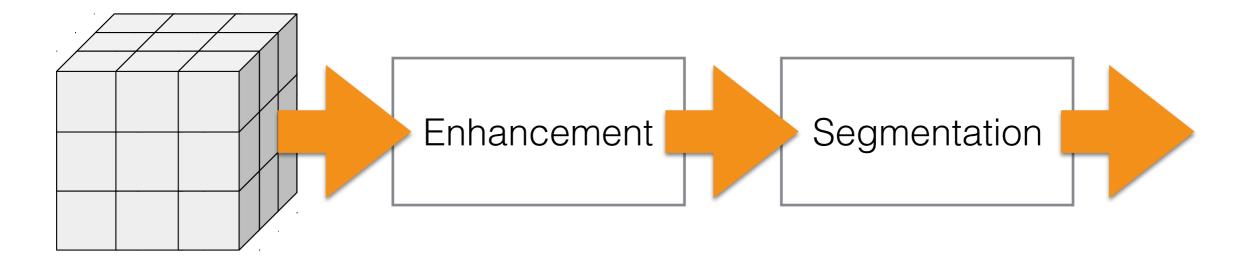
### 3D from Volume: Part I

Dr. Francesco Banterle, francesco.banterle@isti.cnr.it banterle.com/francesco

### The Main Pipeline

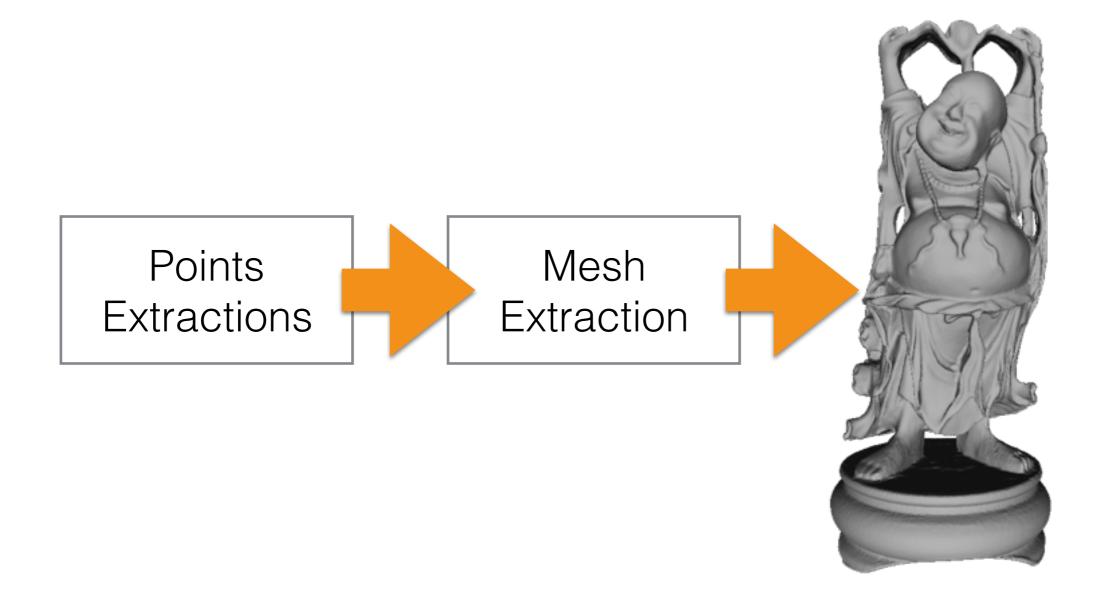


## The Processing Pipeline



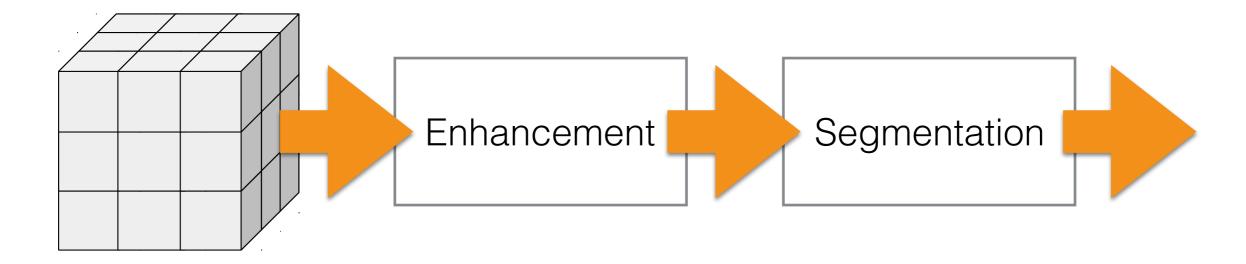
**RAW Volume** 

## The Processing Pipeline



3D Mesh

## The Processing Pipeline



**RAW Volume** 

### Image Enhancement

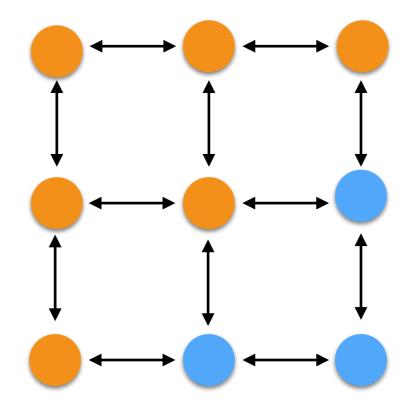
- Step 1: we have to improve the dynamic range of the input images in the volume; i.e., increase/ decrease it.
- **Step 2**: we have to filter the image in order to elicit some features and/or to remove noise (salt-and-pepper, Gaussian noise, etc).

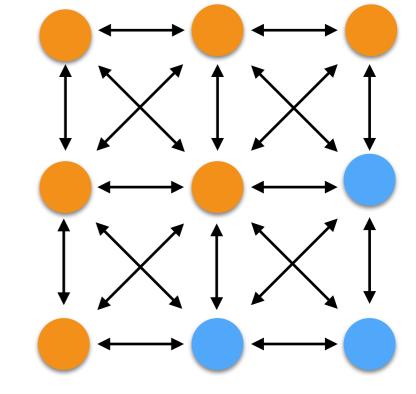
2D Images

### 2D Images

#### • A 2D image is a graph:







8-connected pixel adjacency graph

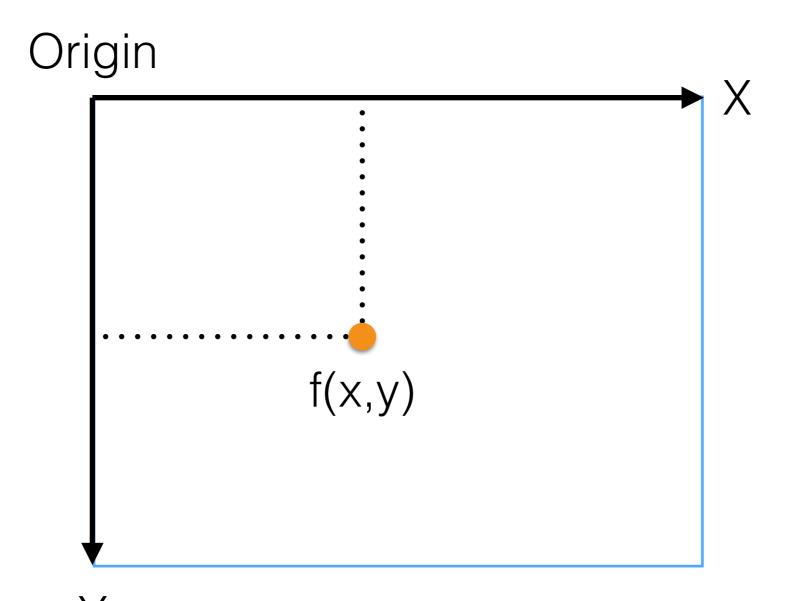
4-connected pixel adjacency graph

2D Image, 3x3 pixels

### A Graph

- A graph is a pair G = (V,E), where:
  - V is a set of vertices. Each element of V is called a vertex of G.
  - E is a pairs of elements in V; e.g, (V<sub>1</sub>; V<sub>2</sub>), etc.
     Each element of E is called an edge of G.

### Image Coordinate System



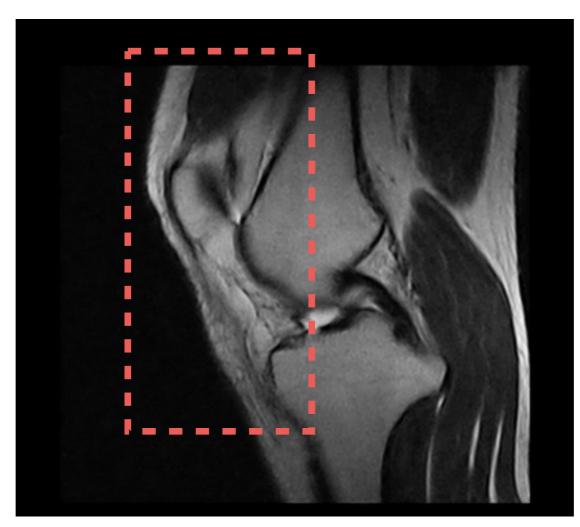
#### Image Coordinate System: MATLAB

- MATLAB origin —> (1,1)
- Given an image, img, as m-n matrix to access to f:

#### f = img(y, x)

# Region Of Interest (ROI)

- We may be interested to process not the full image/ volume but an area/volume.
- This area is typically called region of interest (ROI).



- Images are not perfect: device, patient moves, etc.
- What we really see is:

 $f(x,y) \approx f'(x,y)$  $f(x,y) = [(f'+n_T) \otimes h](x,y) \cdot g(x,y) + n(x,y)$ 

- Images are not perfect: device, patient moves, etc.
- What we really see is:

$$f(x,y) \approx f'(x,y)$$

$$f(x,y) = [(f' + n_T) \otimes h](x,y) \cdot g(x,y) + n(x,y)$$
Other tissues

- Images are not perfect: device, patient moves, etc.
- What we really see is:

$$f(x,y) \approx f'(x,y)$$
  
$$f(x,y) = [(f' + n_T) \otimes h](x,y) \cdot g(x,y) + n(x,y)$$
  
discrete spatial-temporal  
process

- Images are not perfect: device, patient moves, etc.
- What we really see is:

$$f(x,y) \approx f'(x,y)$$
  

$$f(x,y) = [(f'+n_T) \otimes h](x,y) \cdot g(x,y) + n(x,y)$$
  

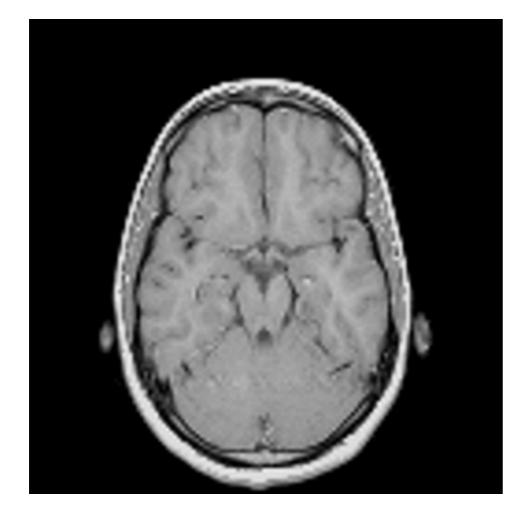
$$\int_{\text{discrete spatial-temporal process}}$$

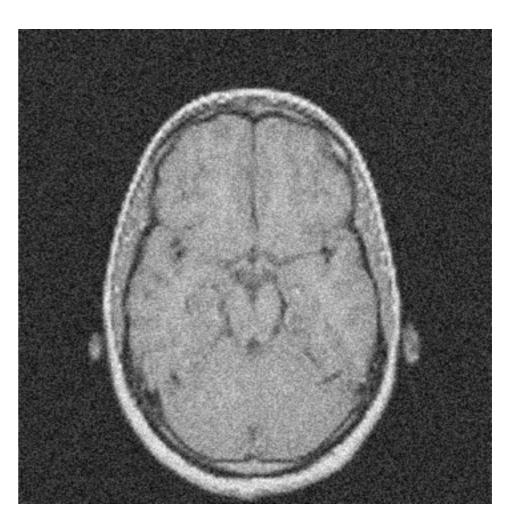
- Images are not perfect: device, patient moves, etc.
- What we really see is:

$$f(x,y) \approx f'(x,y)$$
  
$$f(x,y) = [(f'+n_T) \otimes h](x,y) \cdot g(x,y) + n(x,y)$$
  
$$\int$$
  
device noise

#### Noise Measure

• A classic:  $SNR = \frac{\mu}{\sigma}$ 





SNR = 0.8

SNR = 1.5

#### Medical File Format

### DICOM

- Digital Imaging and COmmunications in Medicine
- It is a standard for producing, storing, displaying, printing, and sending, retrieving, and querying medical images
- Data: 2D images (may be compressed using JPG/ JPG2000)
- Metadata: patient's personal information, date of the exam, position of the patient, etc.
- Issue: many extra fields, which are filled without consistency amongst different software/scanners

### DICOM

- File extension: name\_file.dcm
  - The media format does not allow files to have and extension; the folders structure gives meaning to the file!
- Standard official web-site: <u>http://DICOM.nema.org</u>
- MATLAB and Slicer can open them natively.

### Point-wise Operators

### Point-wise Operators

- An operator takes as input one or two images, and the result is another image.
- Unary operator T<sub>1</sub>:

$$g(x,y) = T_1\left[f(x,y)\right]$$

• Binary operator T<sub>2</sub>:

$$g(x,y) = T_2\left[f(x,y);h(x,y)\right]$$

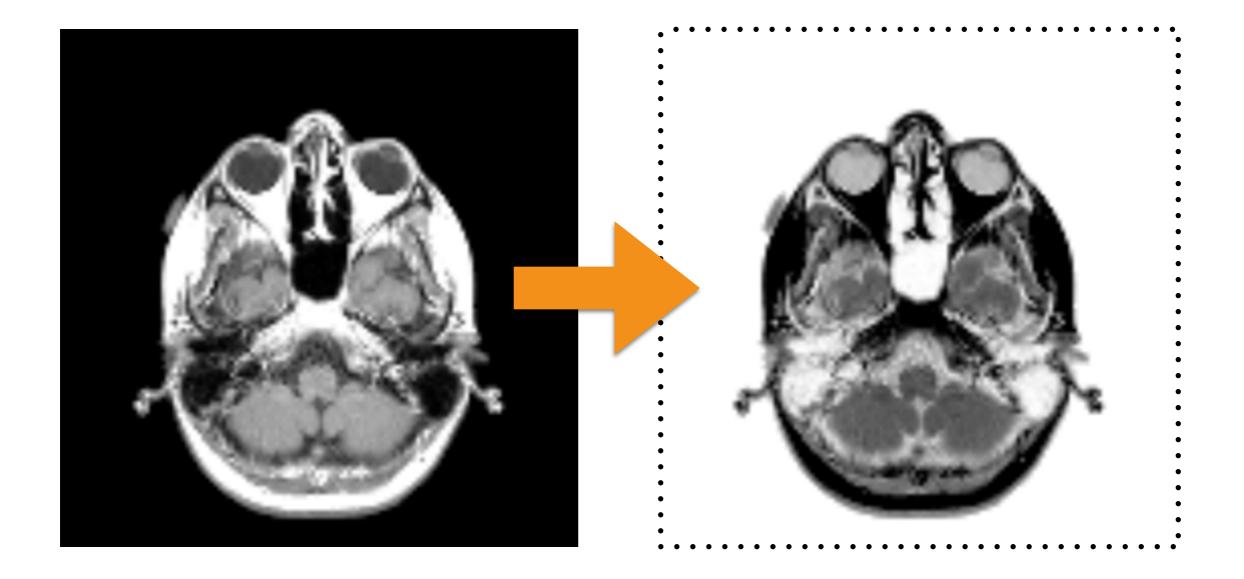
#### Unary Operators: Negative

• Negative or inverter:

$$g(x, y) = Neg[f(x, y)] = 1.0 - f(x, y)$$

- It is usually helpful to highlight some structures.
- Note: this operator assumes images' values are in the rage [0,1].

### Negative: Example



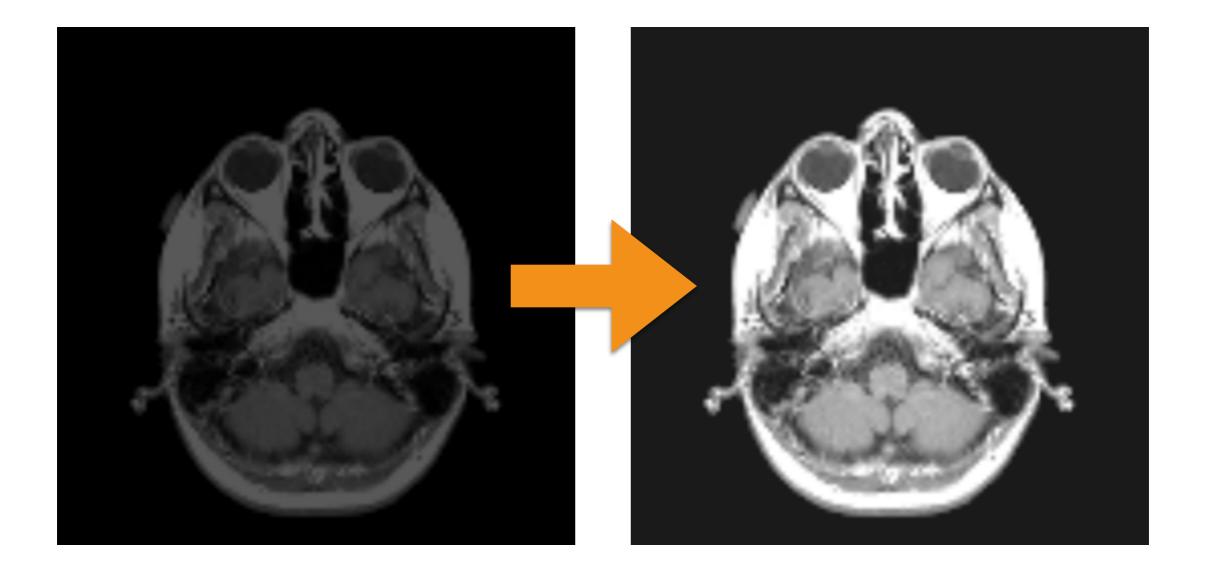
#### Unary Operators: Contrast Stretching

• This operator increases the dynamic range of the input image linearly:

$$g(x,y) = \operatorname{CS}\left[f(x,y); E_{\min}; E_{\max}\right] =$$
$$= (f(x,y) - \min(f)) \frac{E_{\max} - E_{\min}}{\max(f) - \min(f)} + E_{\min}$$

• It is useful when the contrast is low.

#### Contrast Stretching Example



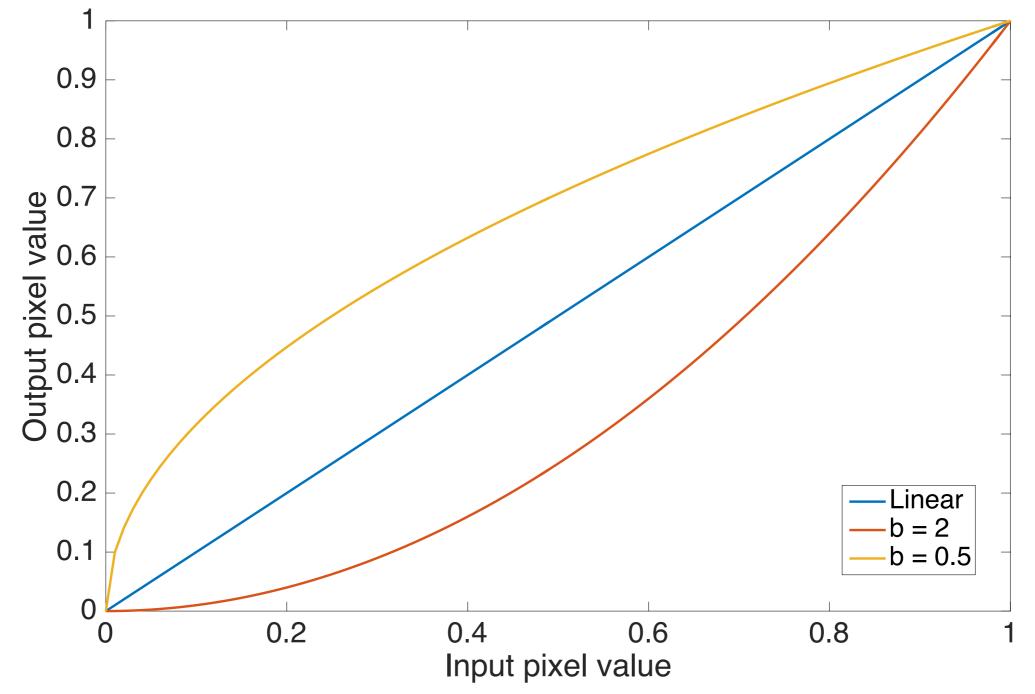
#### Unary Operators: Gamma

Another method for increasing the dynamic range:

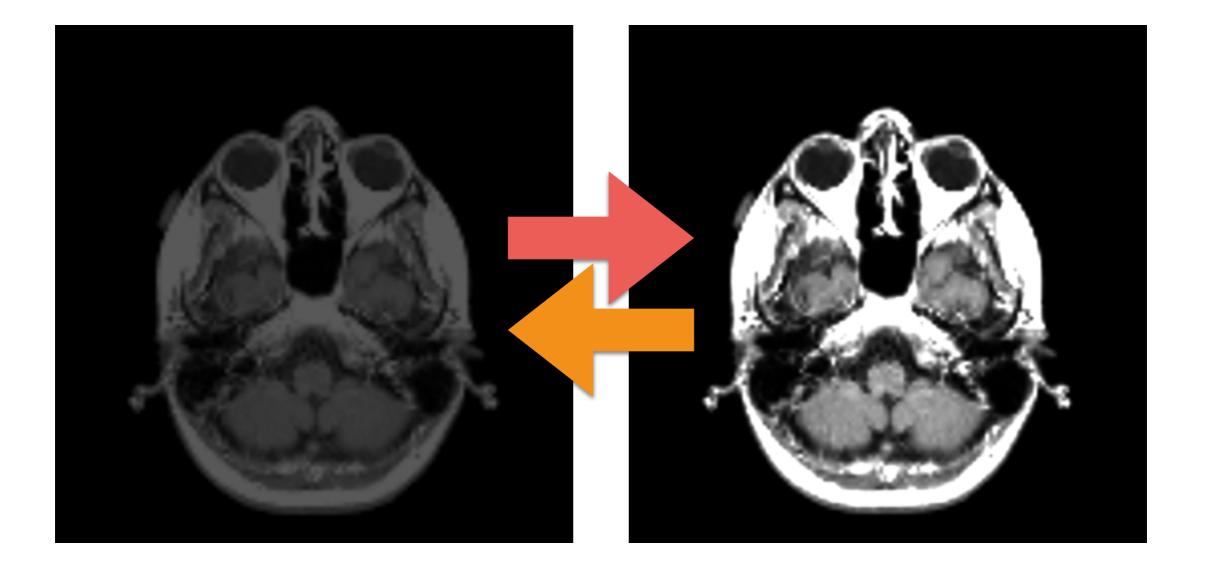
$$g(x, y) = G[f(x, y); k; \gamma] =$$
$$= k \cdot f(x, y)^{\gamma}$$

• It is more intuitive.

#### Unary Operators: Gamma



### Gamma Example

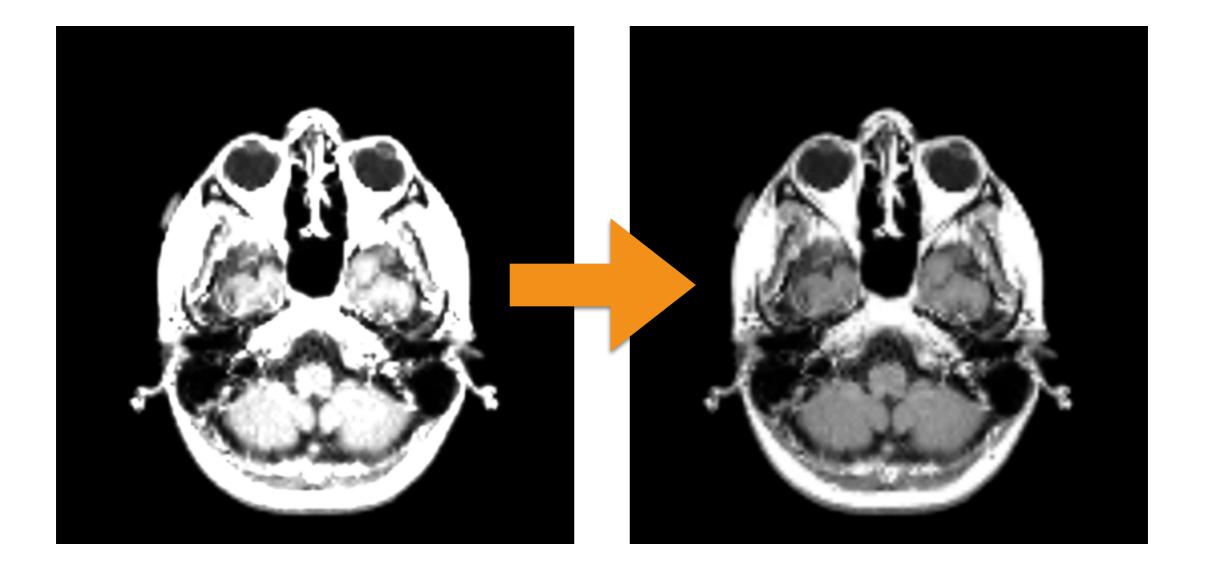


#### Unary Operators: Logarithmic Operator

- The dynamic range may be too large, (16-bit), and most monitors handle only 8-bit!
- The operator is defined as

$$g(x, y) = \log[f(x, y); E_{\min}; E_{\max}] = \\ = (E_{\max} - E_{\min}) \cdot \frac{\log(1 + f(x, y))}{\log(1 + \max(f))} + E_{\min}$$

### Logarithmic Example



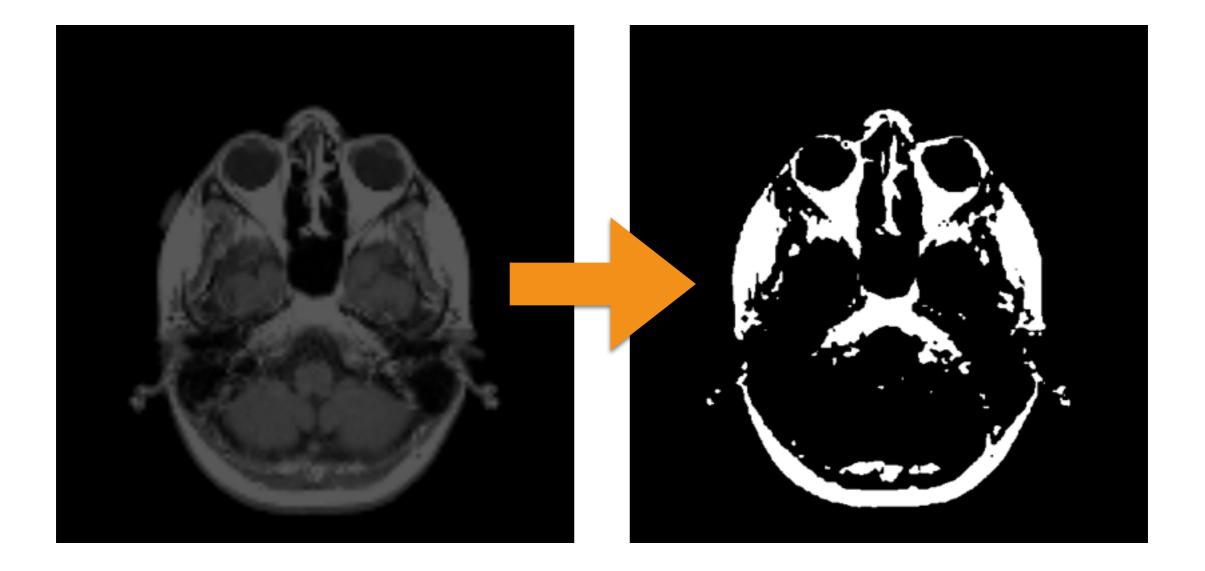
#### Unary Operators: Thresholding

• This operator creates a mask (0 or 1 values):

$$g(x,y) = \operatorname{Thr}[f(x,y);a;b] = \begin{cases} 1 & \text{if } f(x,y) \in [a,b], \\ 0 & \text{otherwise.} \end{cases}$$

• It can be used for segmentation.

### Thresholding Example



# Binary Operators

Binary operators are typically the classic arithmetic operators defined over images:

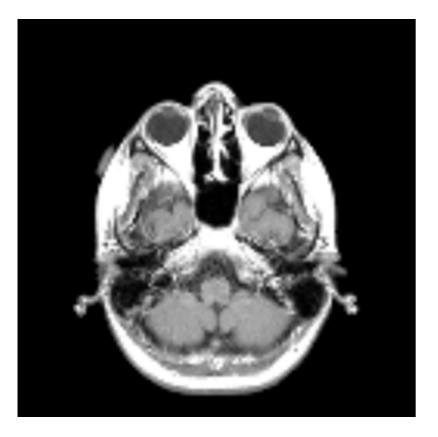
- Note that using + and \*, we can increase the dynamic range and obtaining values in [0,2]:
  - Logarithmic operator
  - Linear scaling in [0,1]

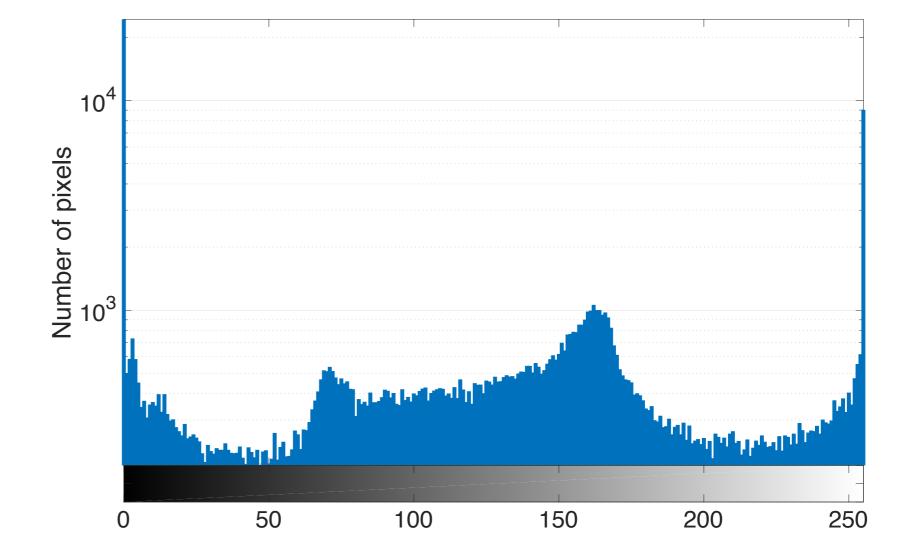
Histograms

# Image Histogram

- The distribution of intensity pixel values.
  - This can be seen as the probability of a pixel of having a given intensity value.
- How to compute?
  - For each unique intensity value J:
    - Count how many pixels have J as intensity
  - MATLAB: **imhist** built-in function

### Example

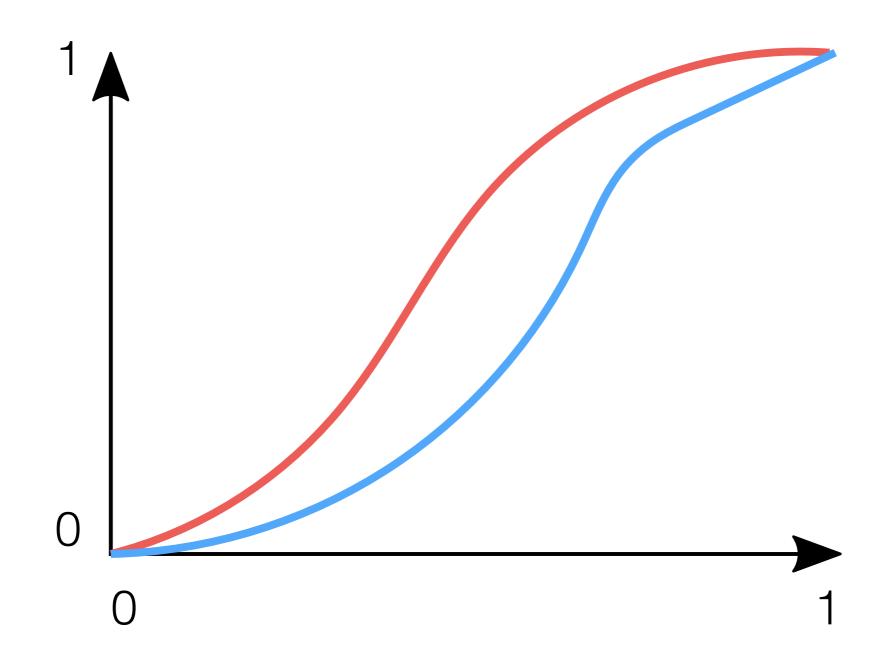


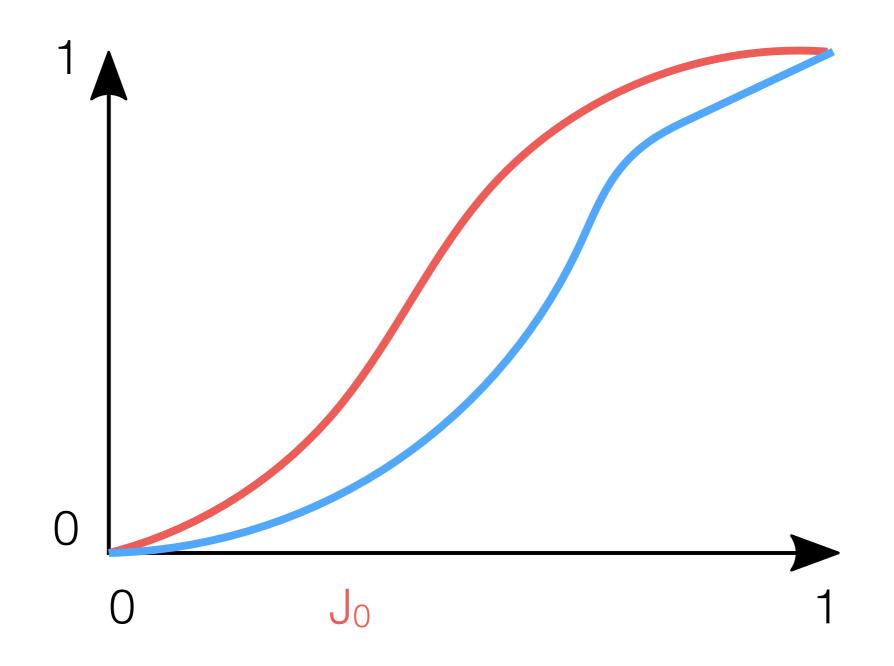


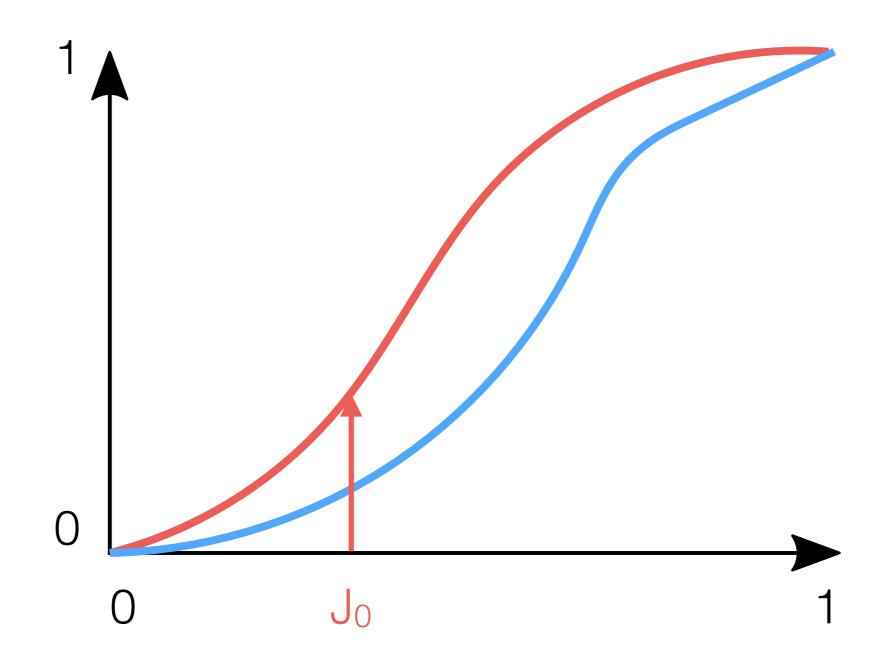
Each intensity value is called "bin"

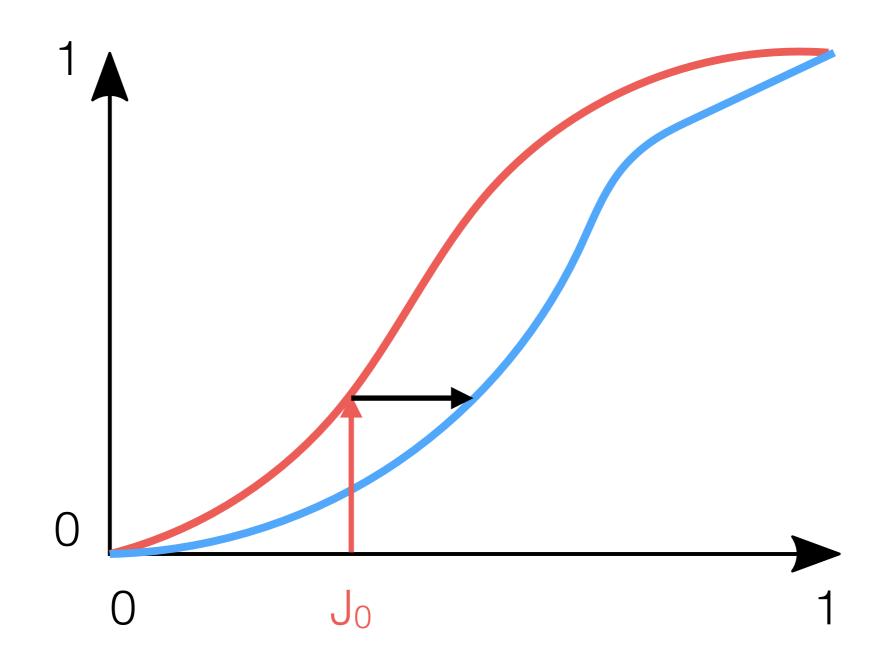
# Histogram Equalization

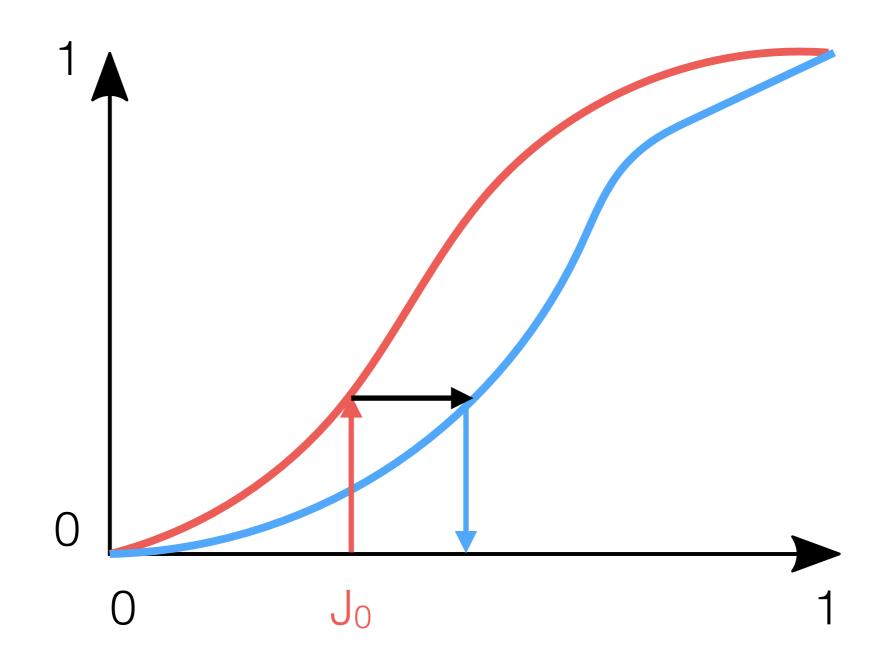
- A technique to improve automatically the contrast of the image.
- The main idea is to have an histogram in which each intensity value J (or bin) has the **same** (more or less) number of pixels:
  - $H[J] = n_{pixels_image} / 2^{n_{bit}}$
- How? Matching the CDF (cumulative distribution function) of the histogram with the CDF of a uniform histogram.
  - This uniform CDF  $\longrightarrow y(x) = x$  with x in [0,1]

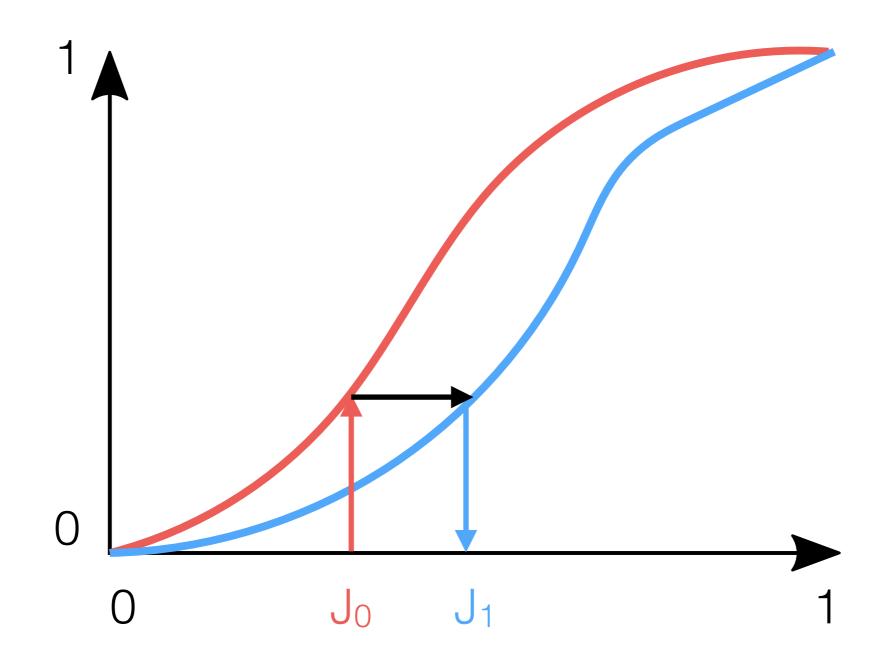


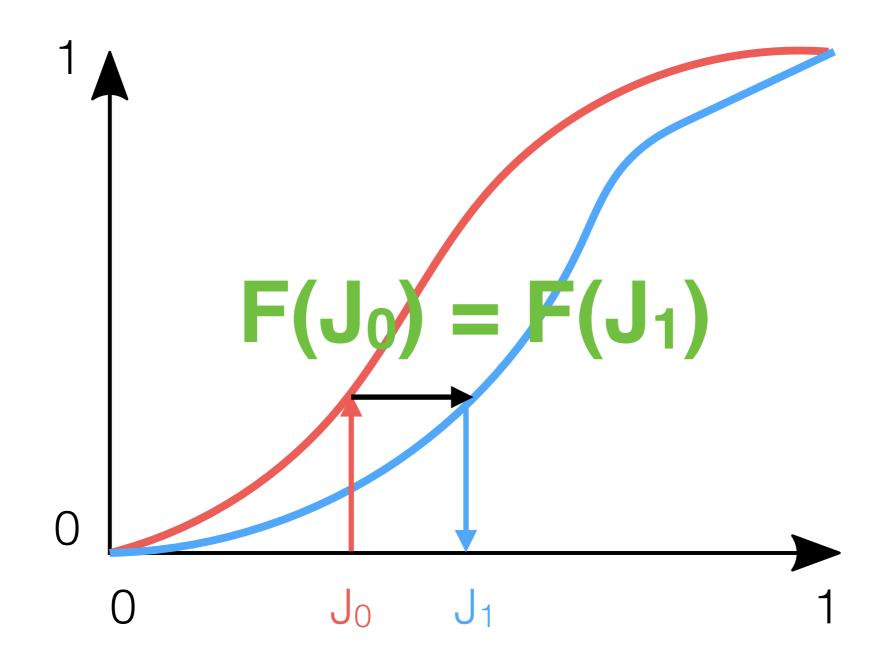


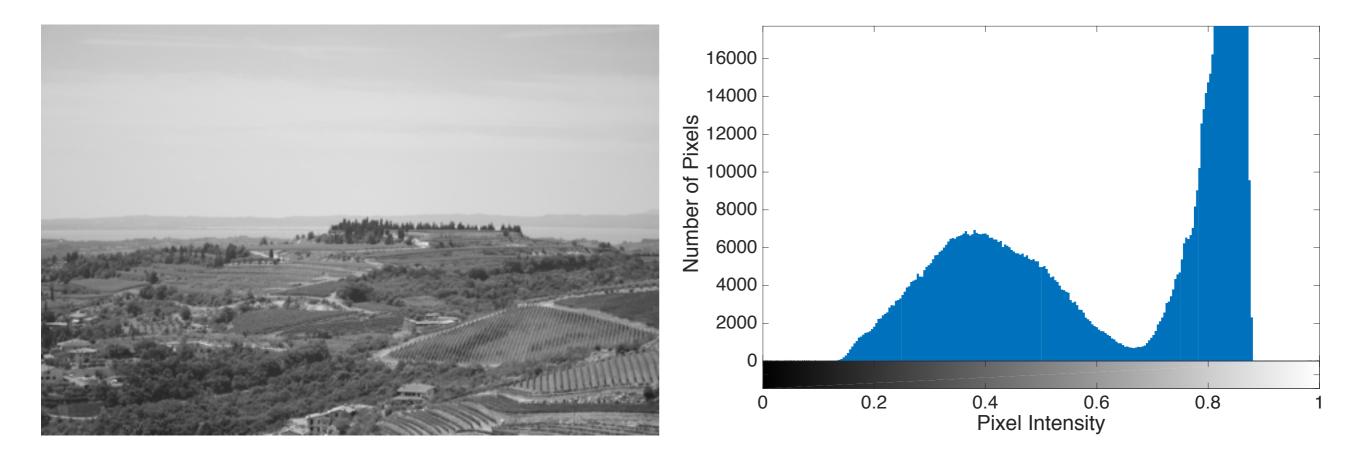


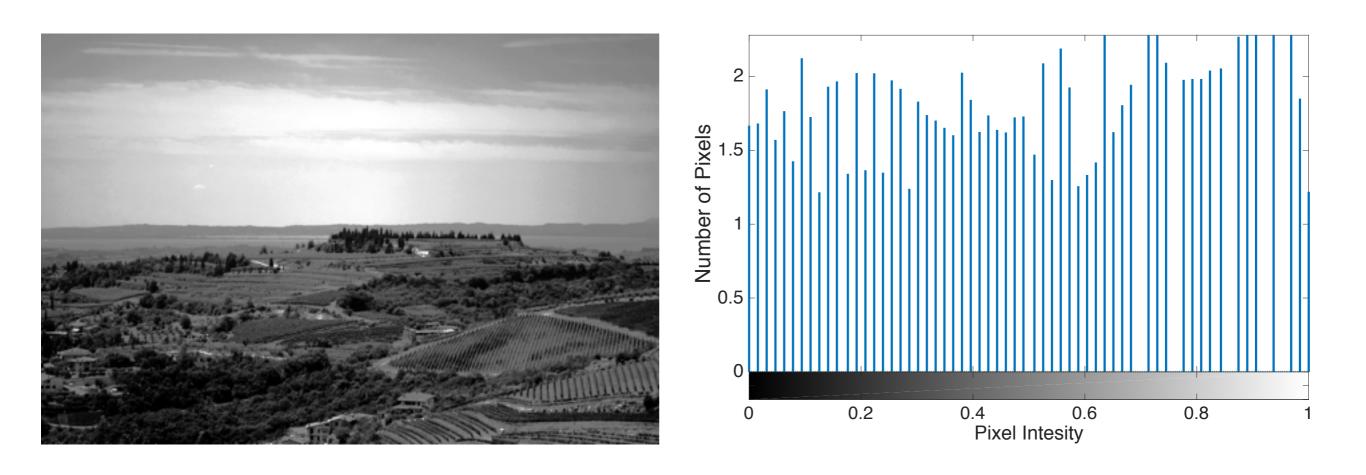


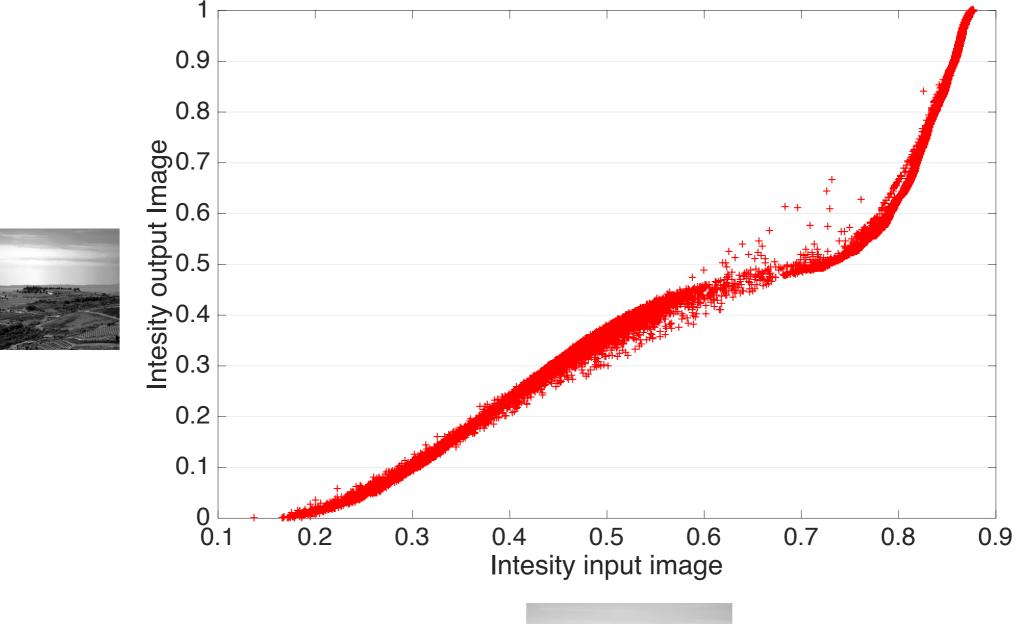




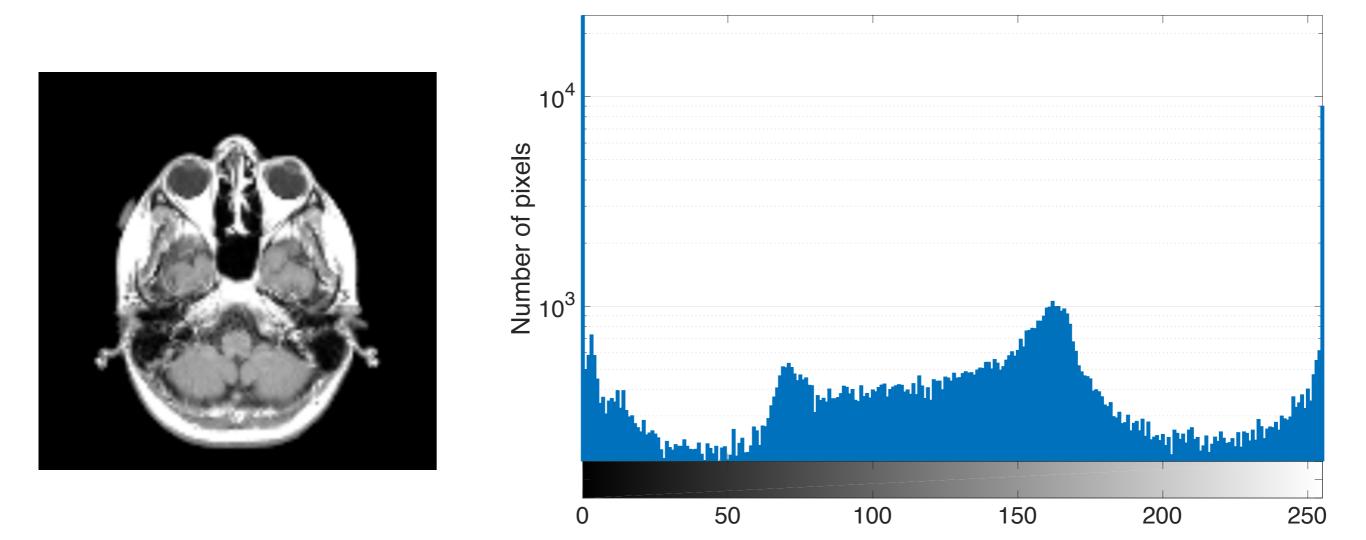












ROI helps in cases of huge peaks (see I=0)

#### Linear Filters

### 1D Convolution

 Given two functions f and g, f convolved g is defined as

$$(f \otimes g)[x] = \int_{-\infty}^{+\infty} f(t) \cdot g(t-x) dx =$$
$$= \int_{-\infty}^{+\infty} f(x-t) \cdot g(x) dx$$

• In the discrete world, this leads to:

$$(f \otimes g)[i] = \sum_{j=-N}^{N} f[i-j] \cdot g[j]$$

### 2D Convolution

• In the 2D world, this leads to:

$$(f \otimes g)[i,j] = \sum_{k=-N}^{N} \sum_{l=-M}^{M} f[i-k,j-l] \cdot g[k,l]$$

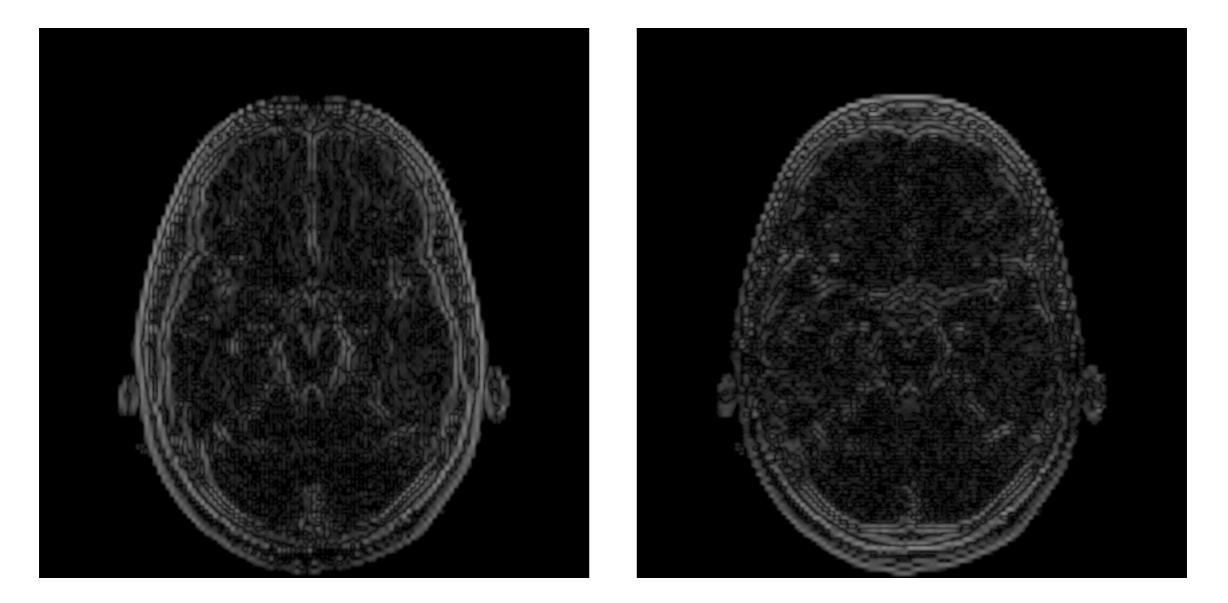
- where g is a (2n)-(2m) matrix, called kernel.
  - For sake of simplicity, let's assume negative addresses!
- MATLAB: conv (1D convolution), and conv2 (2D convolution) built-in functions

### Gradient Filter

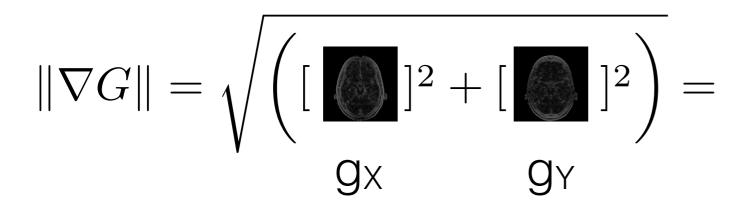
- The gradient of an image is an important piece of information:
  - Where it is high implies we may have an edge; i.e., a boundary between two different regions.
- Typically, kernels for computing gradients are defined as

$$g_X = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad g_Y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

#### Gradient Operator Example

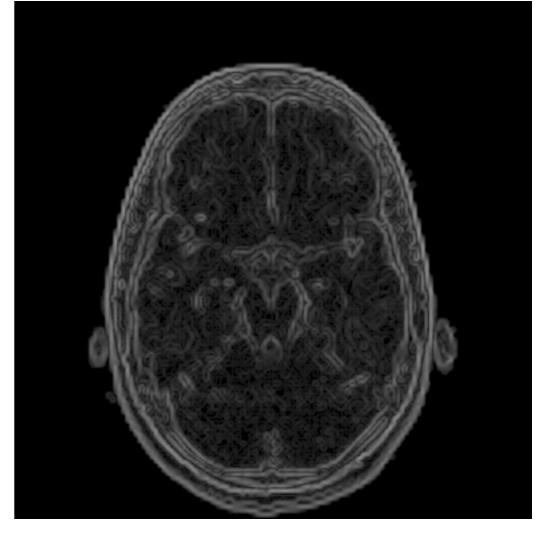


#### Gradient Operator Example



#### Gradient Operator Example

$$\|\nabla G\| = \sqrt{\left( \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}^2 + \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}^2 \right)} = \mathbf{g}_{\mathsf{X}} \qquad \mathbf{g}_{\mathsf{Y}}$$

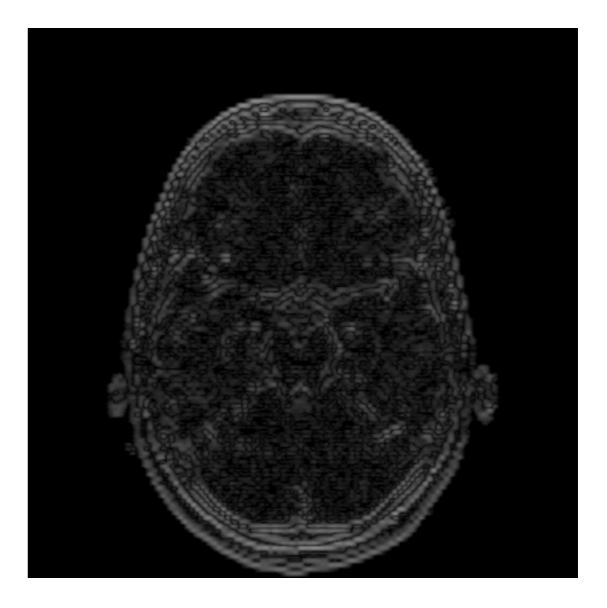


## Sobel Gradient Operator

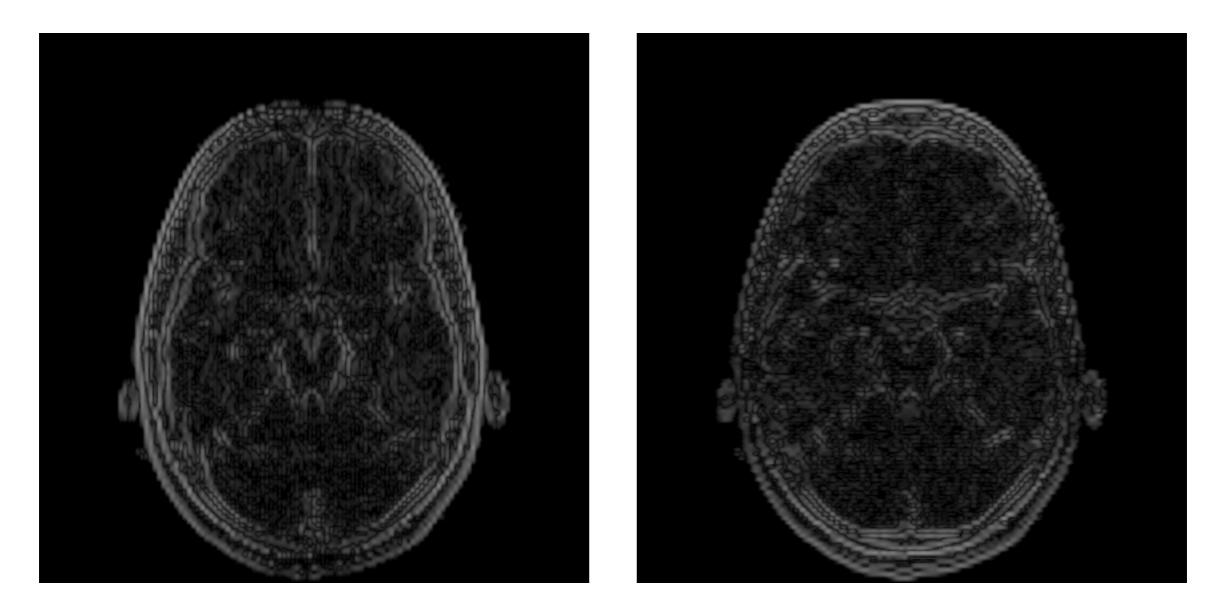
- Technically speaking, it is just another discrete differential operator!
- It emphasizes more edges, which is good.

$$g_X = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad g_Y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & 1 \end{bmatrix}$$

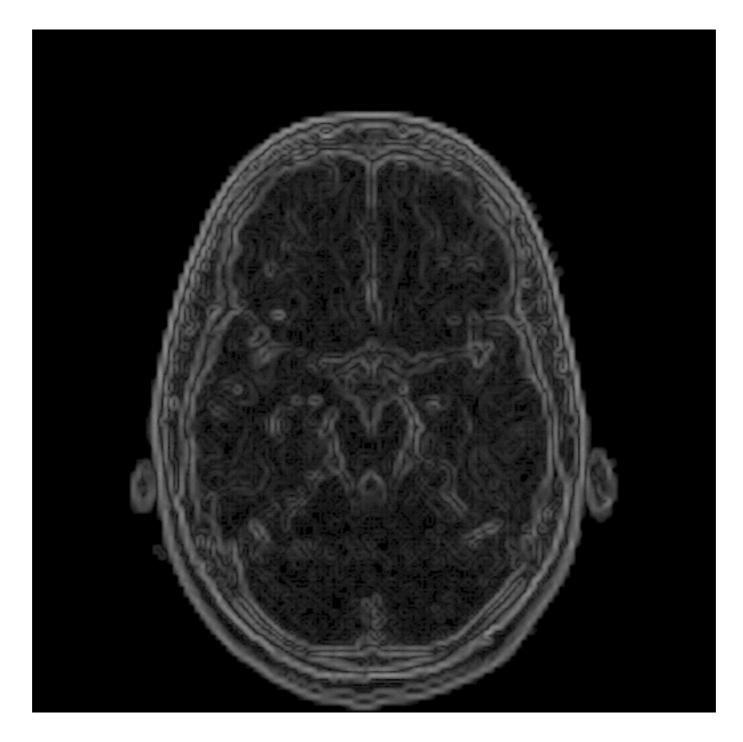
#### Sobel Gradient Operator Example



#### Sobel Gradient Operator Example



#### Sobel Gradient Operator Example



## Edge Detectors

- Edges are can be helpful for defining regions
- They help in the visualization of what we want to segment

# Edge Detectors

- Steps:
  - Compute gradients (magnitude and angle of orientation [atan2])
  - Non-maximum suppression —> remove low power stuff
  - Apply double thresholding; classification: strong, weak, and no edge
  - Edge tracking; a weak edge is a strong one if it is connected to a strong edge!

# Edge Detector Example



thr = 0.001

thr = 0.01

thr = 0.1

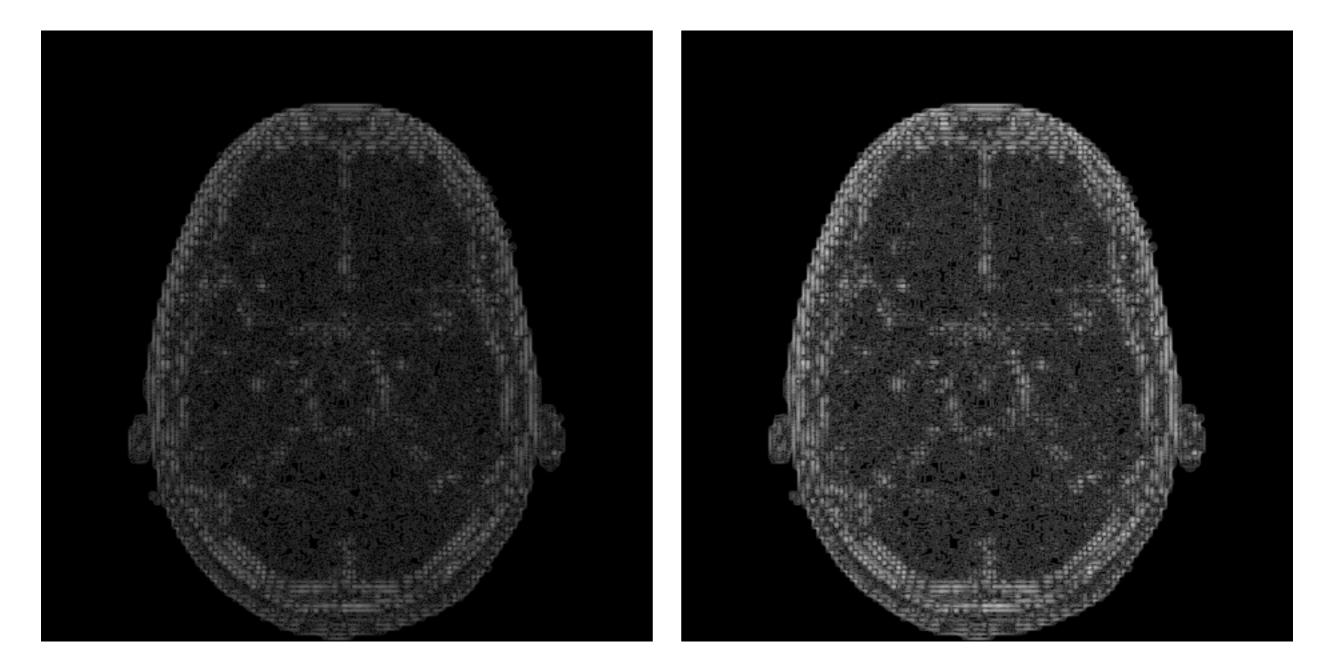
## Laplacian Filter

- If you really want... we can also define a Laplacian operator... Why?
  - The Laplacian of an image highlights regions of rapid intensity change and is therefore often used for edge detection
  - oh, jolly good!

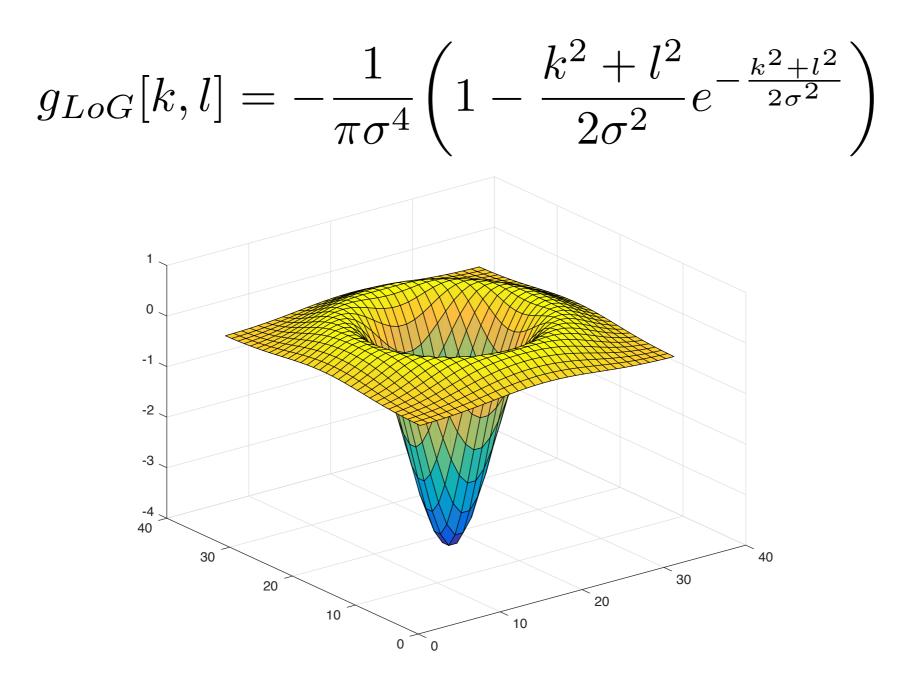
### Laplacian Filter

$$g_{L_4} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad g_{L_8} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

### Laplacian Filter Example



### Laplacian Filter Extra

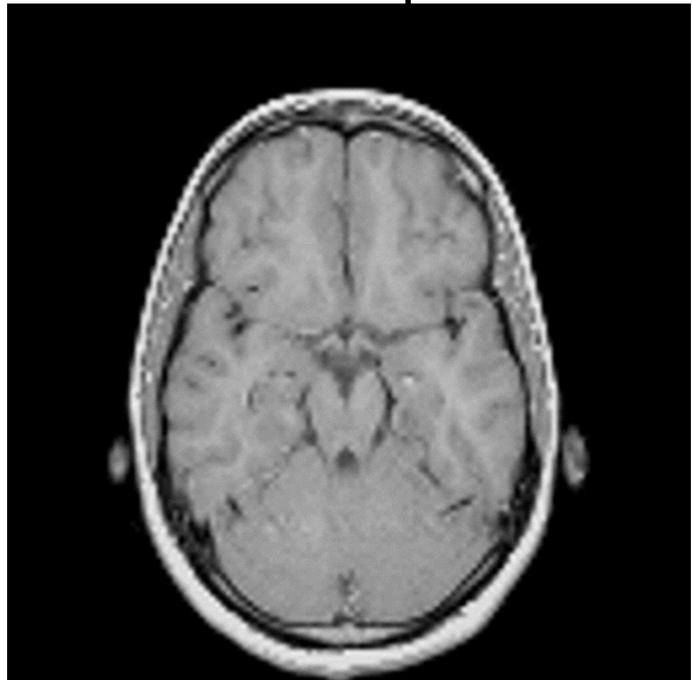


## Laplacian Filter Bonus

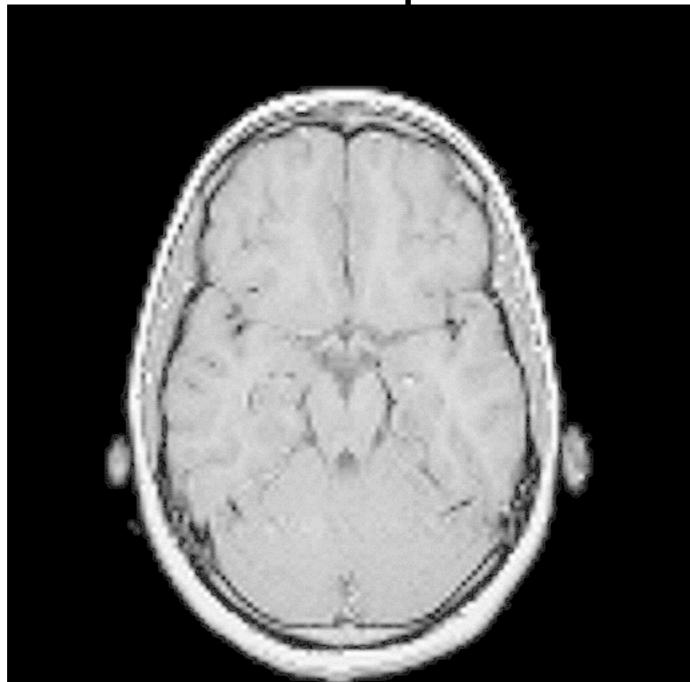
• With a small change (+1) we can increase sharpness in the image:

$$g_{\rm sharp} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

#### Laplacian Filter Bonus Example



#### Laplacian Filter Bonus Example



### Box Filter

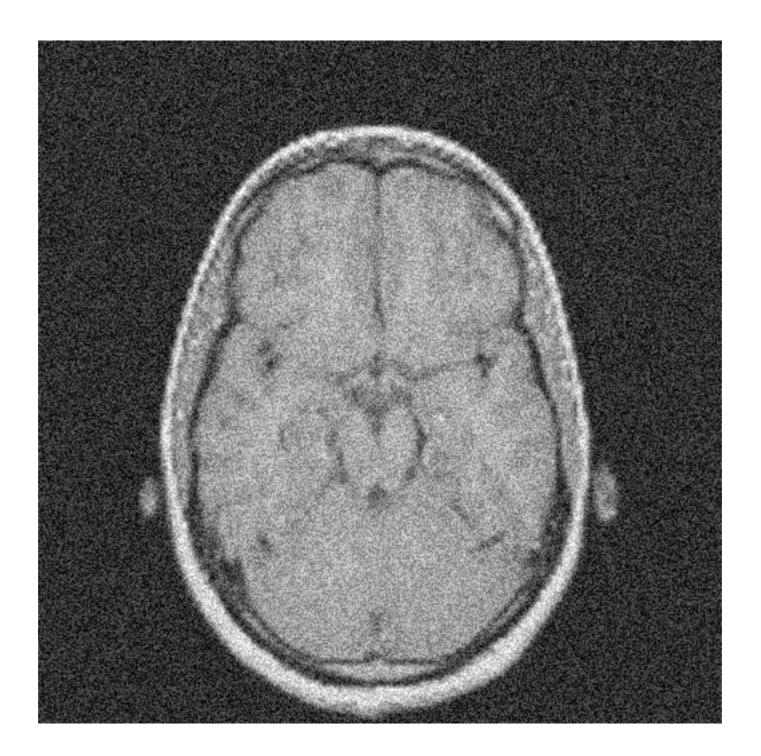
• This is a very simple filter low-pass filter:

$$g[k,l] = 1 \quad \forall k \land \forall l$$

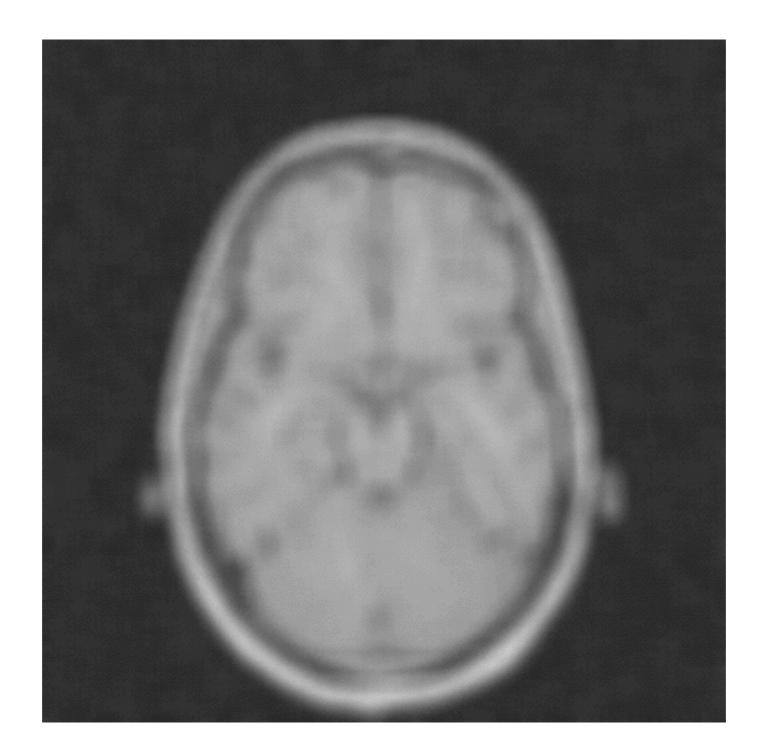
- What does it do? It blurs the signal!
- This kernel has to be normalized:

$$g[k, l] = \frac{g[k, l]}{\sum_{k=-N}^{N} \sum_{l=-M}^{M} g[k, l]}$$

# Box Filter Example



# Box Filter Example



## Gaussian Filter

• We use a Gaussian kernel defined as

$$g[k,l] = G\left(\sqrt{k^2 + l^2}\right)$$

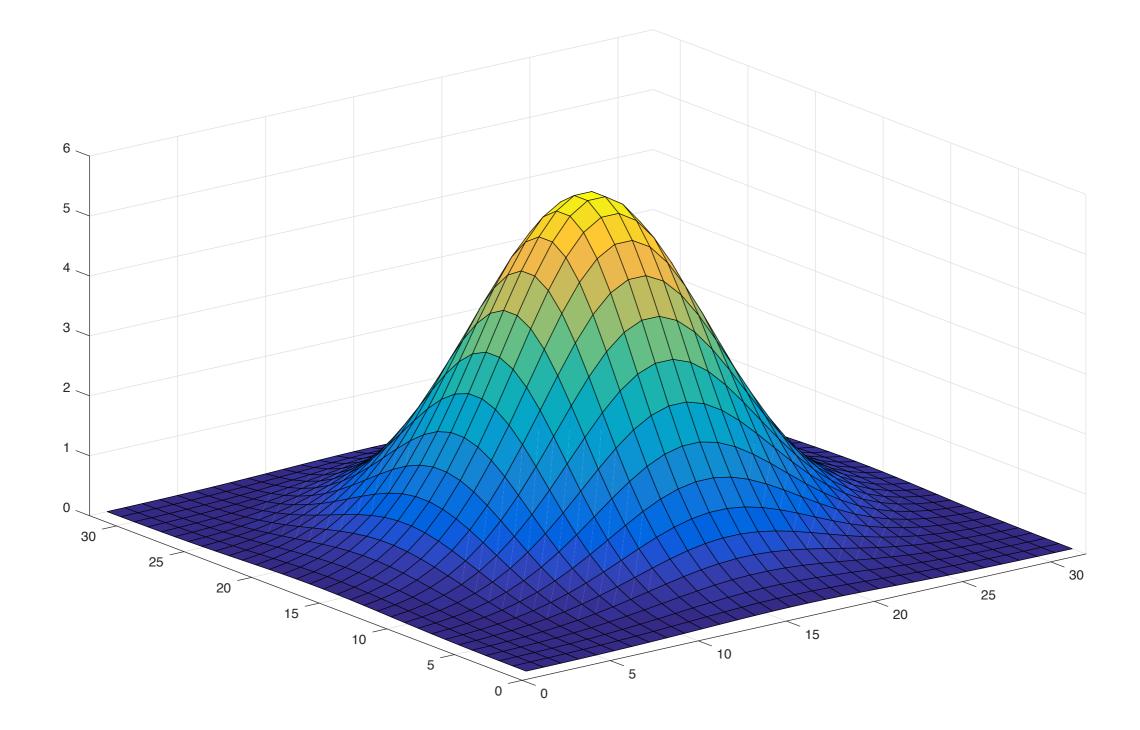
• where G is:

$$G(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

• Note that g has to be normalized:

$$g[k, l] = \frac{g[k, l]}{\sum_{k=-N}^{N} \sum_{l=-M}^{M} g[k, l]}$$

## Gaussian Filter



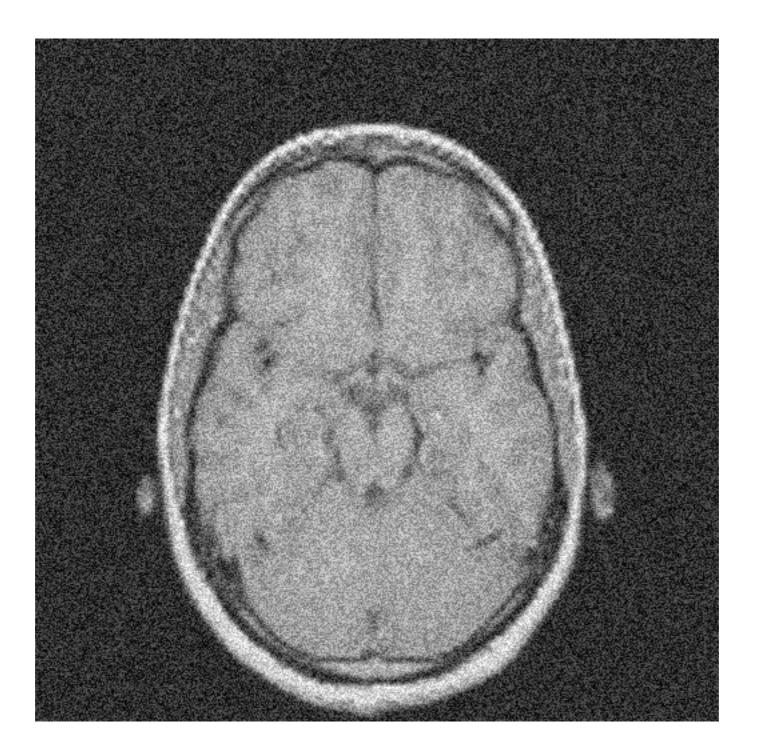
## Gaussian Filter: how large?

- Typically, we have N = M;
- N and M depends on the sigma parameter:

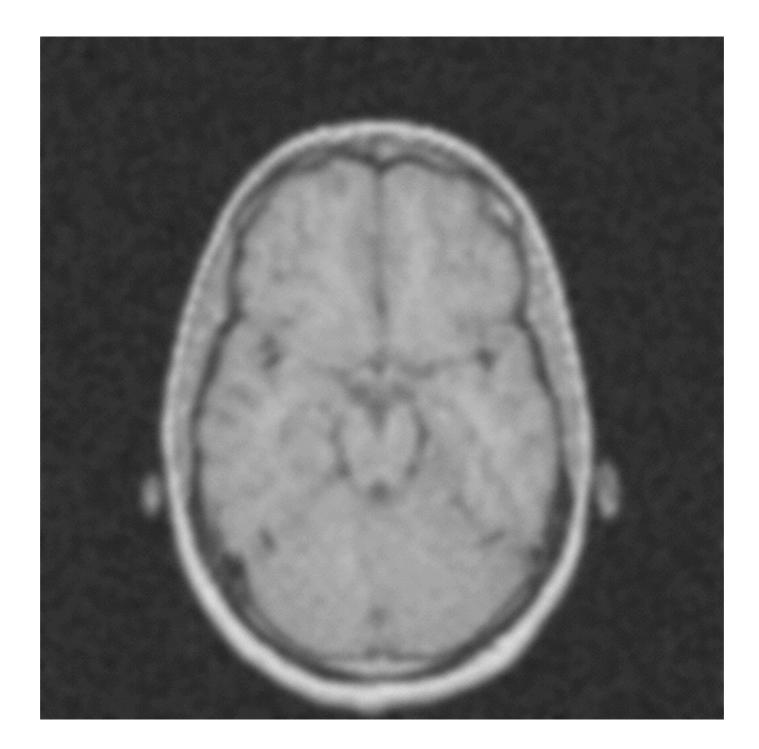
$$N = M = \frac{5}{2} \cdot \sigma \longrightarrow 98\%$$
 of energy

- Larger sigma the better but the slower!
- Note: when sigma is too large (e.g., more than 128 pixels) it is better to work in the Fourier domain!

## Gaussian Filter Example

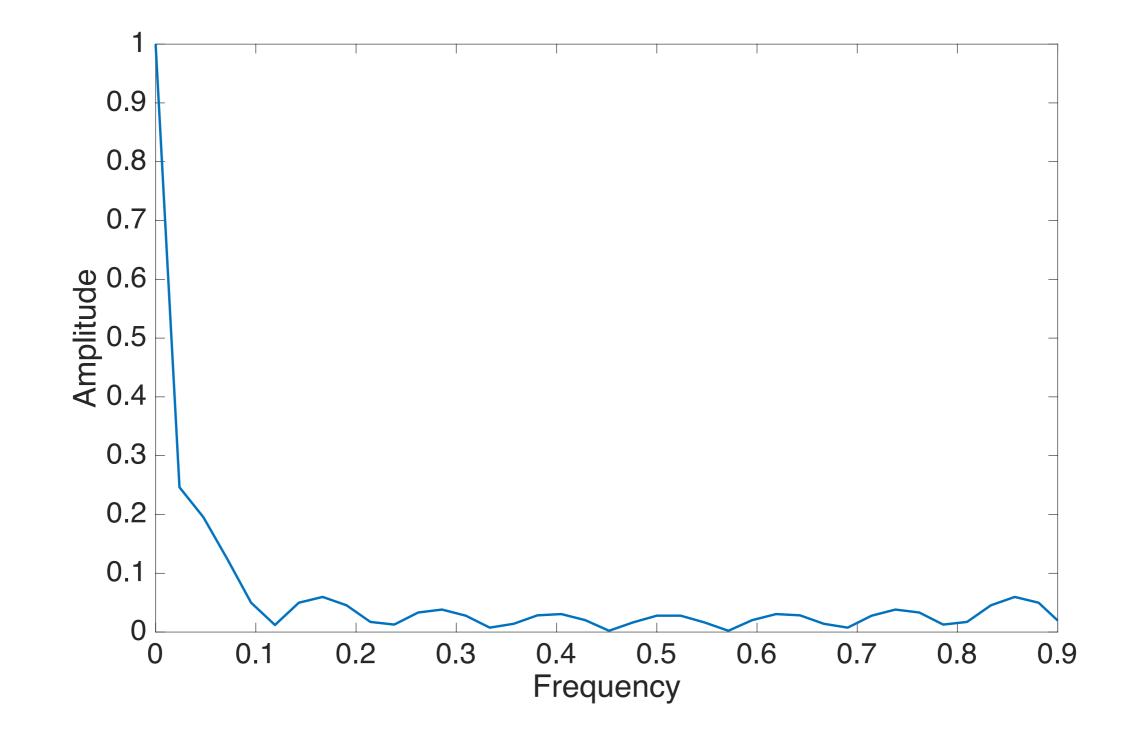


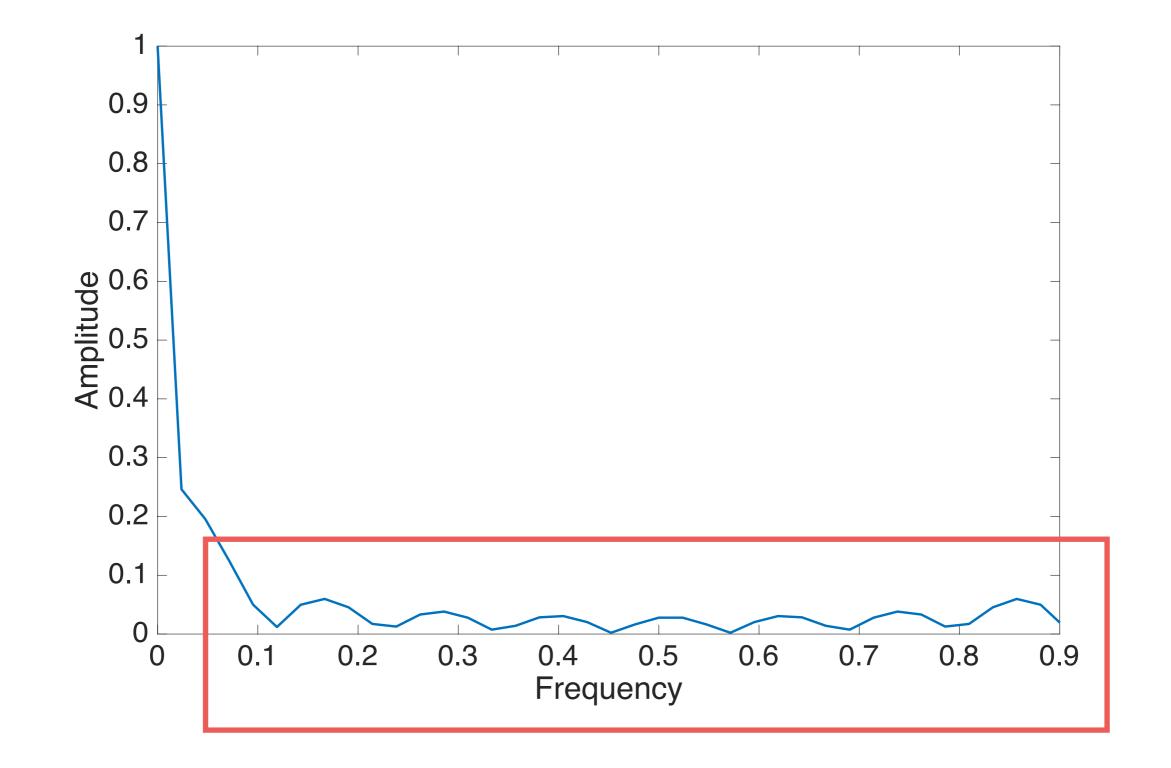
## Gaussian Filter Example

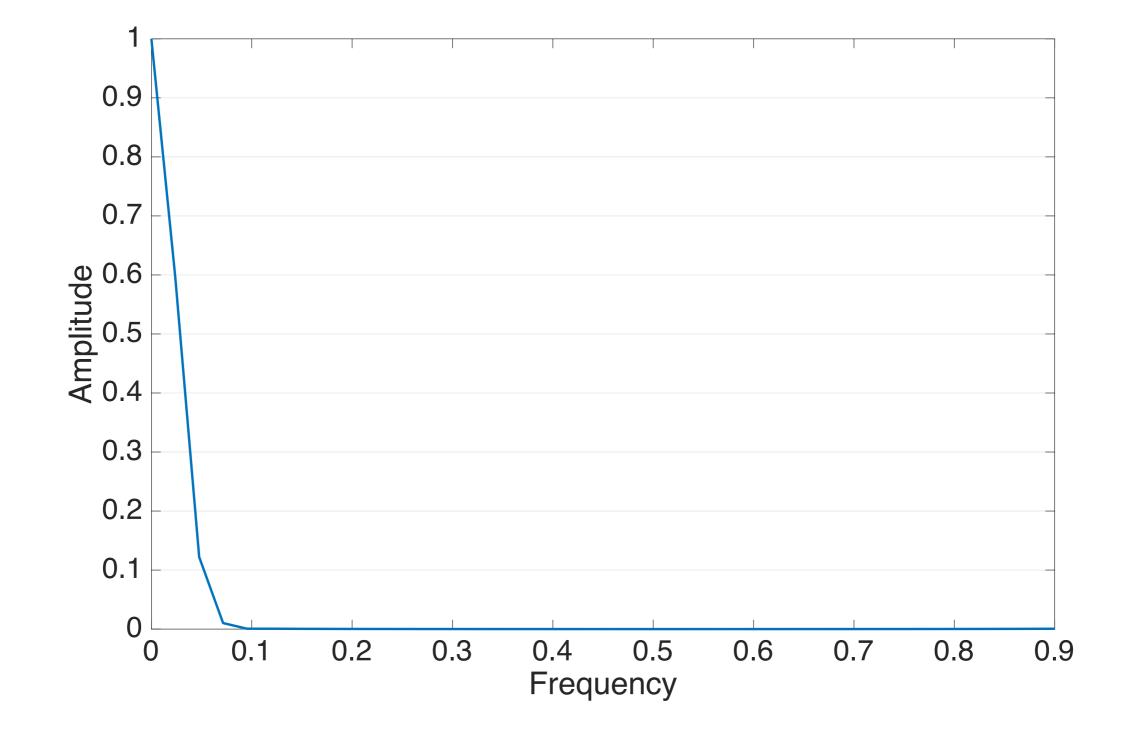


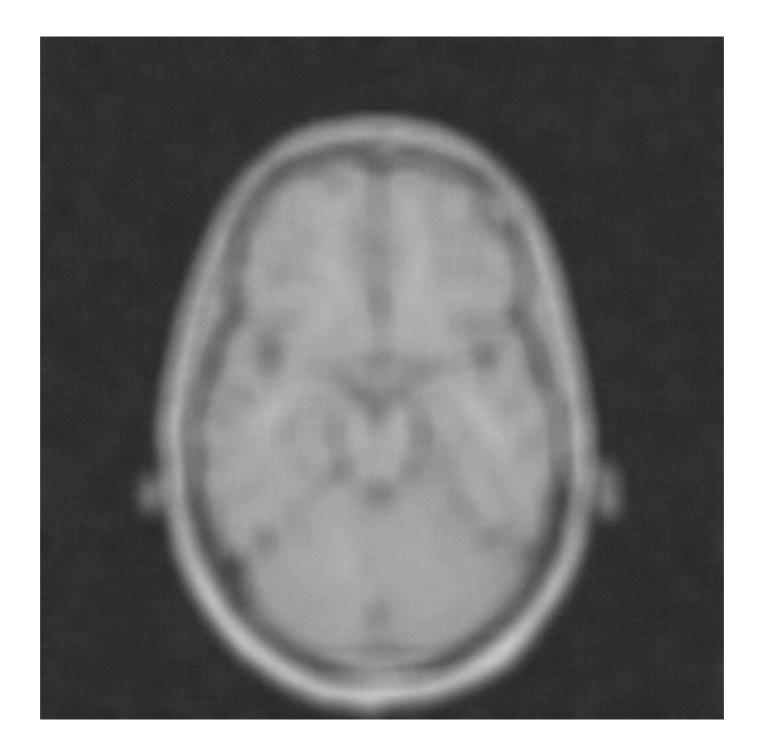
# Box vs Gaussian

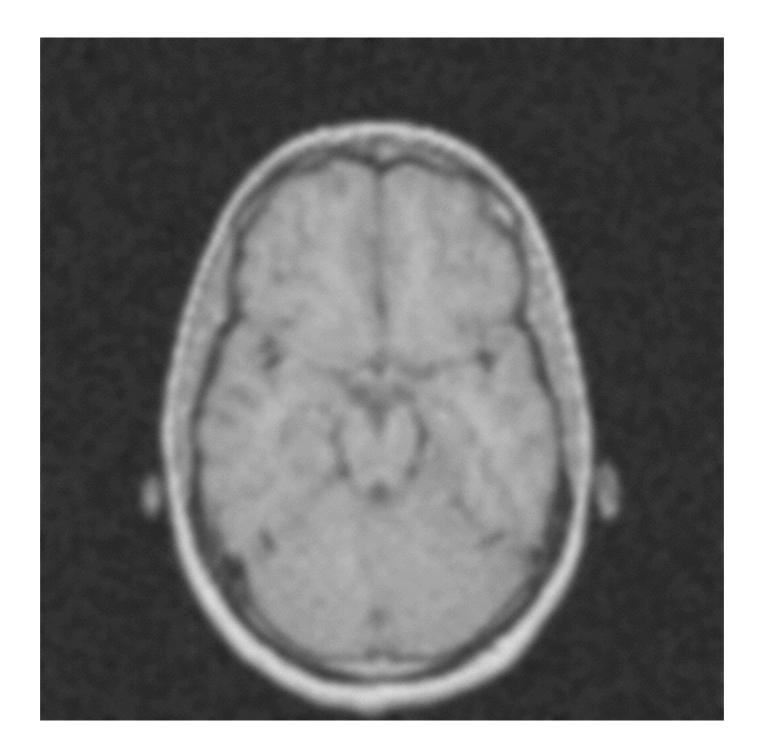
- As you probably know...
- The box filter cuts primarily high frequencies but it has oscillations for some low frequencies.
  - What does it mean? That is BAD!
- The Gaussian filter cuts mostly high frequencies!
  - That is GOOD!





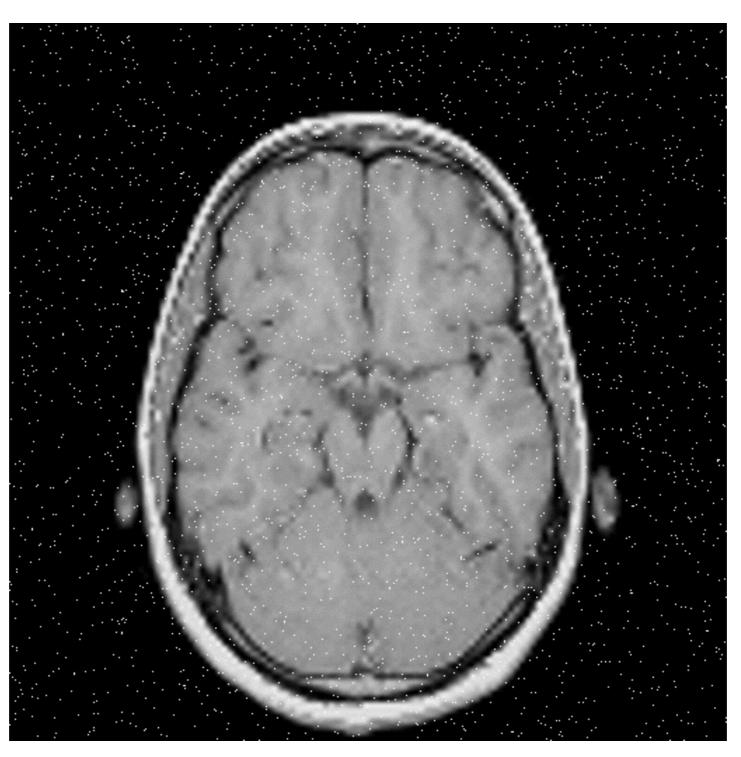




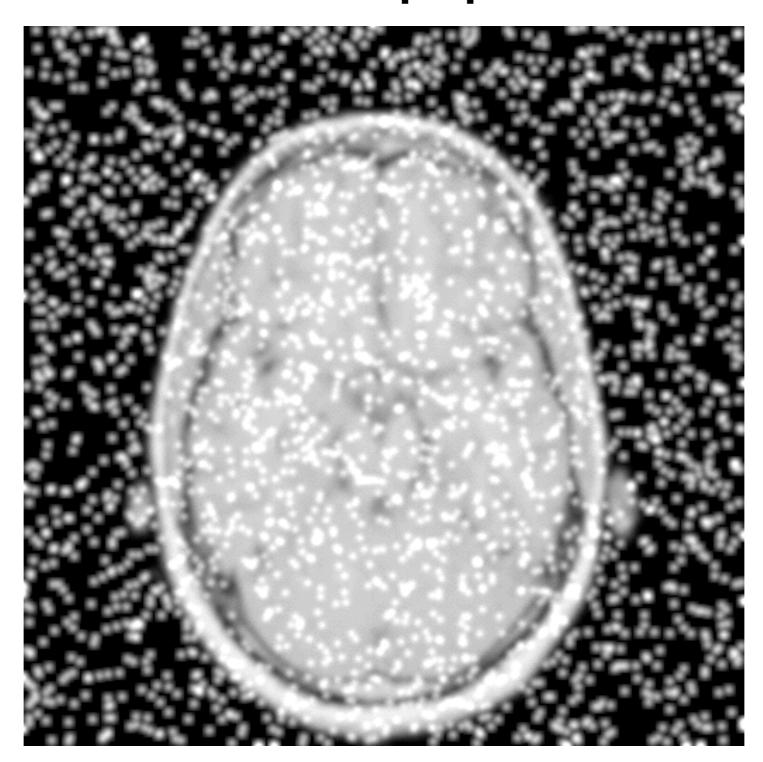


## Non-Linear Filters

# Salt and Pepper Noise



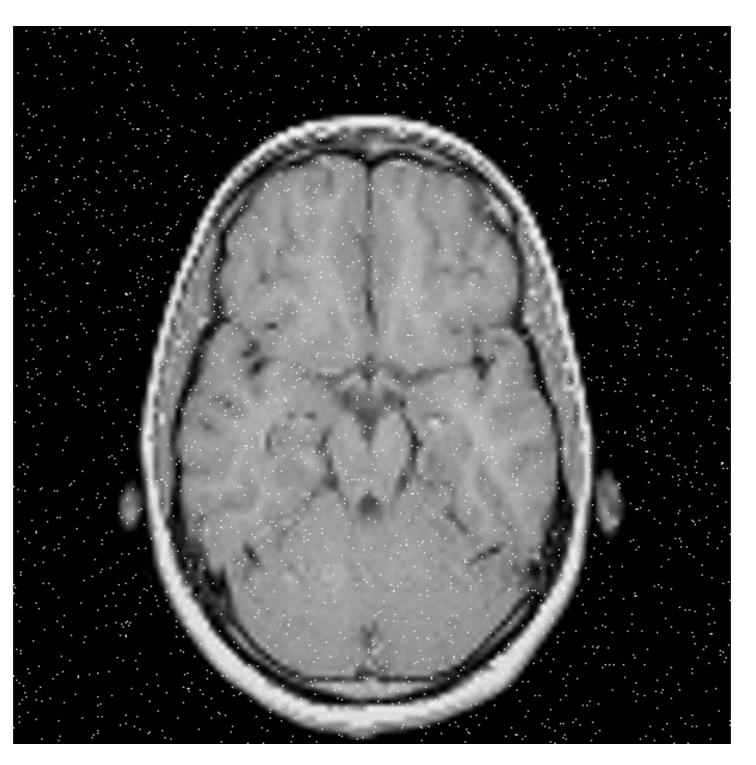
# Salt and Pepper Noise



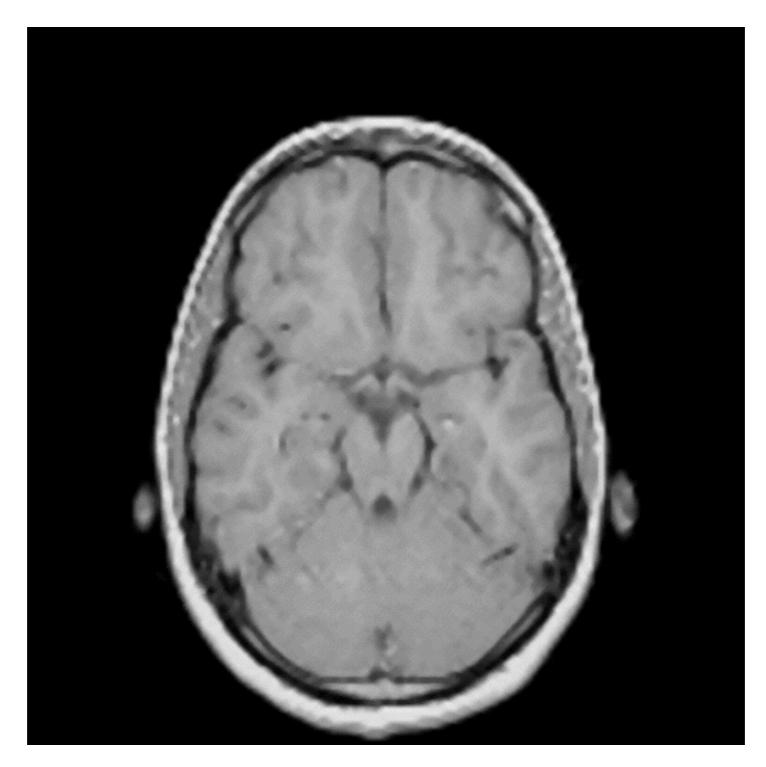
# Median Filter

- This filter is mostly meant for tackling salt-andpepper noise!
  - Linear filters do a mess with salt-and-pepper!
- It exploits the fact that median is robust in separating the higher half of data sample from the lower part! Classist isn't it?

## Median Filter Example



## Median Filter Example



- It is a non-linear filter, oh really?
- It works both spatial domain and intensity/range domain of the image.
- Basically, it is an adaptive linear filter:
  - It behaves as a linear filter in flat regions;
  - At strong edges (step-edge), filtering is "limited".

$$BF[I](\mathbf{x}, f_s, g_r) = \frac{1}{K(\mathbf{x}, f_s, g_r)} \sum_{\mathbf{y} \in \Omega(\mathbf{x})} I(\mathbf{y}) f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$
$$K[I](\mathbf{x}, f_s, g_r) = \sum_{\mathbf{y} \in \Omega(\mathbf{x})} f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$

**Spatial Function** 

$$BF[I](\mathbf{x}, f_s, g_r) = \frac{1}{K(\mathbf{x}, f_s, g_r)} \sum_{\mathbf{y} \in \Omega(\mathbf{x})} I(\mathbf{y}) f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$
$$K[I](\mathbf{x}, f_s, g_r) = \sum_{\mathbf{y} \in \Omega(\mathbf{x})} f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$

$$BF[I](\mathbf{x}, f_s, g_r) = \frac{1}{K(\mathbf{x}, f_s, g_r)} \sum_{\mathbf{y} \in \Omega(\mathbf{x})} I(\mathbf{y}) f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$
$$K[I](\mathbf{x}, f_s, g_r) = \sum_{\mathbf{y} \in \Omega(\mathbf{x})} f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$

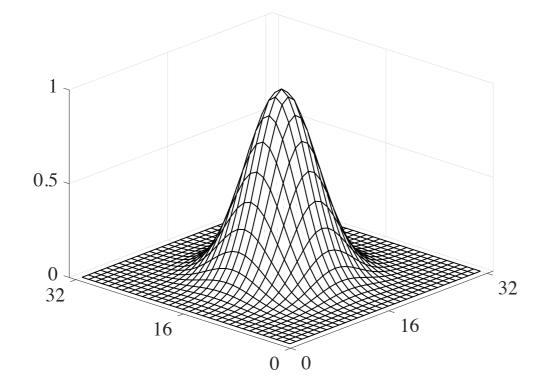
**Range Function** 

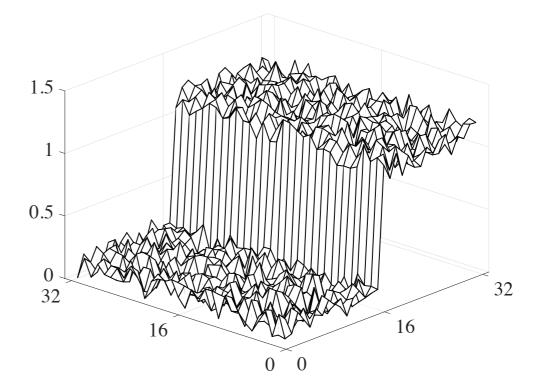
$$BF[I](\mathbf{x}, f_s, g_r) = \frac{1}{K(\mathbf{x}, f_s, g_r)} \sum_{\mathbf{y} \in \Omega(\mathbf{x})} I(\mathbf{y}) f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$
$$K[I](\mathbf{x}, f_s, g_r) = \sum_{\mathbf{y} \in \Omega(\mathbf{x})} f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$

$$BF[I](\mathbf{x}, f_s, g_r) = \frac{1}{K(\mathbf{x}, f_s, g_r)} \sum_{\mathbf{y} \in \Omega(\mathbf{x})} I(\mathbf{y}) f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$
$$K[I](\mathbf{x}, f_s, g_r) = \sum_{\mathbf{y} \in \Omega(\mathbf{x})} f_s(\|\mathbf{x} - \mathbf{y}\|) g_r(\|I(\mathbf{y}) - I(\mathbf{x})\|),$$

- Spatial function: a Gaussian function
- Range function: a Gaussian function
- How large is the kernel?
  - If the spatial function is a Gaussian:

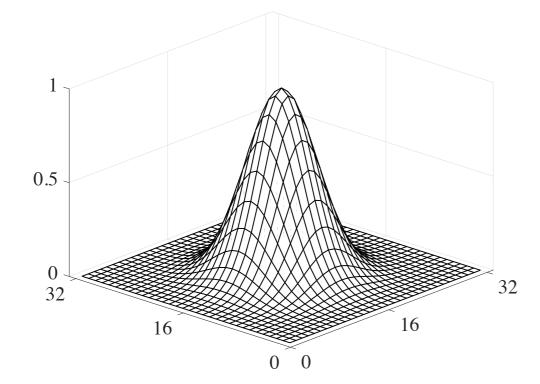
$$N = M = \frac{5}{2}\sigma_s$$

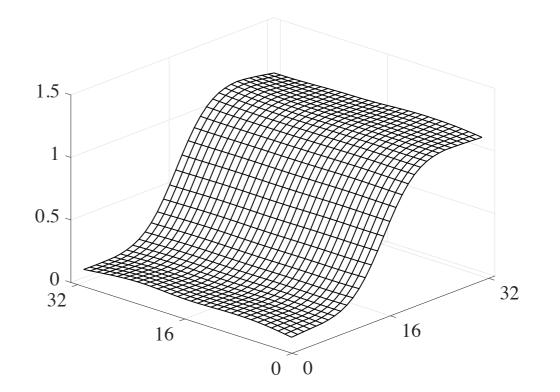




Kernel

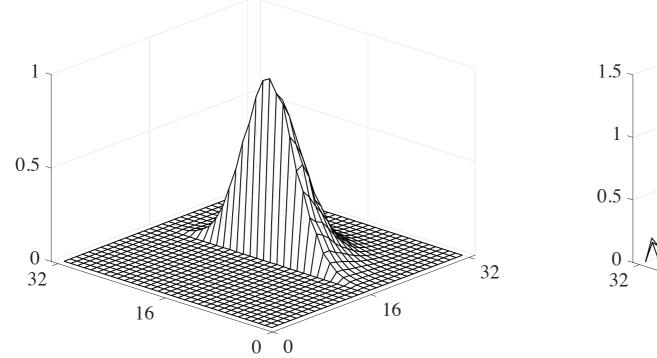
Image

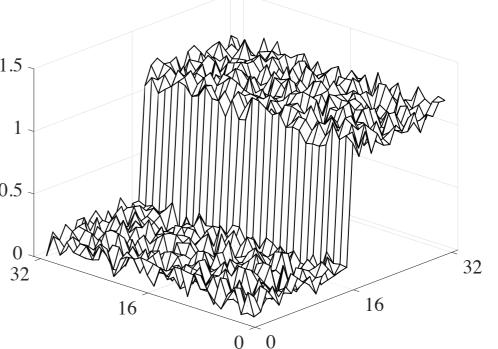




Kernel

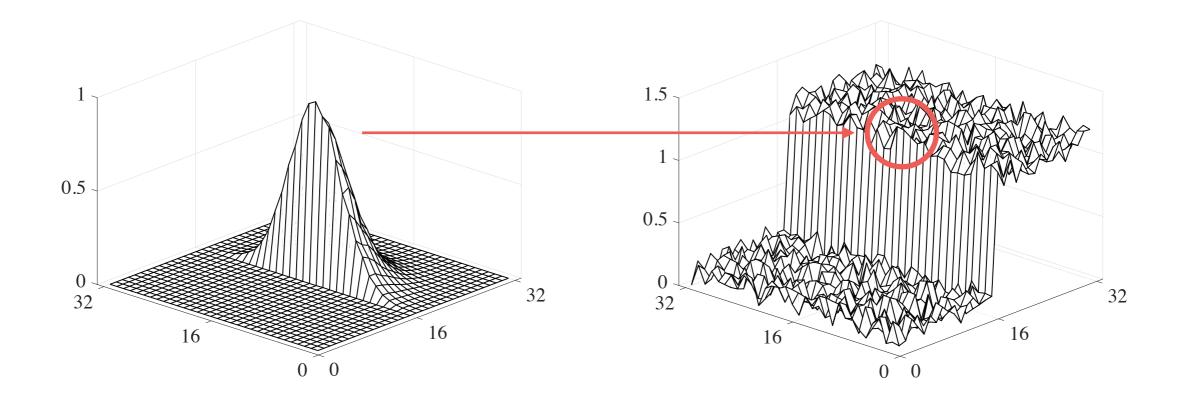
Image





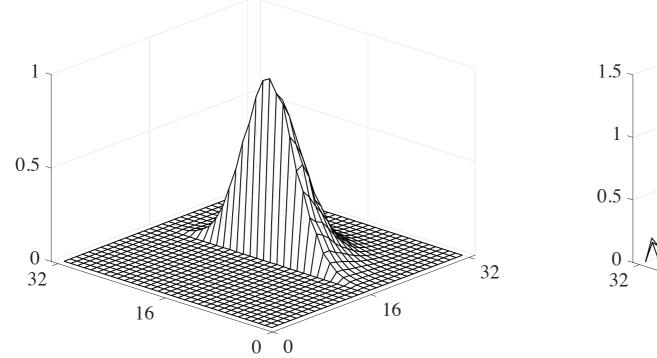
Kernel (change for each pixel!!)

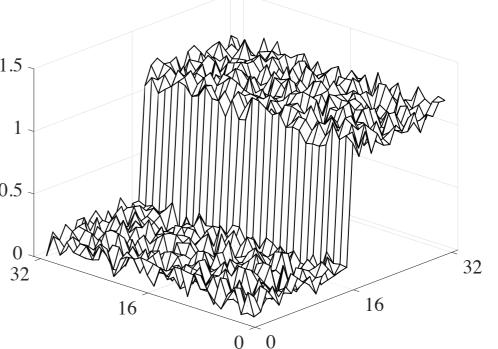
Image



Kernel (change for each pixel!!)

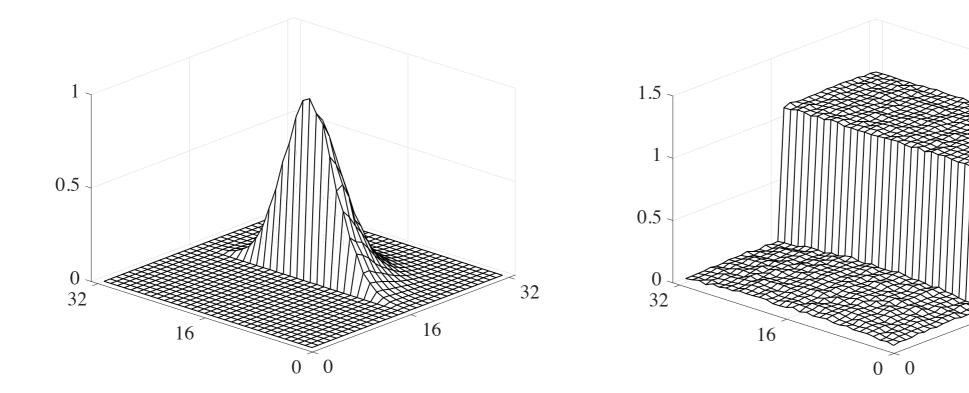
Image





Kernel (change for each pixel!!)

Image

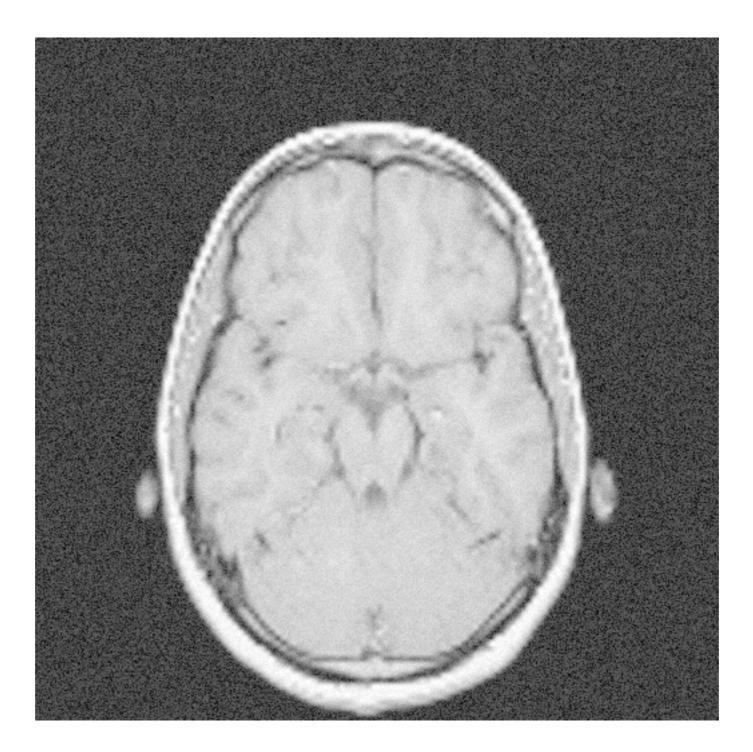


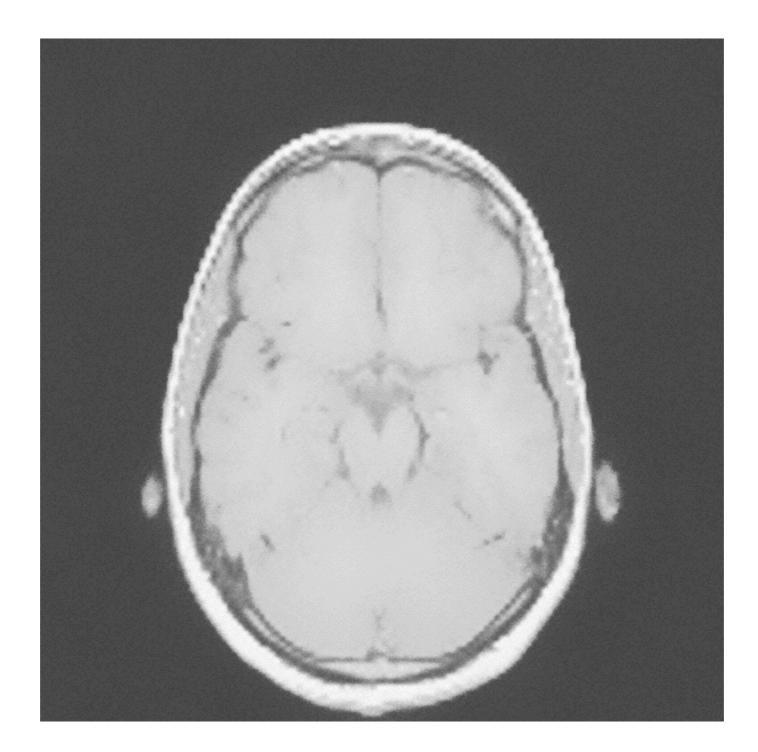
Kernel (change for each pixel!!)

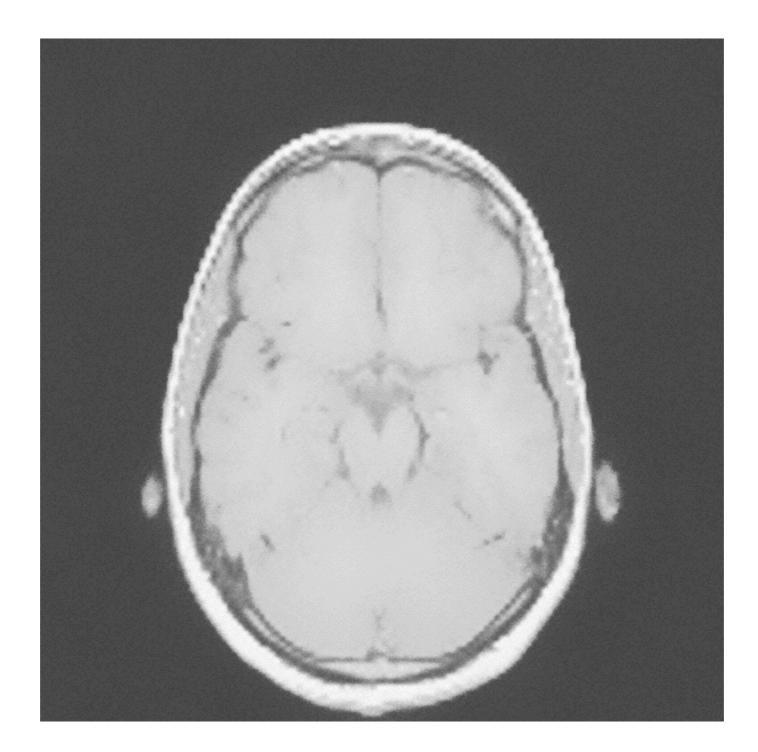
Image

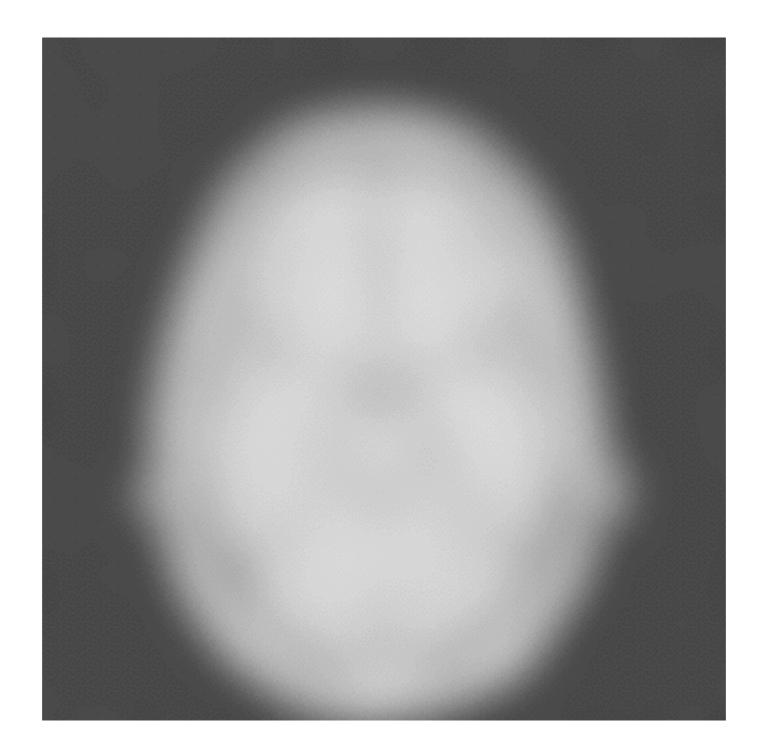
32

16







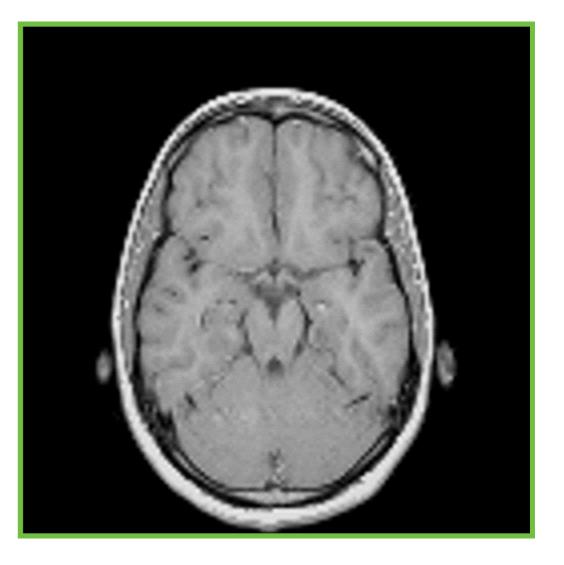


#### Local Contrast Enhancement

- Before, we have seen how to increase local contrast using the sharpening operator.
- We can achieve better results using a more general framework!

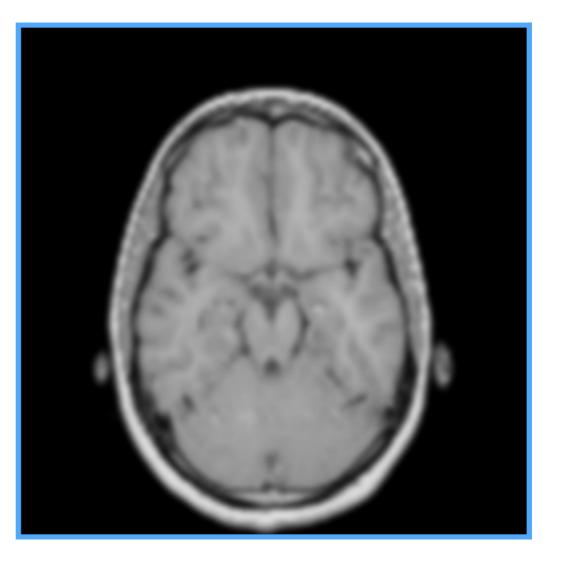
$$O[i,j] = f[i,j] \cdot \left(\frac{f[i,j]}{(f \otimes g)[i,j]}\right)$$

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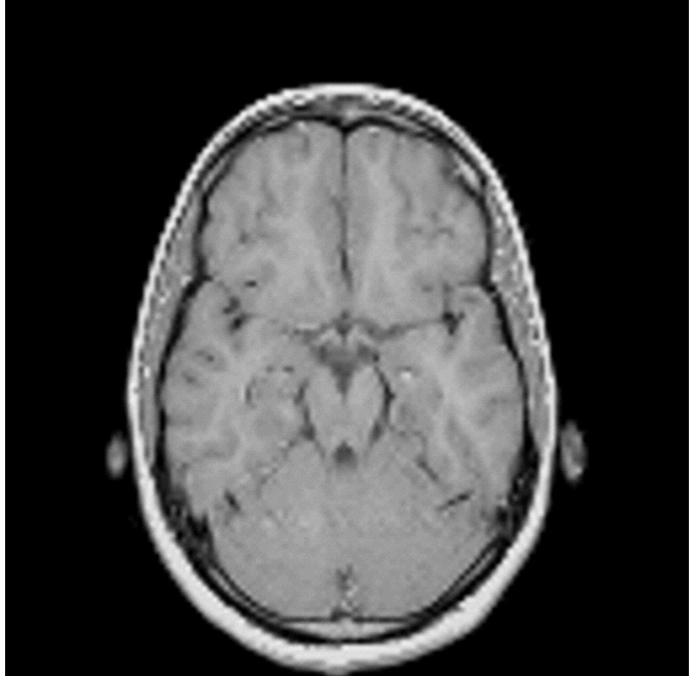
$$O[i,j] = f[i,j] \cdot \left( \frac{f[i,j]}{(f \otimes g)[i,j]} \right)$$

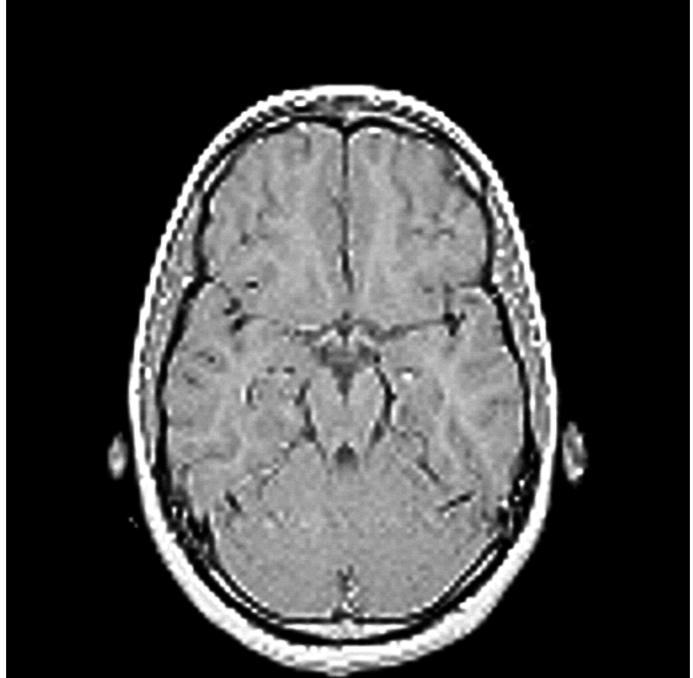


$$O[i,j] = f[i,j] \cdot \left(\frac{f[i,j]}{(f \otimes g)[i,j]}\right)$$

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### Bonus: Deconvolution

$$\begin{cases} I_0 = 0.5 \\ I_{i+1} = I_i \cdot \left( \frac{J}{I_i \otimes K} \otimes K^\top \right) \end{cases}$$

Richardson–Lucy deconvolution

### Bonus: Deconvolution

$$\begin{cases} I_0 = 0.5 \text{ Input Blurred Image} \\ I_{i+1} = I_i \cdot \left( \underbrace{J}_{I_i \otimes K} \otimes K^\top \right) \end{cases}$$

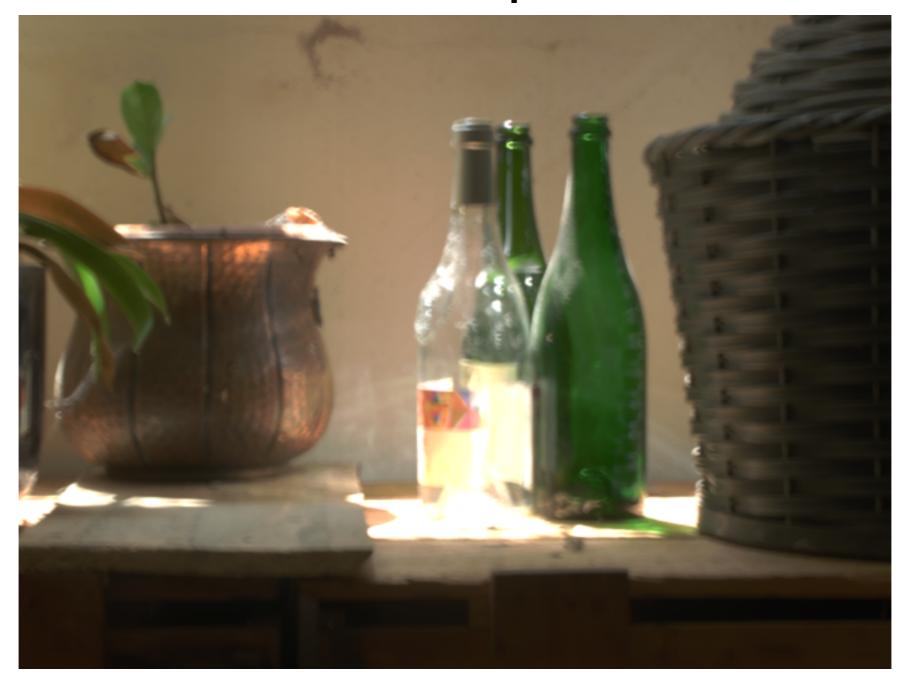
Richardson–Lucy deconvolution

### Bonus: Deconvolution

$$\begin{cases} I_0 = 0.5 \\ I_{i+1} = I_i \cdot \left( \frac{J}{I_i \otimes K} \otimes K^\top \right) \end{cases}$$

Richardson–Lucy deconvolution

#### Bonus: Deconvolution Example



#### Bonus: Deconvolution Example



#### Local Contrast Enhancement

- When using linear filters we may introduce halos!
  - halos —> BIAS!
- It is better to use non-linear filters such as the bilateral filter, the guided filter, WLS, etc.

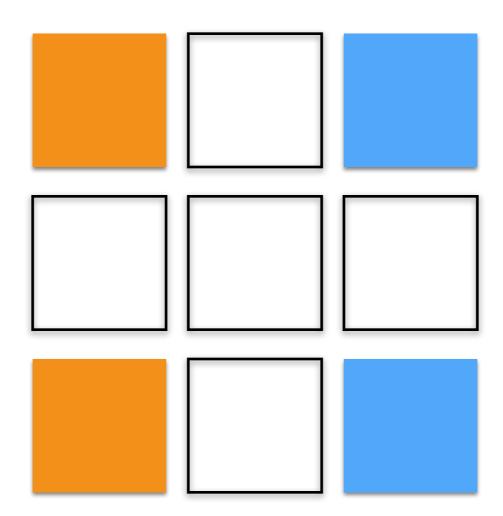
# Image Upsampling

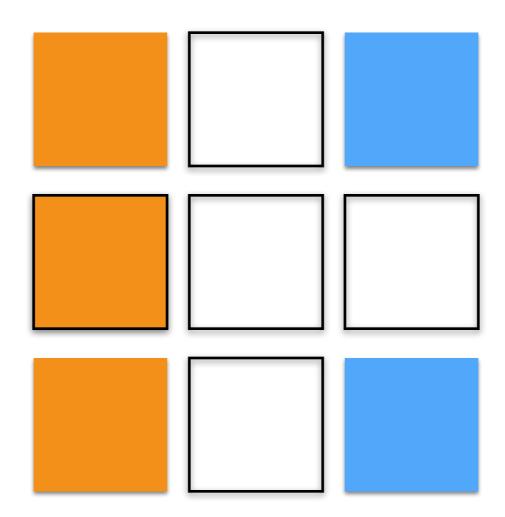
# Why Upsampling?

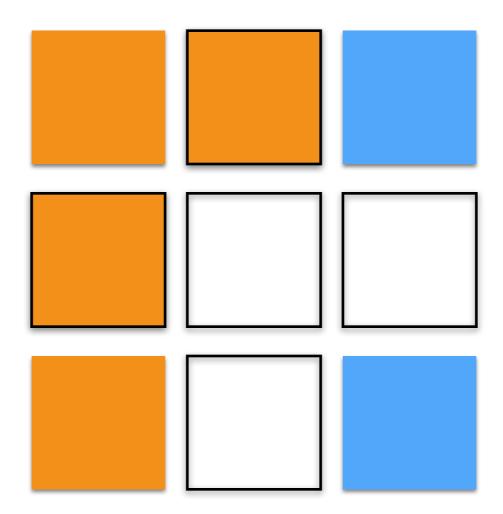
- The main reason why we want to upsample (we invent data basically) our input data is that they have a very low resolution
- Forget 4K for your flicks, we have 512x512 resolution in happy days

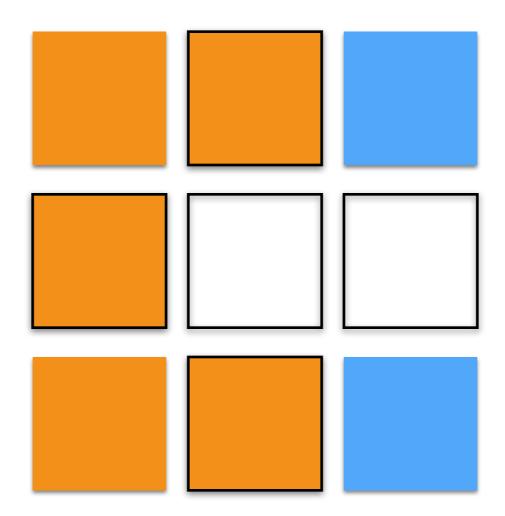
# Upsampling

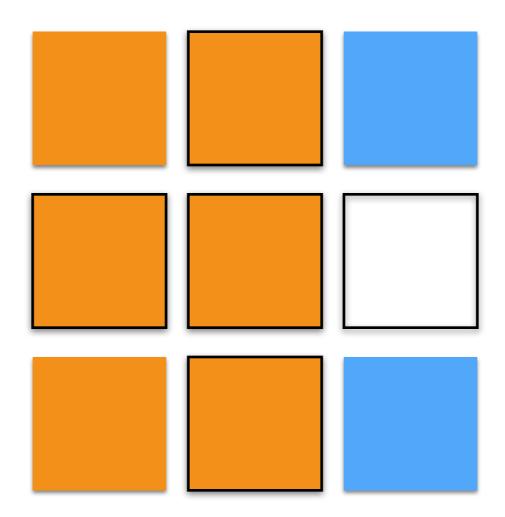
- When we upsample we need to invent the pixel in between the original ones...
- Basic solution:
  - For each missing pixel:
    - find the closest (norm 1, 2, whatever) "real" pixel with intensity/color Cn
    - Set the intensity/color of the missing pixel equals to Cn

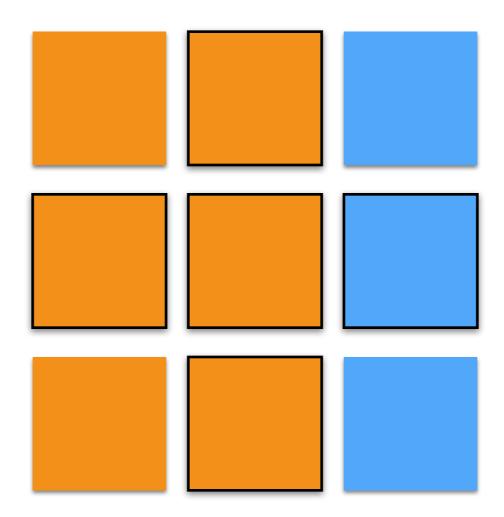




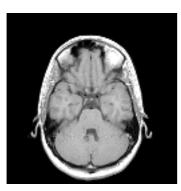




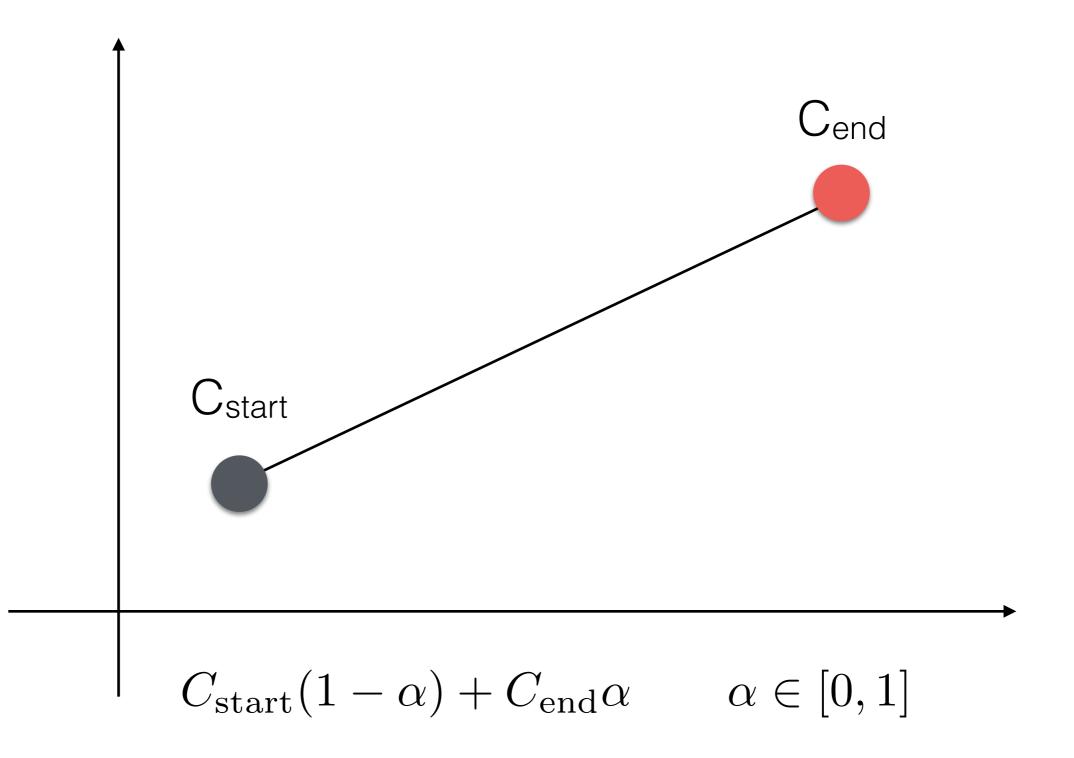




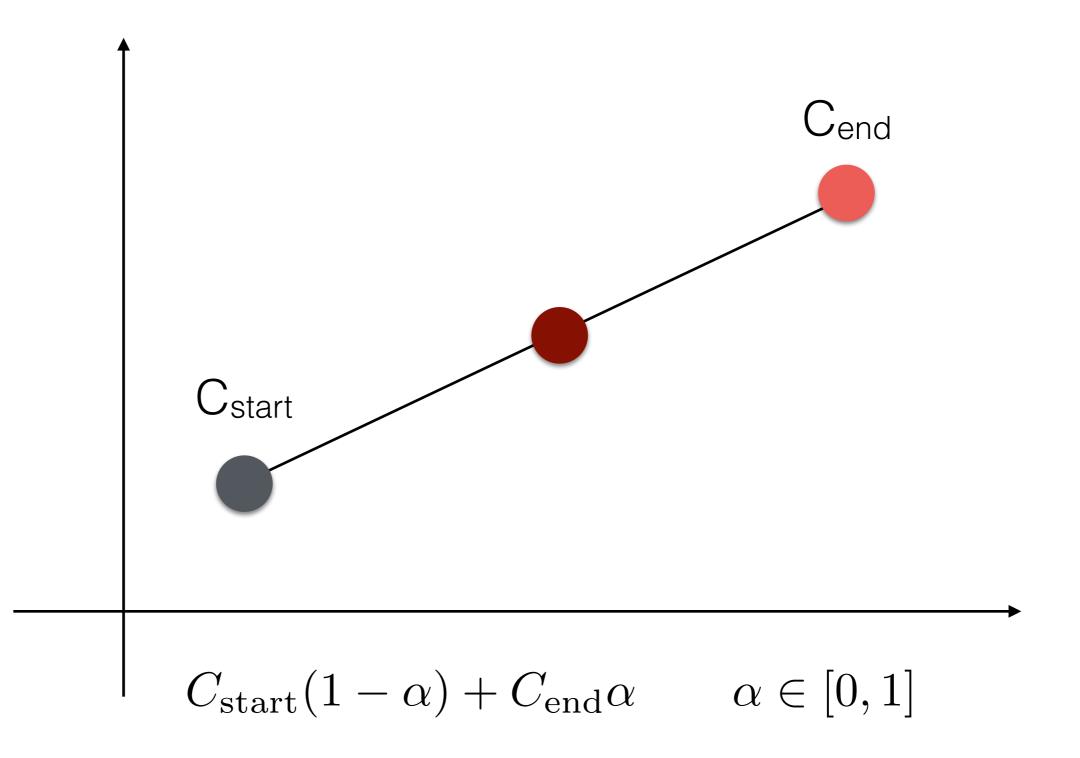




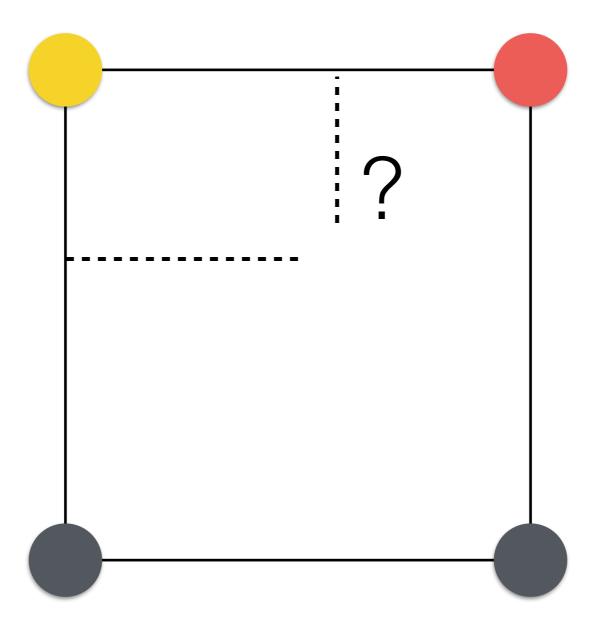
#### Upsampling 1D: Linear Interpolation

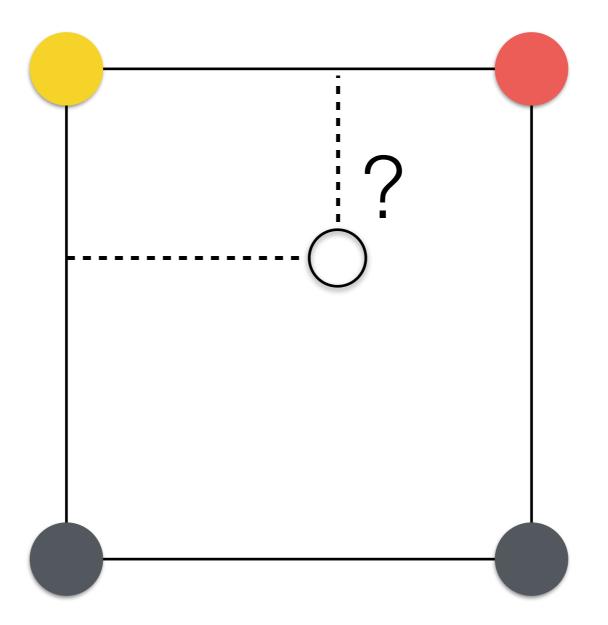


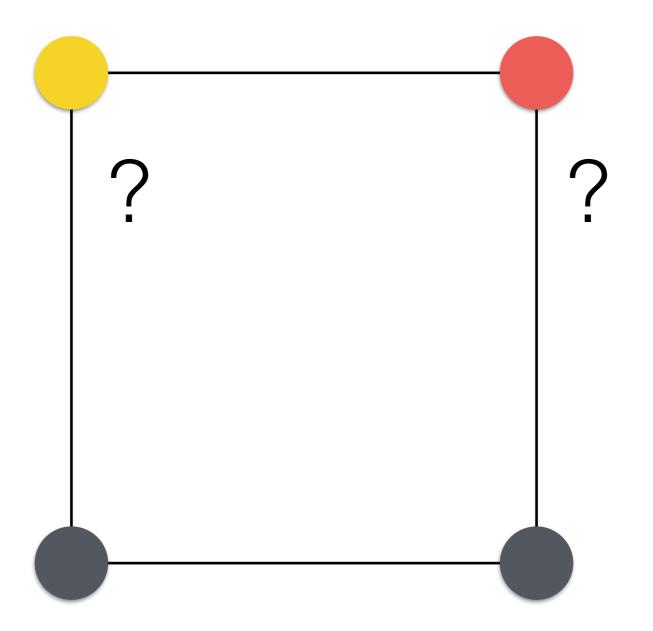
#### Upsampling 1D: Linear Interpolation

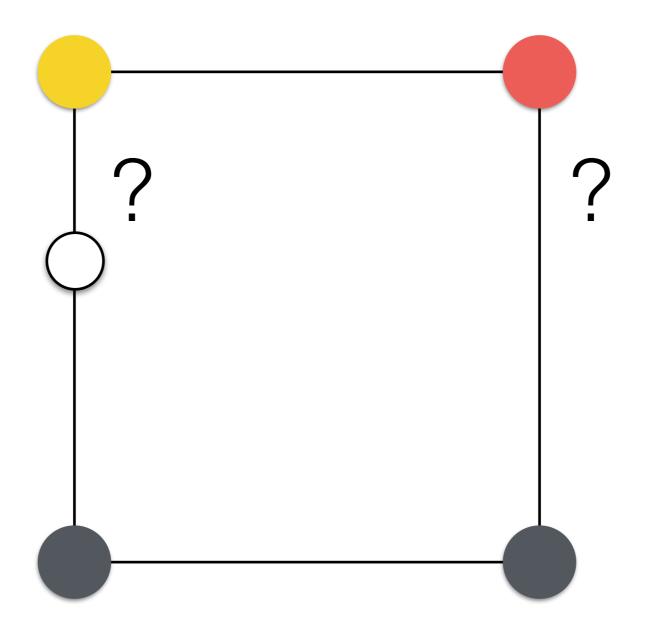


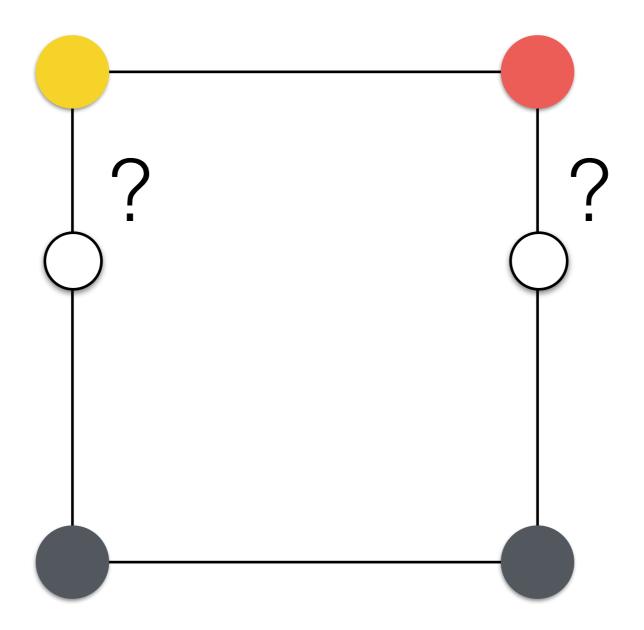
This becomes a bi-linear interpolation in 2D!

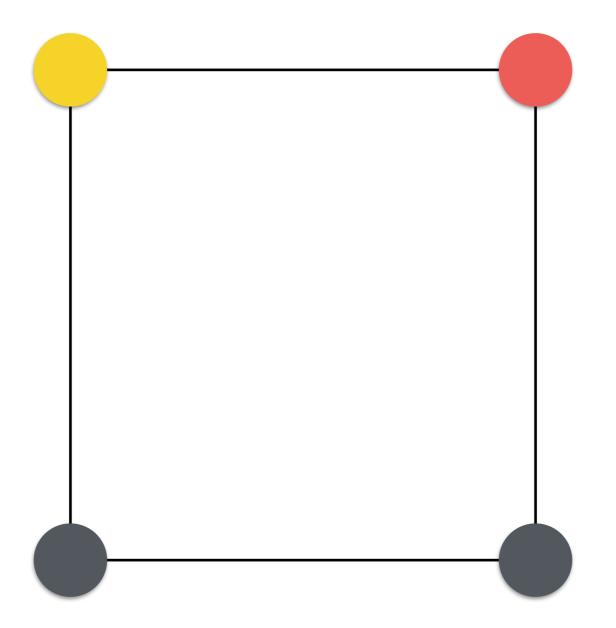


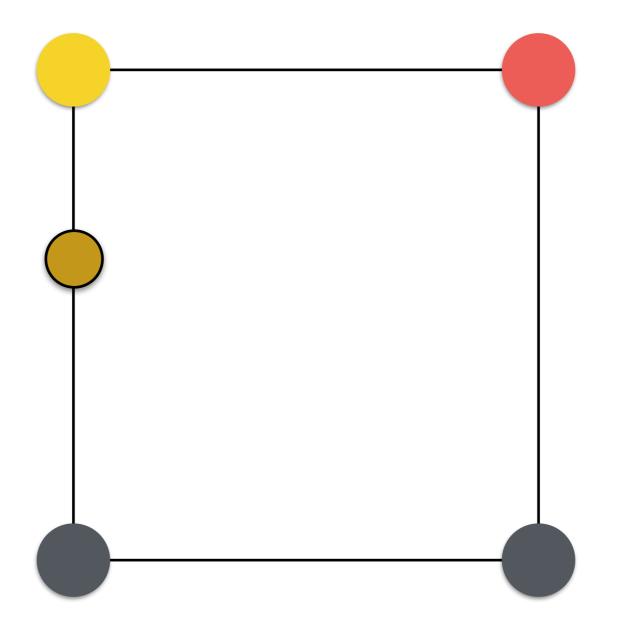


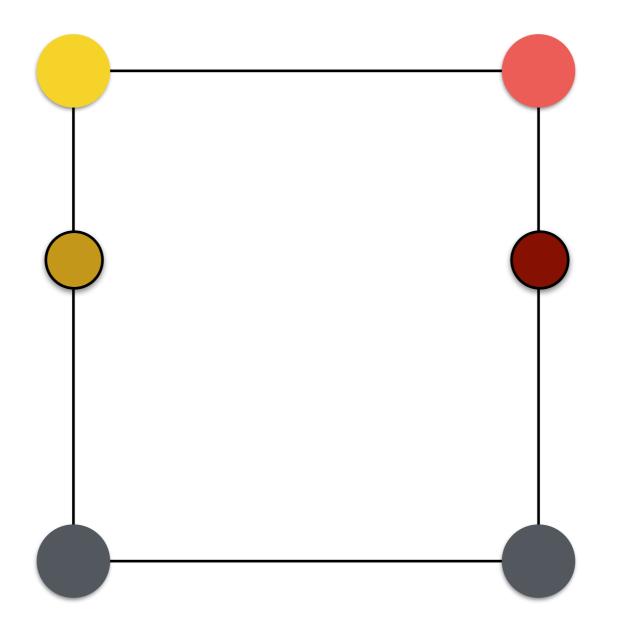


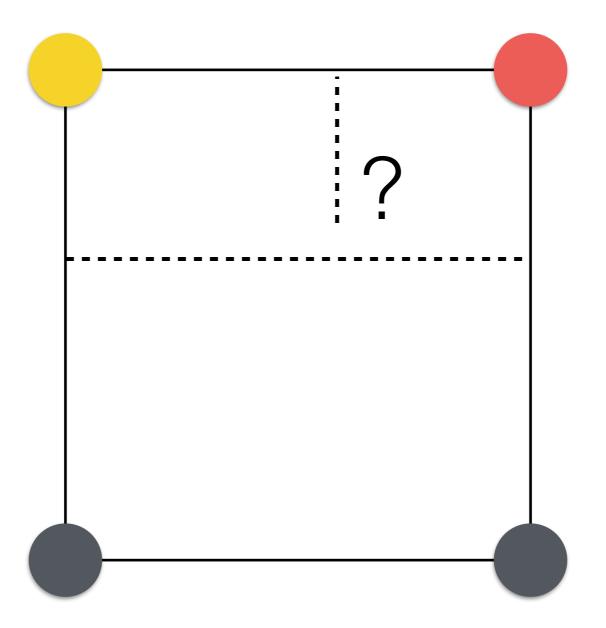


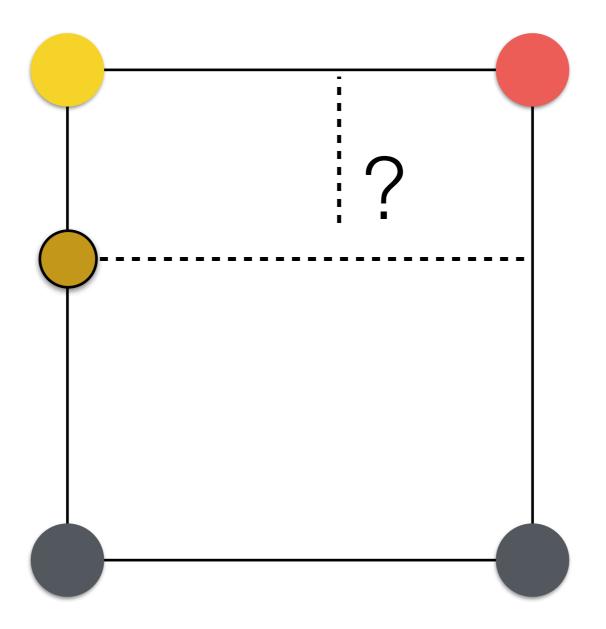


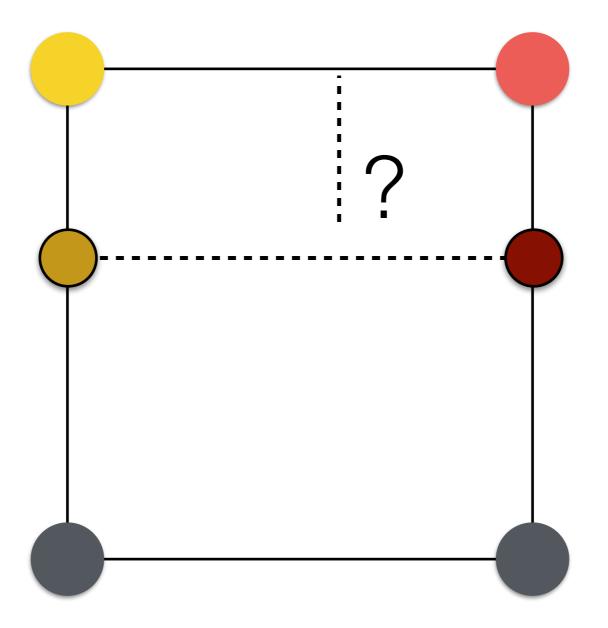


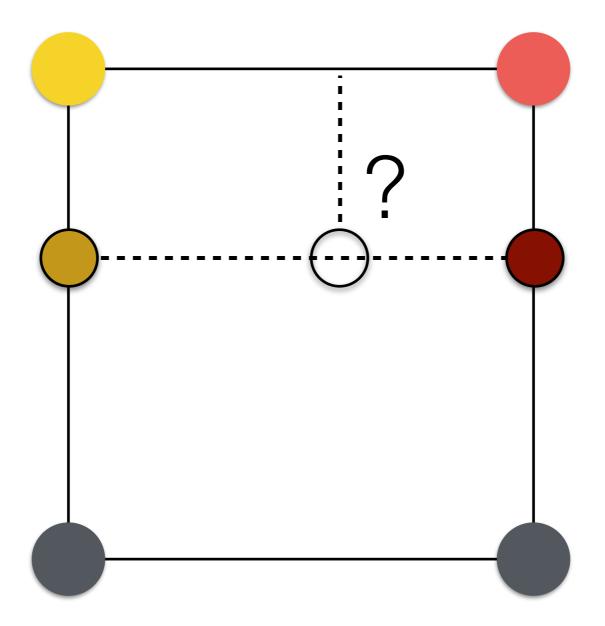


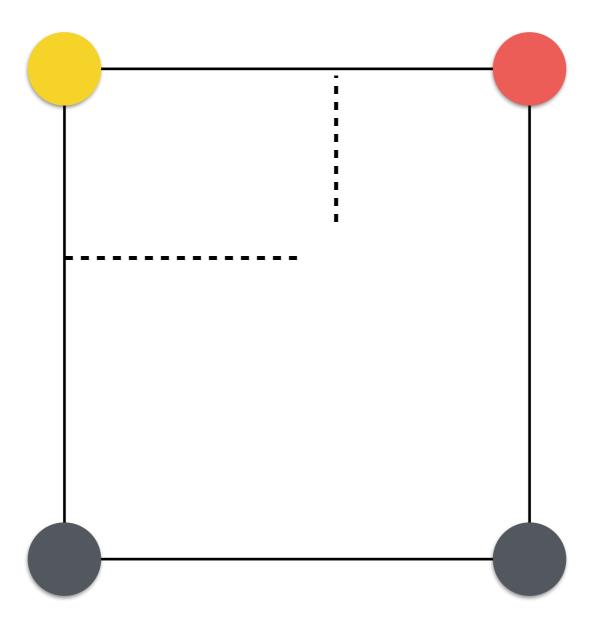


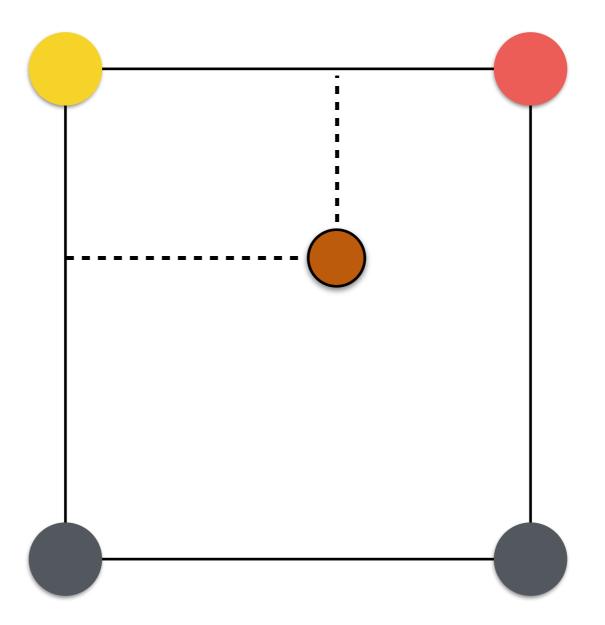




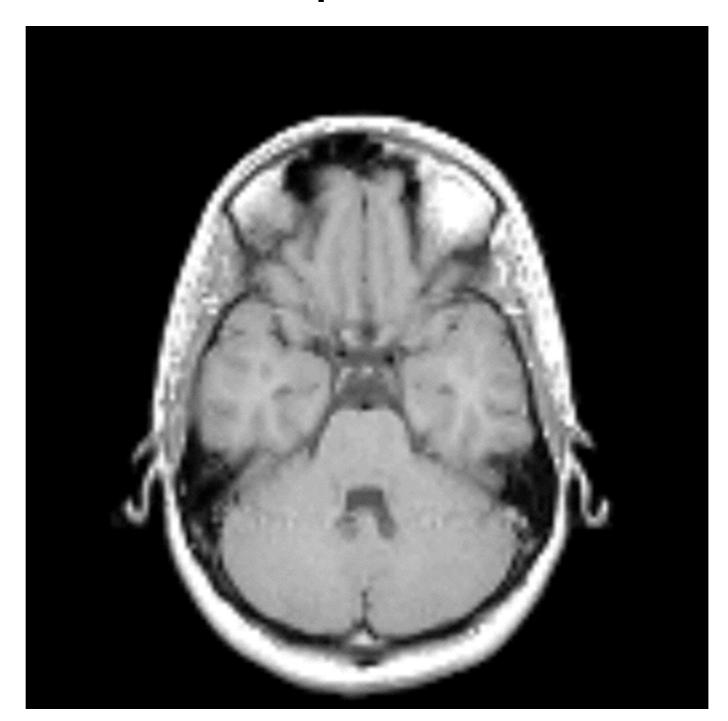








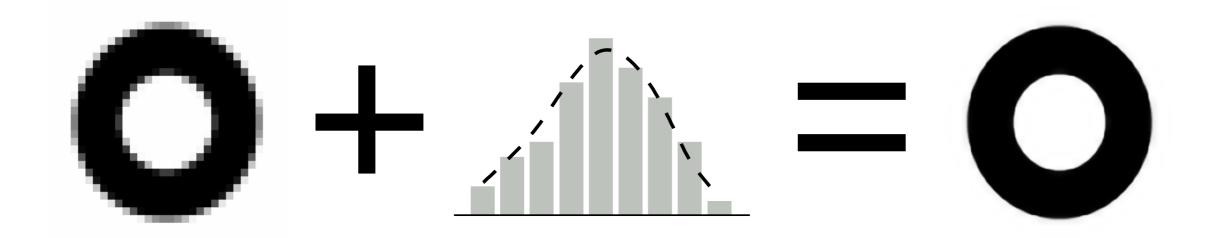
#### Bilinear Upsampling Example





### Can we do it better?

### Upsampling



#### that's all folks!