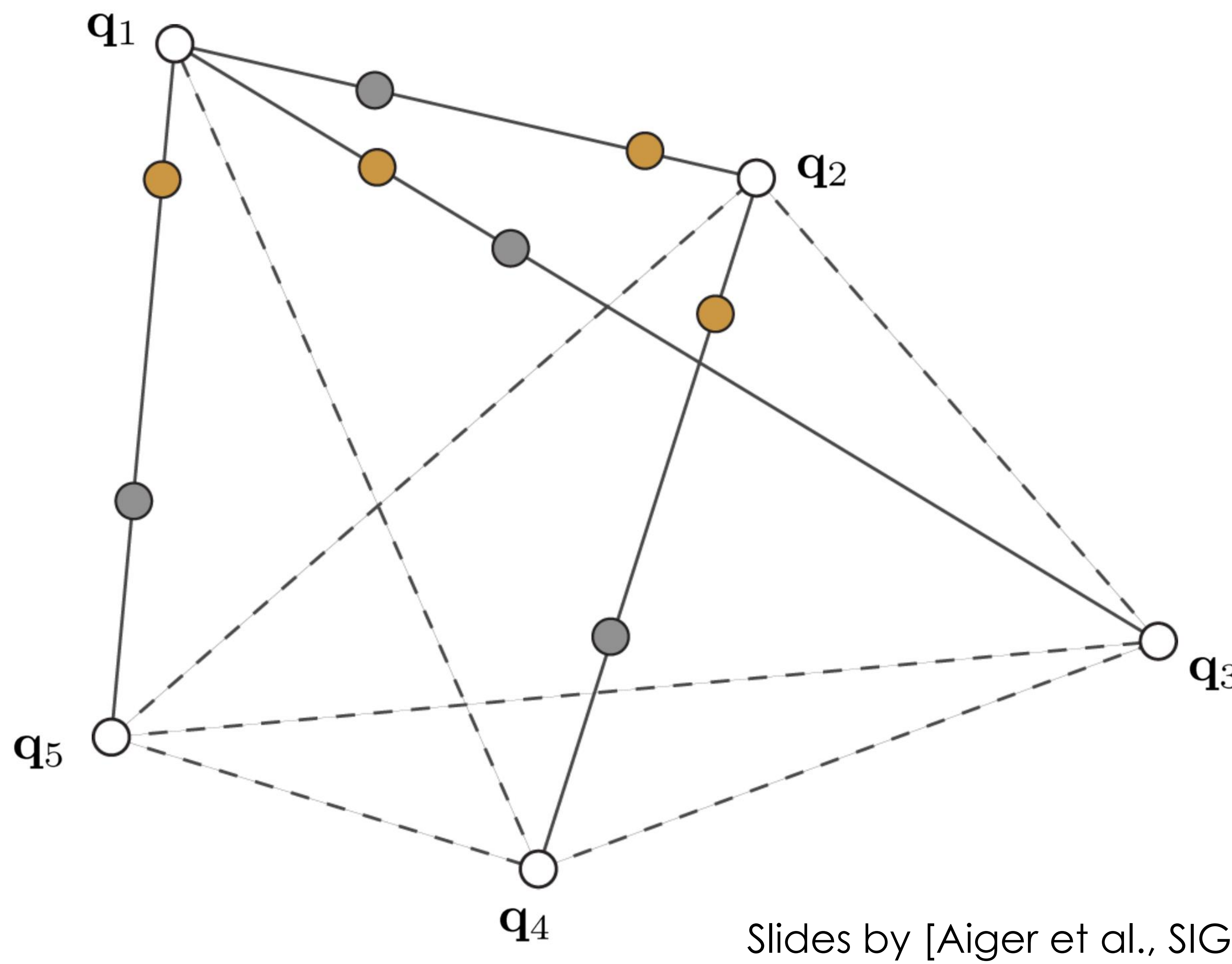
4 Point Congruent Sets

- Extracting Congruent 4-points



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4 Point Congruent Sets

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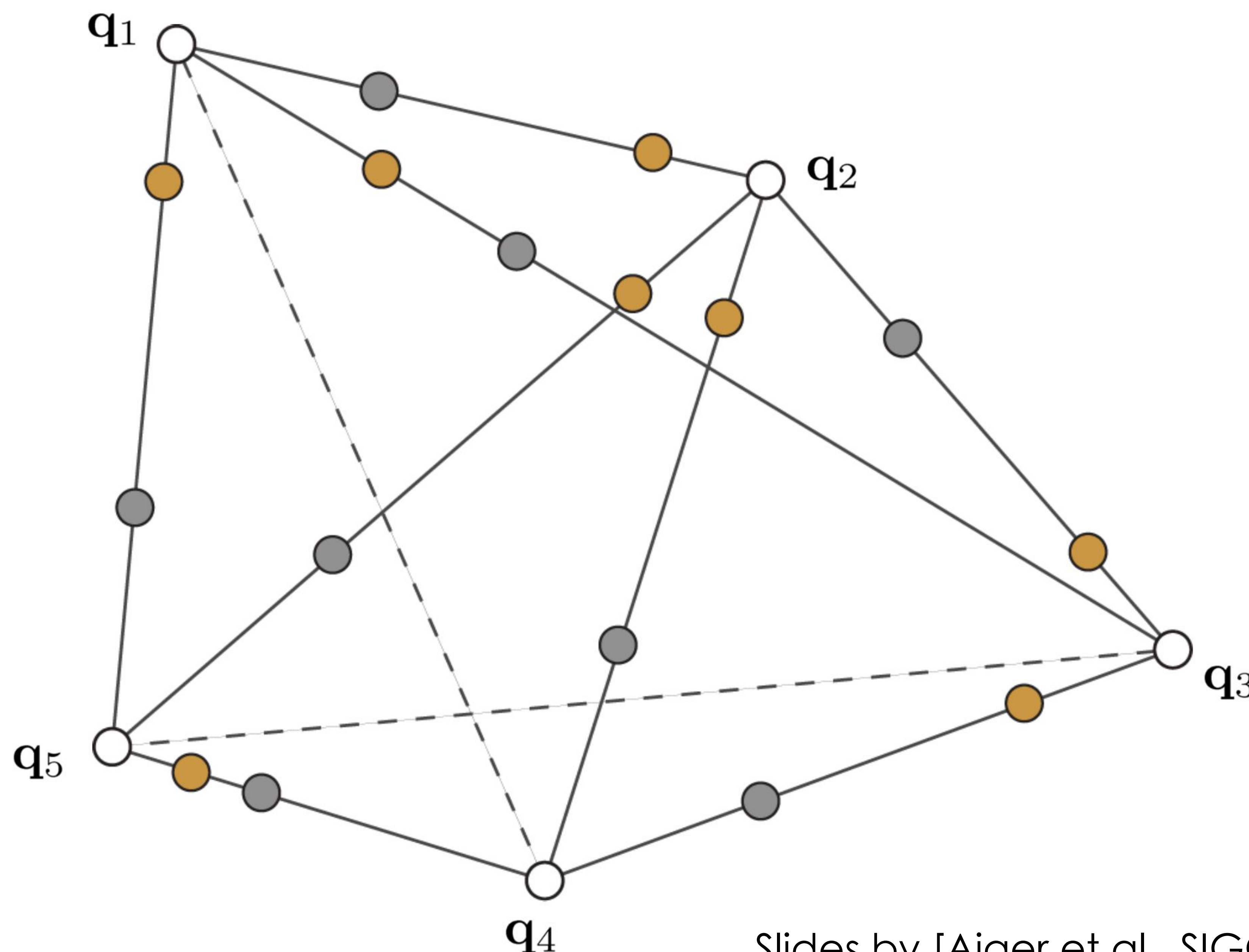

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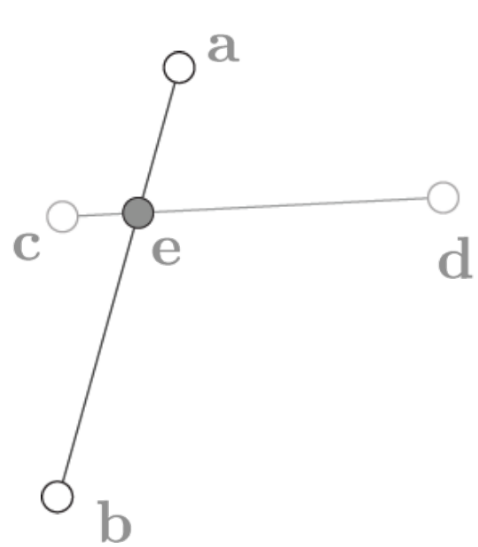
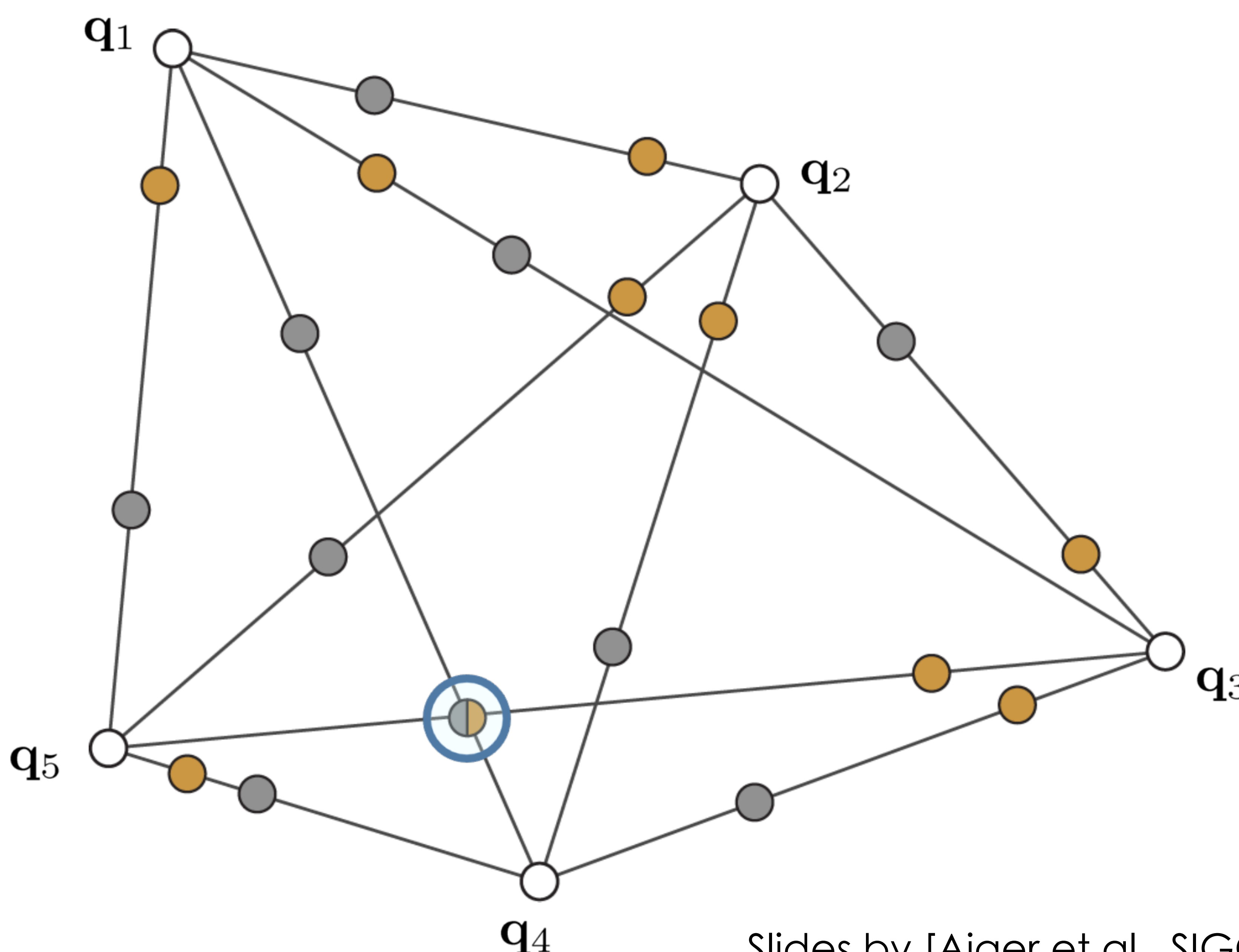
4 Point Congruent Sets

- Extracting Congruent 4-points



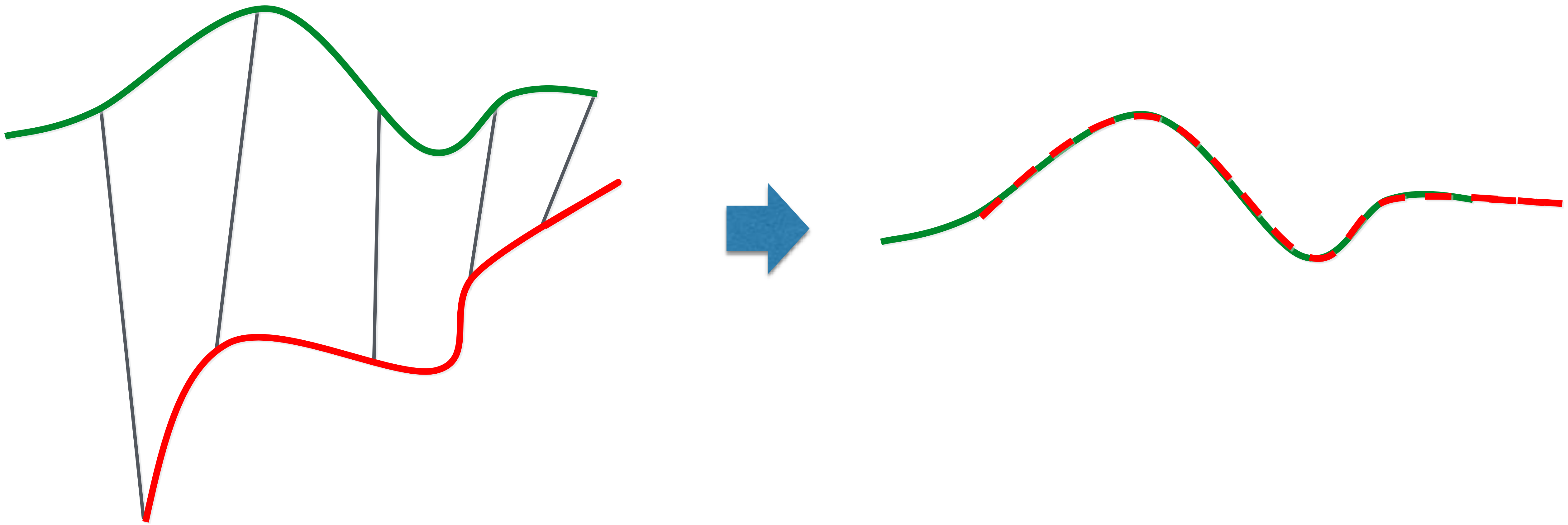
$$\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\} \equiv \{\mathbf{q}_5, \mathbf{q}_3, \mathbf{q}_4, \mathbf{q}_1\}$$



Slides by [Aiger et al., SIGGRAPH 08]

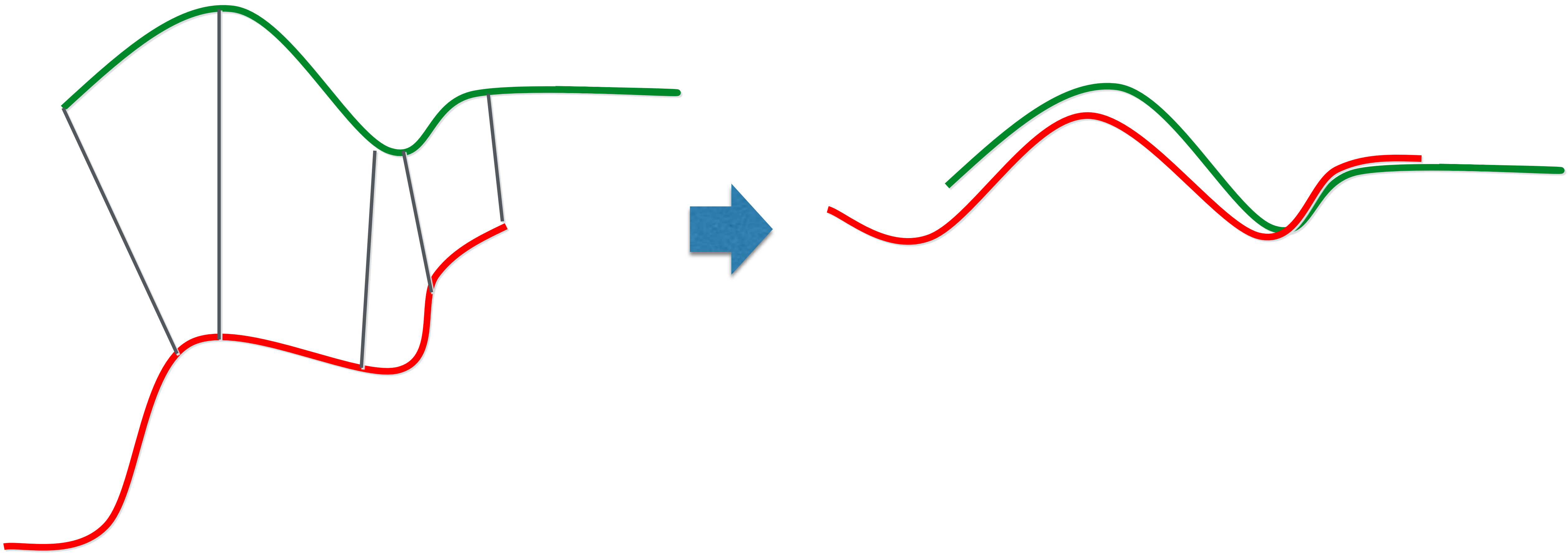
Align 3D data

- If correct correspondences are known, we can find correct relative rotation/translation



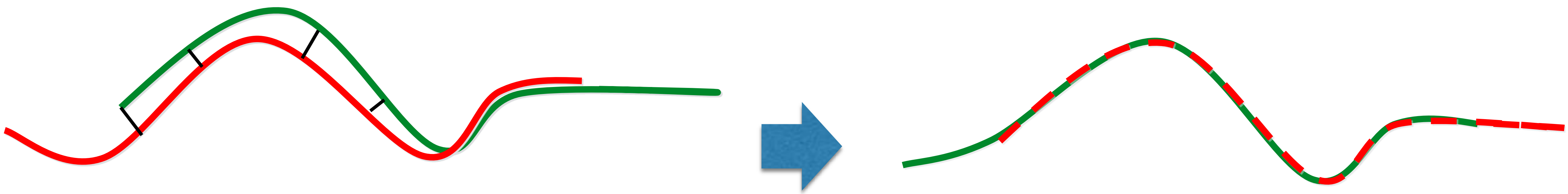
Align 3D data

- How to find correspondences: user input? feature detection?
- Alternative: assume closest points correspond



Align 3D data

- … and iterate to find alignment - Iterative Closest Points (ICP) [Bels et al., PAMI92]
- Converges if starting position "close enough"



ICP

- Pairwise alignment of mesh P and Q
  1. Select sample point from one mesh or both
  2. Match each to closest point on other scan
  3. Reject bad correspondences
  4. Compute transformation that minimizes the error metric
  5. Iterate until convergence

ICP

- Pairwise alignment of mesh P and Q

1. **Select sample point from one mesh or both**
2. Match each to closest point on other scan
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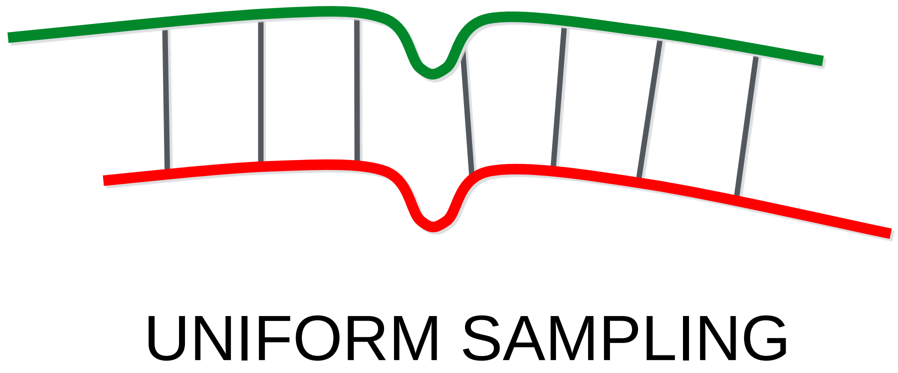
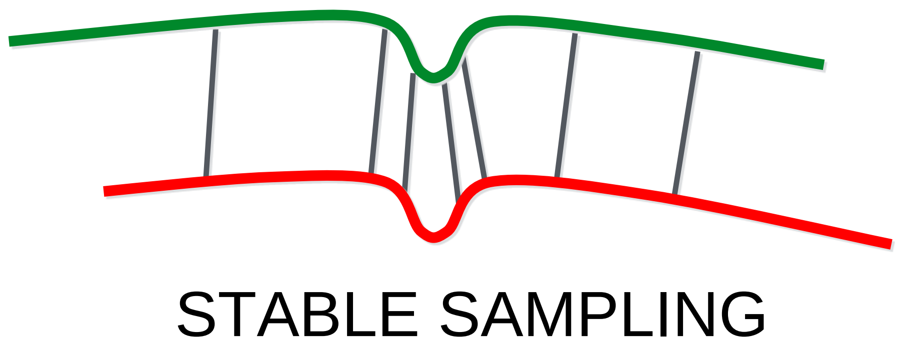
Sample selection

- All the points
- Random Sampling
- Uniform Sampling
- Stable Sampling
  - Select samples that constrain all degrees of freedom of the rigid-body transformation

    [Gelfand et al., 3DIM03]





Stable sampling

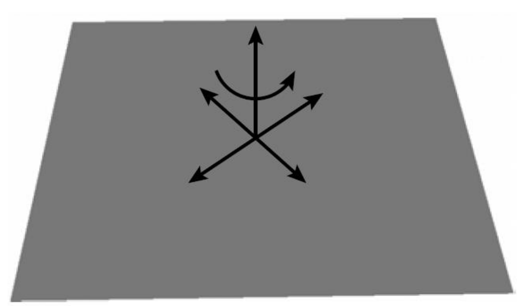
- Aligning transform is given by $\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} (\mathbf{p}_1 \times \mathbf{n}_1)^T & \mathbf{n}_1^T \\ (\mathbf{p}_2 \times \mathbf{n}_2)^T & \mathbf{n}_2^T \\ \cdots & \cdots \end{bmatrix} \in \mathbb{R}^{6\times n} \quad \mathbf{x} = \begin{bmatrix} \mathbf{t} \\ \mathbf{r} \end{bmatrix} \in \mathbb{R}^6 \quad \mathbf{b} = \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{q}_1)^T\mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2)^T\mathbf{n}_2 \\ \cdots \end{bmatrix} \in \mathbb{R}^n$$

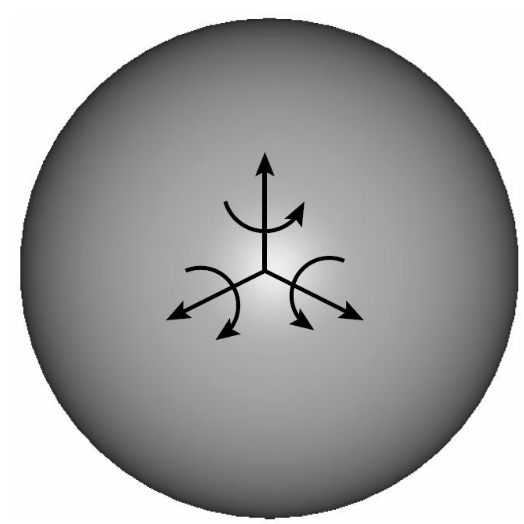
- Covariance matrix $C = \mathbf{A}^T\mathbf{A}$ determines the change in error when surfaces are moved from optimal alignment

Stable sampling

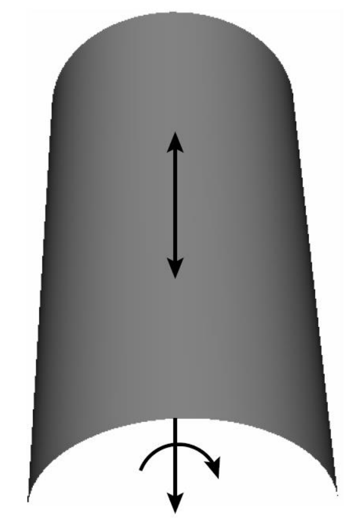
- Eigenvectors of C with small eigenvalues correspond to sliding transformations



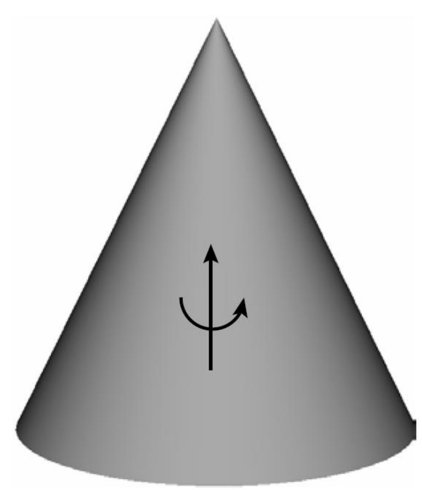
2 translations, 1 rotation



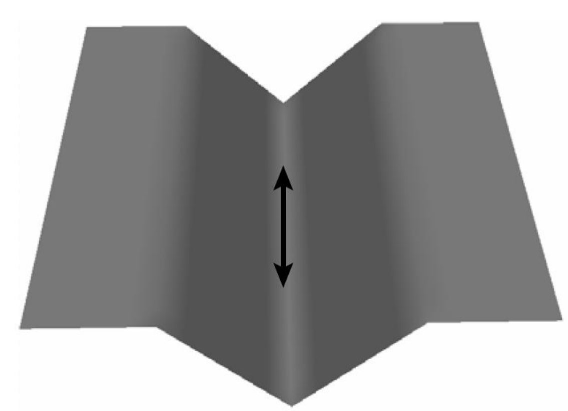
3 rotations



1 rotation, 1 translation



1 rotation



1 translation

[Gelfand et al., 3DIM03]

Stable sampling

- Select points to prevent small eigenvalues
  - Based on C obtained from sparse sampling

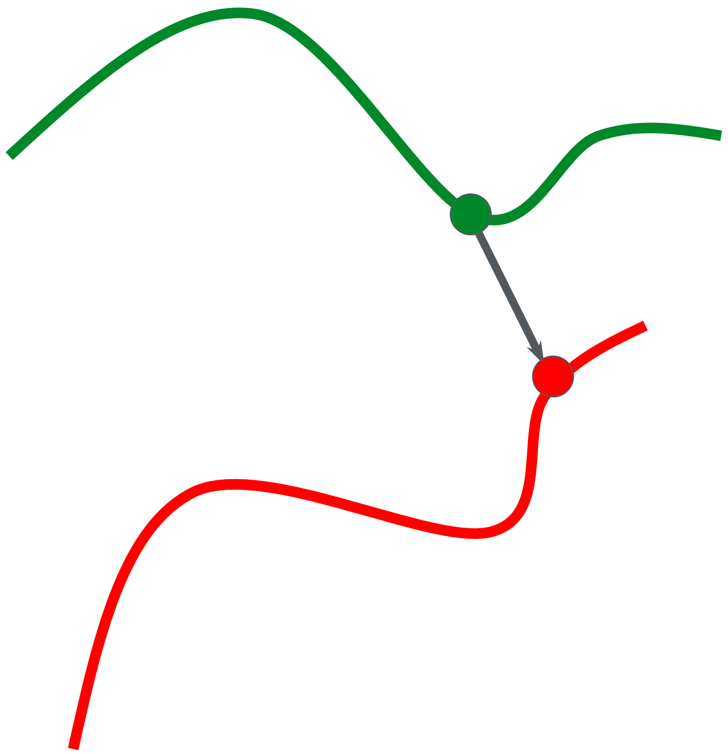
- Simpler variant: normal-space sampling select points with uniform distribution of normals
  - Pro: faster, does not require eigenanalysis
  - Con: only constrains translation

ICP

- Pairwise alignment of mesh P and Q
  1. Select sample point from one mesh or both
  2. **Match each to closest point on other scan**
  3. Reject bad correspondences
  4. Compute transformation that minimizes the error metric
  5. Iterate until convergence

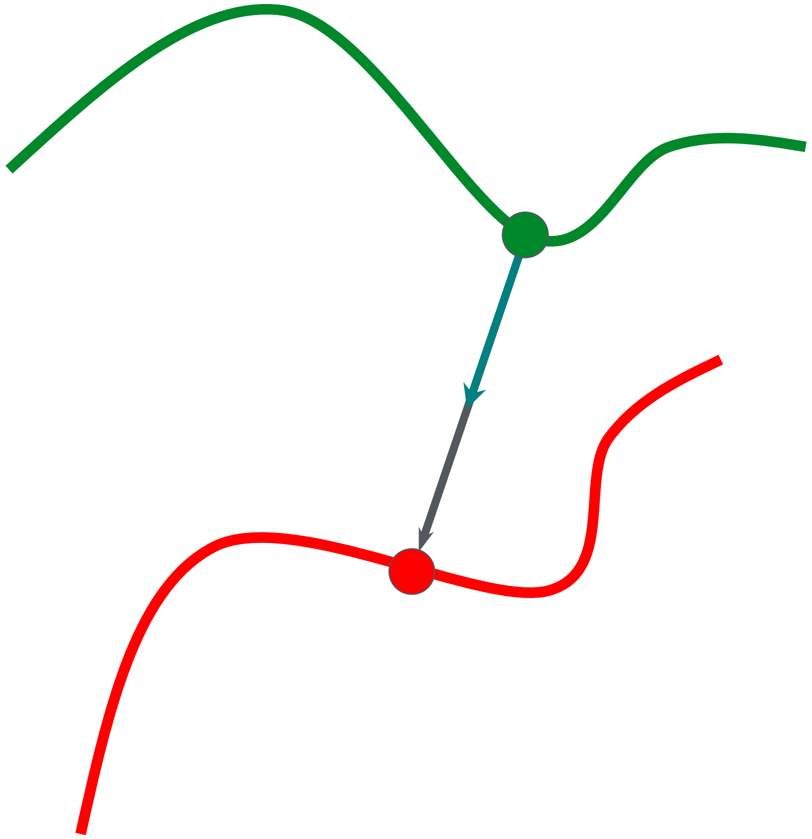
Points Matching

- Finding Closest point is most expensive stage of the ICP algorithm
  - Brute force search – O(n)
  - Spatial data structure (e.g., k-d tree) – O(log n)



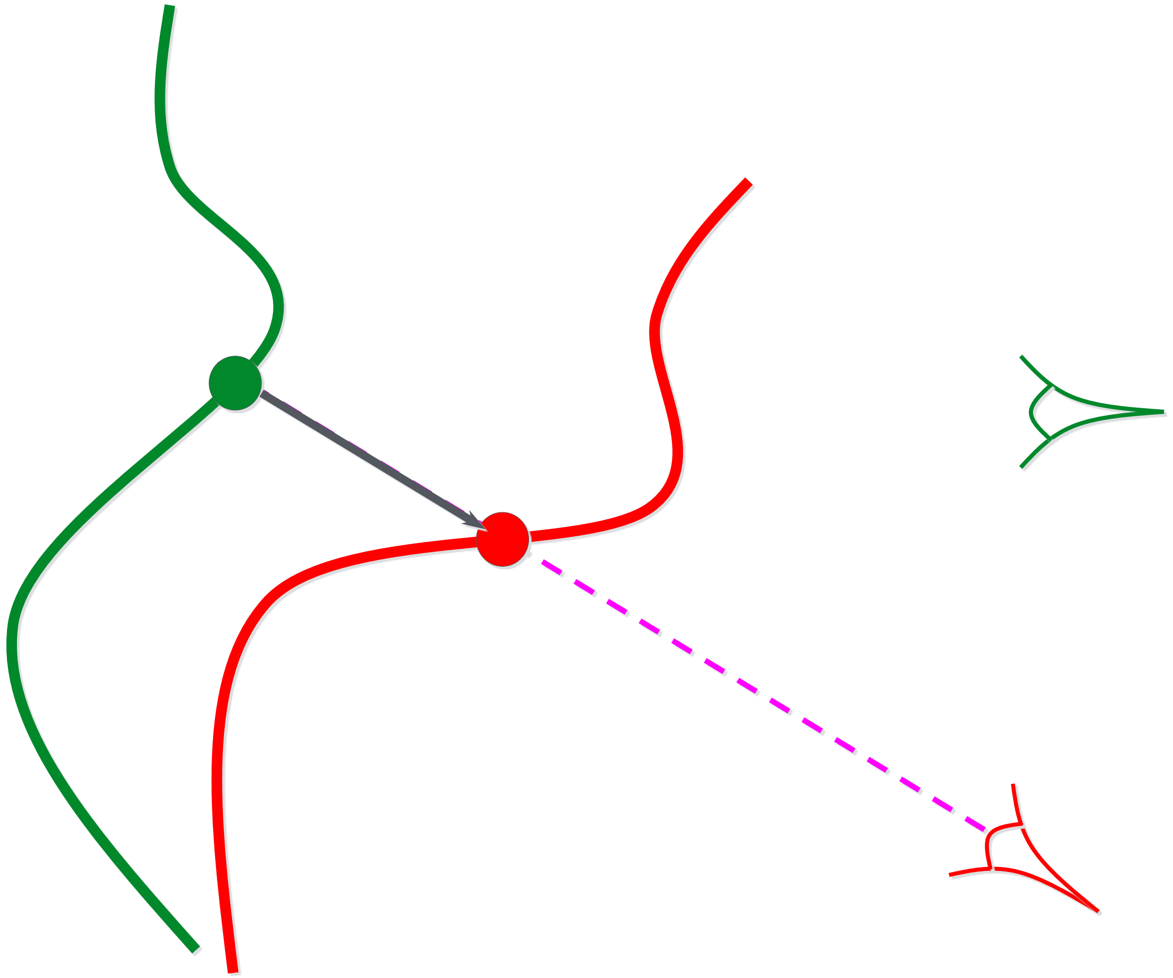
Points Matching

- Alternative: Normal Shooting



Points Matching

- Alternative for range map: Projection
  Project the point onto the destination mesh from the point of view of the destination camera

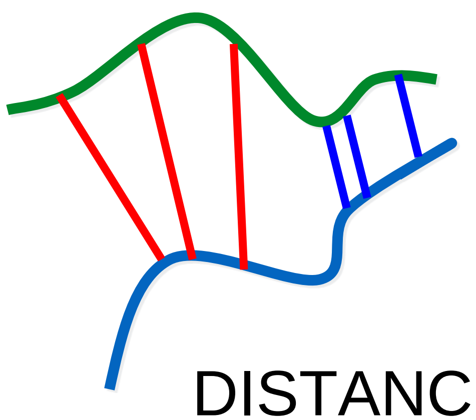


ICP

- Pairwise alignment of mesh P and Q
  1. Select sample point from one mesh or both
  2. Match each to closest point on other scan
  3. **Reject bad correspondences**
  4. Compute transformation that minimizes the error metric
  5. Iterate until convergence

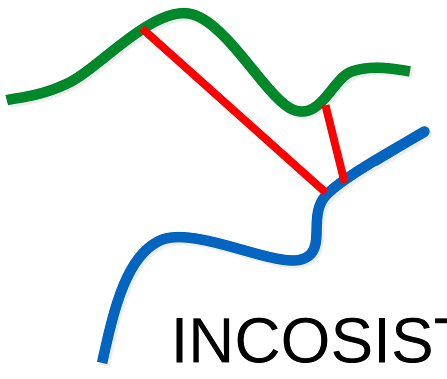
Rejecting Pairs

- Corresponding points with point to point distance higher than a given threshold.
- Rejection of worst n% pairs based on some metric.
- Pairs containing points on mesh border.
- Rejection of pairs that are not consistent with their neighboring pairs
- Pair with not consistent normal



DISTANCE

BORDER



INCOSISTENT PAIRS

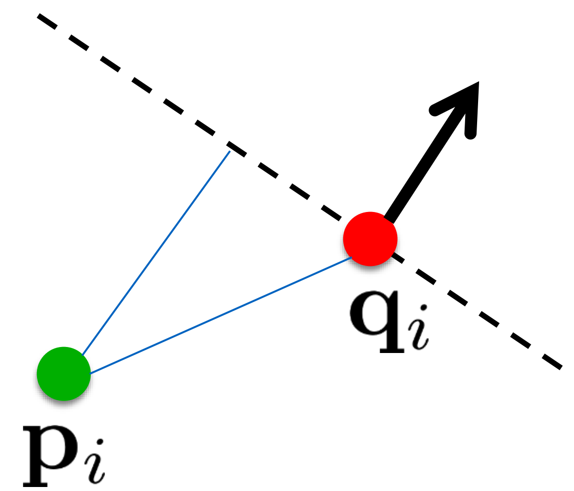
ICP

- Pairwise alignment of mesh P and Q
  1. Select sample point from one mesh or both
  2. Match each to closest point on other scan
  3. Reject bad correspondences
  4. **Compute transformation that minimizes the error metric**
  5. Iterate until convergence

Error metric

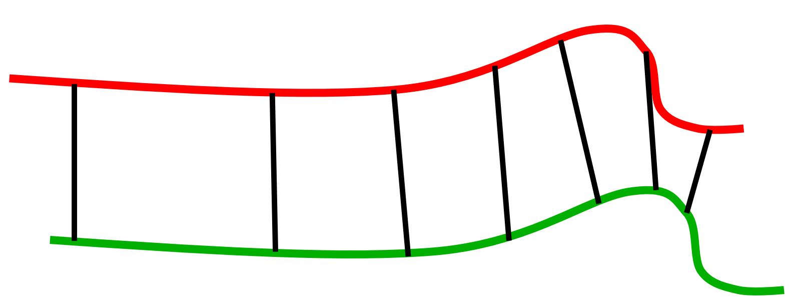
- Point-to-Point error metric (minimization with a direct method)

$$E = \sum_{i=1}^{n} \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i\|^2$$



- Point-to-Plane error metric (flat regions can slide along each other, then faster convergence)

$$E = \sum_{i=1}^{n} ((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)\mathbf{n}_i)^2$$



Point-to-Plane error metric

- Doesn't exist a closed form to minimize this error metric because rotation is not a linear function

$$E = \sum_{i=1}^{n} \left((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)\mathbf{n}_i \right)^2$$

- We can make the problem linear assuming small rotation

$$\cos\theta \approx 1$$

$$\sin\theta \approx \theta$$

Point-to-Plane error metric

- We can make the problem linear assuming small rotation

$$\mathbf{R} = \mathbf{R}_z(\gamma)\mathbf{R}_y(\beta)\mathbf{R}_x(\alpha)$$

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \quad \mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad \mathbf{R}_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta \approx 1$$
$$\sin\theta \approx \theta$$

$$\mathbf{R} \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix}$$

Point-to-Plane error metric

- Linearize

$$E \approx \sum_{i=1}^{n} \left((\mathbf{p}_i - \mathbf{q}_i)^T \mathbf{n}_i + \mathbf{r}^T (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t}^T \mathbf{n}_i \right)^2$$

$$\mathbf{r}^T = (\alpha, \beta, \gamma)$$

- Result: overconstrained linear system

Point-to-Plane error metric

- Overconstrained linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} (\mathbf{p}_1 \times \mathbf{n}_1)^T & \mathbf{n}_1^T \\ (\mathbf{p}_2 \times \mathbf{n}_2)^T & \mathbf{n}_2^T \\ \cdots & \cdots \end{bmatrix} \in \mathbb{R}^{6 \times n} \quad \mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{t} \end{bmatrix} \in \mathbb{R}^6 \quad \mathbf{b} = \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{q}_1)^T \mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2)^T \mathbf{n}_2 \\ \cdots \end{bmatrix} \in \mathbb{R}^n$$

- Solve using least squares

$$\mathbf{A}^T \mathbf{A}\mathbf{x} = \mathbf{A}^T \mathbf{b}$$

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$

Global registration

- Given: n scans around an object
- Goal: align them
- Want method for distributing accumulated error among all scans

Approach 1: Avoid the problem

- In some cases, have 1 (possibly low-resolution) scan that covers most surface
- Align all other scans to this “anchor”
- Disadvantage: not always practical to obtain anchor scan

Approach 2: The Greedy Solution

- Align each new scan to all previous scans [Masuda ’96]
- Disadvantages:
  - Order dependent
  - Doesn’t spread out error

Approach 3: The Brute-Force Solution

```
While not converged:
    For each scan
        For each point
            For every other scan
                Find closest point
```

- Minimize error w.r.t. transforms of **all** scans
- Disadvantage: Solve (6*n*)x(6*n*) matrix equation, where *n* is number of scans

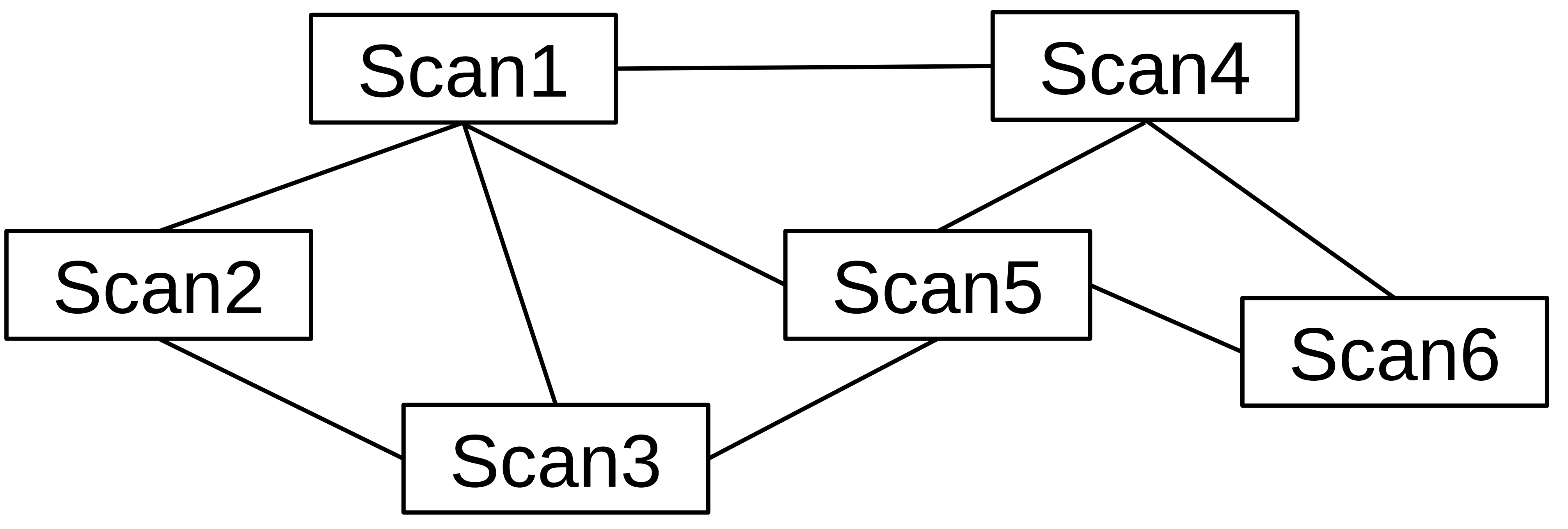
Approach 4: The Less Brute-Force Solution

```
While not converged:
  For each scan:
    For each point:
      For every other scan
        Find closest point
```

- Minimize error w.r.t. transforms of **this** scans
- Faster than previous method (matrices are 6x6)

Graph method

- Many global registration algorithms create a graph of pairwise alignments between scans (an edge for each pair of scans with enough overlapping)



Graph method

[Pulli et al., 3DIM99]

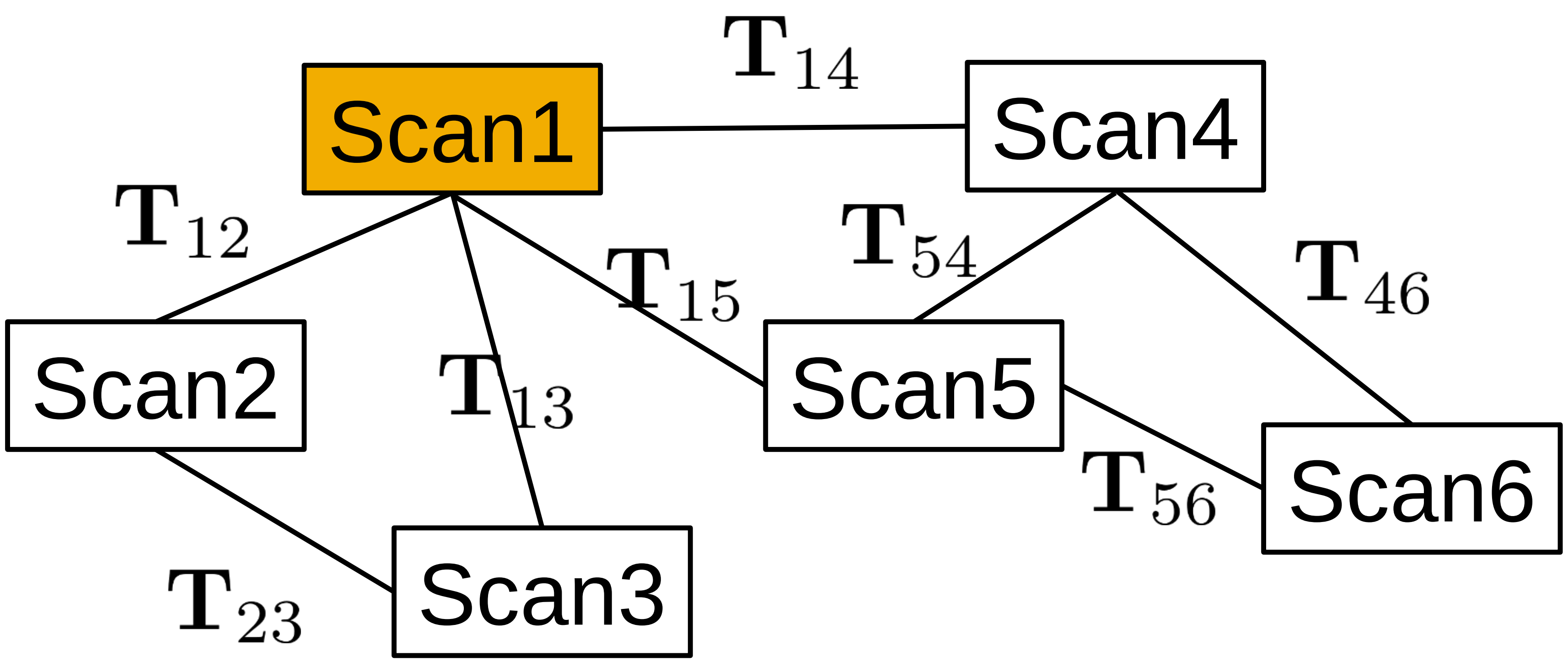
- Pairwise registration between every view and each overlapping neighboring, record the corresponding points
- For each scan, starting with most connected

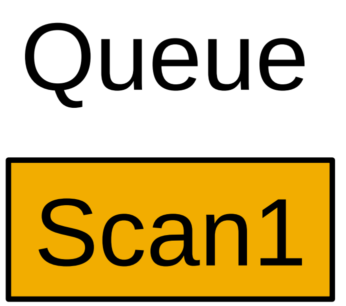
```
Add most connect view in the queue
While (queue is not empty)
  Align the current view with the neighbor in
  the graph
  If (change in error > threshold)
    Add neighbors to the queue
```

- All alignments during global phase use precompute corresponding points.

Graph method

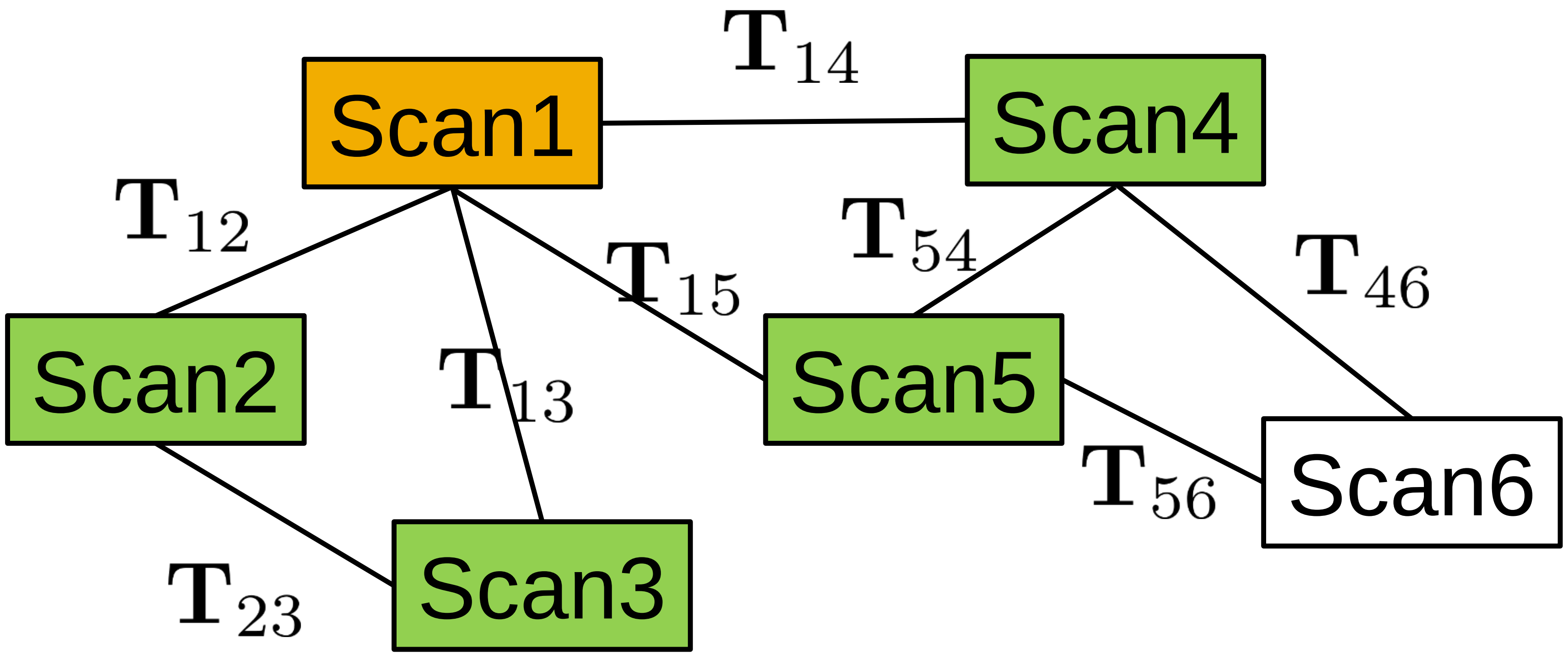
- Put most connected scan in the queue





Graph method

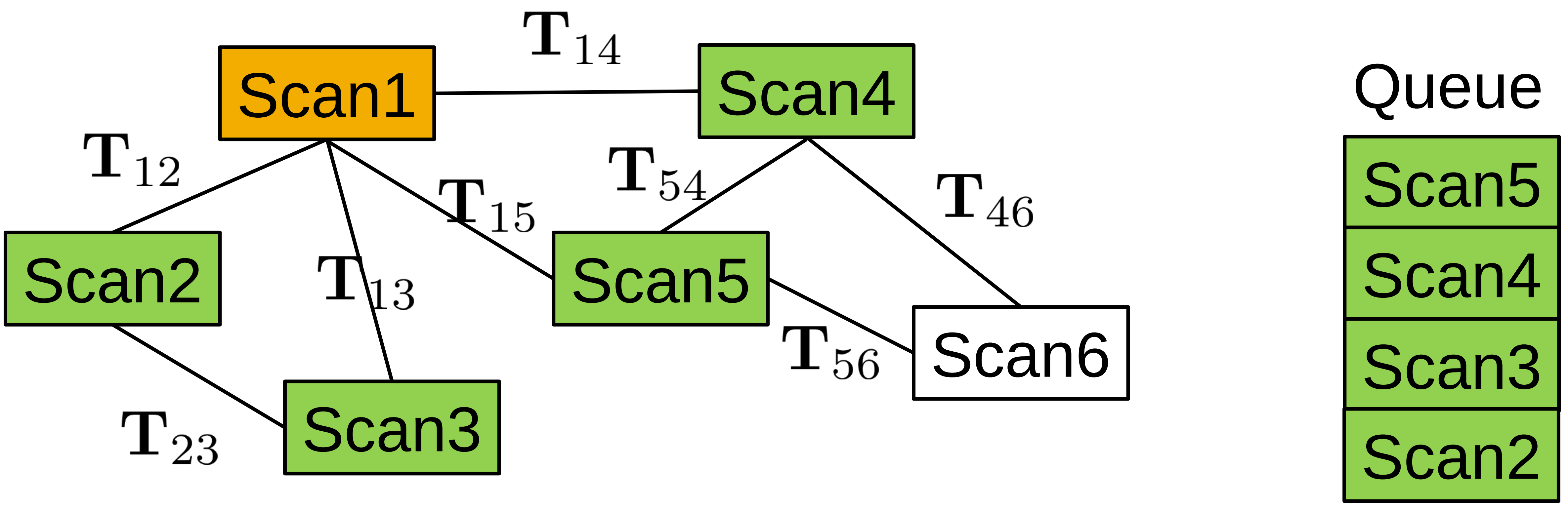
- Select overlapping scans and use the correspondences of the pairwise alignment to estimate the new transformation



Queue

Graph method

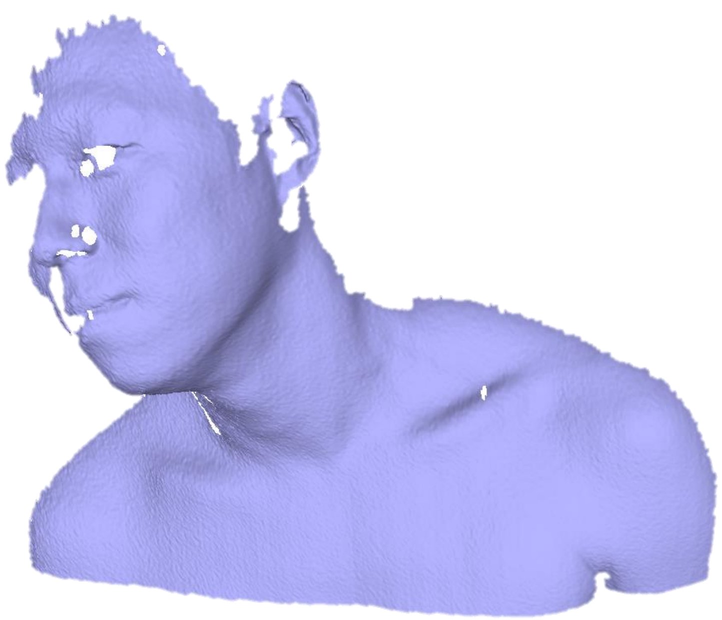
- If the change error is above a threshold put neighbors in the queue and iterate



Non-Rigid Registration

- More difficult problems
  - Deformation
  - Correspondences
  - Overlap

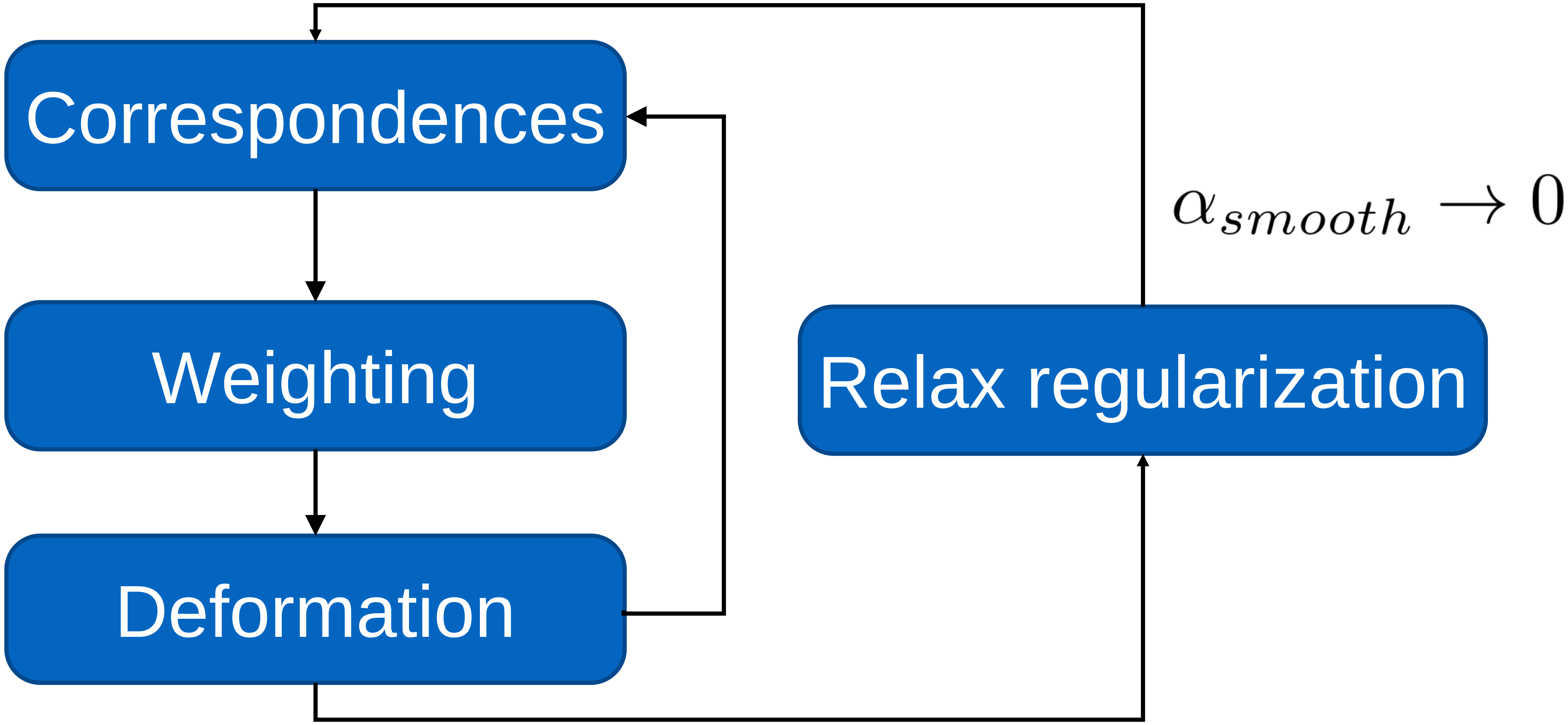
- Solution: global optimization via local refinement
  - Minimize the deformation energy
  - Minimize the alignment error
  - Maximize regions of overlaps



Images by
[Li et al., SGP08]

Non-Rigid Registration

$$E = \alpha_{rigid}E_{rigid} + \alpha_{smooth}E_{smooth} + \alpha_{fit}E_{fit} + \alpha_{conf}E_{conf}$$



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